7B.1 A 3-D VARIATIONAL METHOD FOR SINGLE-DOPPLER VELOCITY RETRIEVAL APPLIED TO A SUPERCELL STORM CASE

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1. INTRODUCTION

The US WSR-88D radar network covers most of the United States, but provides only single-Doppler coverage for most areas. To use the data in numerical models and for diagnostic studies, techniques for single-Doppler velocity retrievals (SDVR) and data assimilation have been developed in recent years (Sun et al. 1991; Qiu and Xu 1992; Shapiro et al. 1995; Laroche et al. 1994; Zhang et al. 1996, Gao et al 2001). These techniques attempt to retrieve all three components of wind given observations of radial velocity and reflectivity.

Gao et al (1999a) recently developed a new 3-D variational single-Doppler velocity retrieval scheme that uses reflectivity conservation (with source terms) and anelastic mass continuity equations as weak constraints. Compared to the so-called simple-adjoint methods (e.g., Qiu and Xu 1992; Gao et al 2001) and those based on full model adjoint, this scheme avoids the time integration of the reflectivity conservation equation and its adjoint therefore has significant computational advantages. The method was tested with a simulated data set of a deep-convection storm in Gao et al (1999a) where the full three-dimensional wind field was retrieved in a dynamically consistent manner. In this paper, that method is improved and tested using data from the May 11, 1981 Oklahoma supercell storm for which dual Doppler observations are available.

2. METHODOLOGY

Our variational analysis procedure attempts to minimize a cost function J defined as the sum of squared errors due to the misfit between observations and analyses subject to certain constraints. Each constraint is weighted by a factor that accounts for its presumed accuracy. The variational method makes use of the derivative of J with respect to the analysis (control) variables and J must therefore be differentiable. In our single-Doppler radar analysis, we define the cost function as follows:

$$J = J_E + J_O + J_B + J_D + J_S,$$

where, the first term,

$$J_{E_r} = \frac{1}{2}\sum_{ijkn} W_E (E)^2,$$

measures the extent to which the reflectivity conservation equation,

$$E \equiv \frac{\partial \eta}{\partial t} + u_m \frac{\partial \eta}{\partial x} + v_m \frac{\partial \eta}{\partial y} + w_m \frac{\partial \eta}{\partial z} - k_h \nabla_H^2 \eta - k_v \nabla_Z^2 \eta + F_m ,$$

is satisfied. Here u_m, v_m, and w_m are temporal mean (over the retrieval period) velocities to be retrieved. The coefficients of eddy viscosity, k_h and k_v, are assumed to be unknown constants and will be retrieved. F_m is a time-mean source that is also to be retrieved and it includes sources and sinks of hydrometeors in association with microphysical processes and the effect of terminal velocity. In order to correctly use the reflectivity equation, a least square method is used to interpolate the reflectivity from observation points (in radar coordinate) to the analysis (Cartesian coordinate) grid.

The second term, J_O is the difference between the analyzed radial velocity V_r and the observed radial velocity V_{rob}^n:

$$J_O = \frac{1}{2}\sum_n (V_r - V_{rob}^n)^2,$$

where n is the index of observations. $V_r = CD(u_m, v_m, w_m)$ where D is a linear interpolation operator that maps u_m, v_m, w_m from the grid to observation points and C the projection operator that projects the winds u_m, v_m, w_m to the radial direction. By doing so, all observed velocities, including their orientation, are used without any directional bias. In another word, an interpolation step that may produce averaged vectors that point in inaccurate directions is avoided.

The other terms in the cost function are defined as follows:

$$J_B = \frac{1}{2}\left[\sum_{ijk} W_{ub}(u_m - u_b)^2 + \sum_{ijk} W_{vb}(v_m - v_b)^2 + \sum_{ijk} W_{wb}(w_m - w_b)^2\right],$$

$$J_D = \frac{1}{2}\sum_{ijk} W_D D^2,$$

$$J_S = \frac{1}{2}\left[\sum_{ijk} W_{us}(\nabla^2 u_m)^2 + \sum_{ijk} W_{vs}(\nabla^2 v_m)^2 + \sum_{ijk} W_{ws}(\nabla^2 w_m)^2\right].$$

Here, J_B measures how close the variational analysis is to the background field, and J_D imposes a weak anelastic

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mass-continuity constraint on the analyzed wind field, where

$$D \equiv \partial \bar{\rho} u_m / \partial x + \partial \bar{\rho} v_m / \partial y + \partial \bar{\rho} w_m / \partial z \ ,$$

and $\bar{\rho}(z)$ is the mean air density in the horizontal level. The last term in the cost function, J_S, represents a smoothness constraint.

To solve the above variational problem, we derive the gradient of the cost function with respect to the control variables u_m, v_m, w_m, k_h, k_v, and F_m, and use the Liu and Nocedal (1989) limited memory, quasi-Newton conjugate gradient method to perform the minimization.

3. TEST WITH SIMULATED DATA

1) Experimental design

To evaluate the performance of our variational analysis method, we apply our method to the 17 May 1981 Arcadia, OK supercell storm. Twelve coordinated dual-Doppler scans were obtained from the Norman and Cimarron, OK S-band Doppler radars over a one hour period spanning the pre-tornadic phase of the storm. Using the variational dual-Doppler analysis technique developed by authors (Gao et al. 1999b), we performed a detailed dual-Doppler analysis of this storm. The analysis grid comprises 83x83x37 grid points and the grid interval is 1 km in the horizontal and 0.5 km in the vertical. This dual-Doppler analysis will be used to verify the single-Doppler retrieval results.

Figure 1 shows horizontal and vertical cross-sections of wind vectors, vertical velocity (vertical section is plotted through line A-B in Fig.1a), and reflectivity for 1641 CST on 17 May. A strong rotating updraft and associated low-level downdraft are evident near the center of the domain. A cold outflow is seen to originate from rear flank downdrafts that exhibit two maximum centers flanking the occlusion point of the gust fronts. Ahead of this outflow is the rear flank gust front which is associated with surface convergence and vertical velocity maximum. The reflectivity field shows a hook-echo pattern that is consistent with the retrieved flow. Such a flow structure is typical of a tornadic supercell storm with strong low-level rotation (Lemon and Doswell 1979).

2) Results

For the single-Doppler velocity retrieval, the analysis domain is the same as dual-Doppler analysis. The parameter settings used are $W_r = 1$, W_E =1x10^2, W_{ub}=W_{vb}, =1.0x10^{-2}, $W_{wb} = 0.$, W_D=$1/(0.5\times10^{-3})^2$, and W_{us} = W_{vs} = W_{ws} = 0.1x10^{-2}, the background field is defined from a nearby sounding from Tuttle, Oklahoma. These values are chosen so that all cost function terms have the same order of magnitude after being multiplied by the coefficients. An initial guess of zero is used in this experiment, and the minimization is stopped after 350 iterations. This large number of iterations is generally needed because of the first guess is zero for all control variables (In operational 3DVAR, the previous forecast is used as the first guess, 50 iterations are usually enough to obtain converged solutions).

Fig. 2 shows the fields obtained from the single Doppler velocity retrieval (see caption for more details). All significant features in the horizontal winds, i.e., the

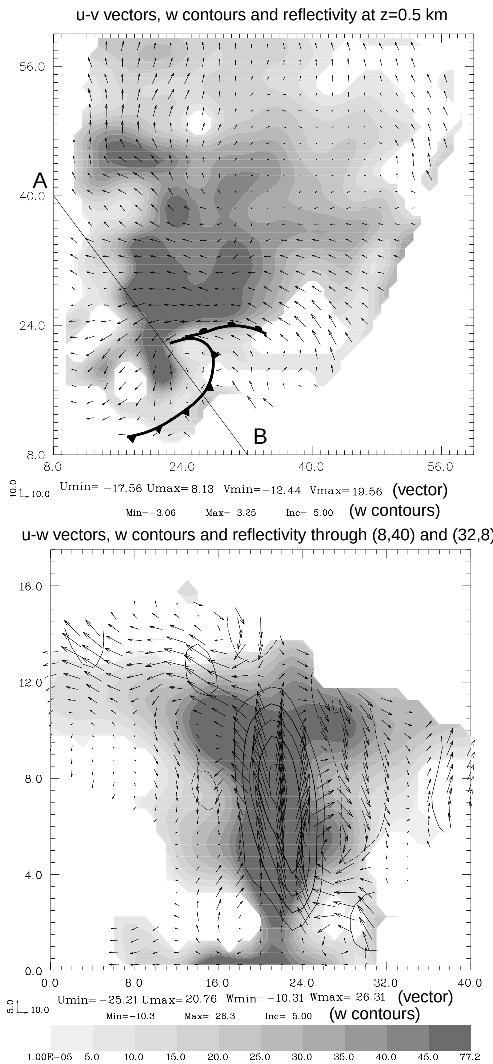

Figure 1. Wind vectors, vertical velocity (contours) retrieved using the variational dual-Doppler analysis method for Arcadia, OK 17 May 1981 tornadic storm. a) Horizontal cross-section at z = 0.5 km. b) Vertical cross-section through line A-B in panel a).

curvature around the rotating updraft and the convergence of wind fields, are well recovered. The main updraft is seen to originate ahead of the low-level gust front and in general matches the areas of maximum reflectivity, but the retrieved maximum updraft is only about 12.83 m/s (Fig 2b), much lower than the dual-Doppler analysis which is about 26.31 m/s (Fig 1b). The main downdraft is located below the updraft core and is collocated with a region of high reflectivity behind the gust front. These features suggest that both the horizontal and vertical flows are kinematically consistent. They does agree very well with the dual-Doppler analysis given in Fig. 1. Using a nearby sounding as a background, it can be seen that there is a smooth transition between the area where the data is provided by the radar and the area where it is only a background sounding is available.

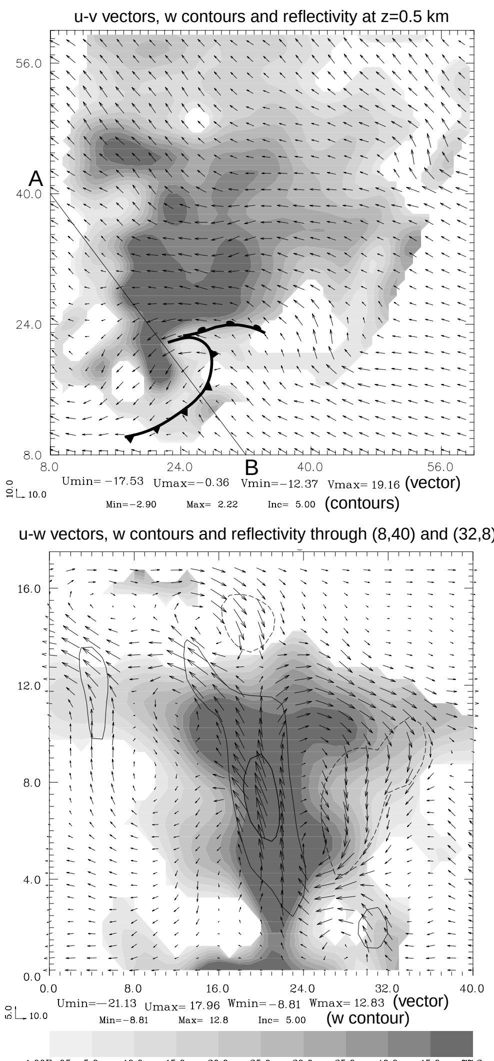

Fig 2. Wind vectors, vertical velocity (contours) retrieved using the variational dual-Doppler analysis method for Arcadia, OK 17 May 1981 tornadic storm. a) Horizontal cross-section at z = 0.5 km. b) Vertical cross-section through line A-B in panel a).

4. SUMMARY

In this paper, we tested a new variational analysis scheme that is capable of obtaining three-dimensional winds from single Doppler observations of convective storms. The method incorporates radar data, background fields, smoothness, mass continuity constraints and the residual of reflectivity conservation in a single cost function. By minimizing this cost function, an analysis with the desired fit to these constraints is obtained in a single procedure. The detailed structure of the storm is well retrieved in comparison with dual-Doppler analysis. Unlike most kinematic methods of retrieval, our method is capable of adequately dealing with data voids. When an analysis background is available, which can be from a proximity sounding, an analysis from conventional data, or a numerical model, the method naturally blends the Doppler radar observations with the background. Smooth transition is obtained between data-rich and data-void areas. These features are considered important for the analysis to be usable for initializing storm-scale numerical models as well as for diagnostic studies of storm structures.

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