

Meteorology 5344, Fall 2003
Computational Fluid Dynamics
Hour Exam #1
September 29, 2003

1. (30%) Issues on supercomputing.

a) Give and briefly discuss the main reasons for the supercomputing industry to move during past decade from conventional shared-memory parallel architectures to distributed-memory parallel systems.

- Distributed-memory systems are cheaper to build and often use off-shelf processors and components
- Distributed-memory systems are more scalable than shared-memory systems
- Of course, multi-processor parallel processing is essentially to solve large problems.
- Standardized tools and libraries (e.g., MPI) has matured and become commonly available
- A large number of codes and algorithms have been developed in the past decade that can run on distributed memory architectures.

b) For a program that contains a fraction s of non-parallelizable (serial) code, what is the maximum possible speedup when run on p number of processors? Comment on your result.

Write down or derive the Amhdal's Law. The speedup is optimal and assumes no overhead. The overall speedup is seriously limited by the fraction of non-parallel code, especially when p is large. E.g., for $p=2$, and $p=1000$,

2. (30%) Given the following three PDE's,

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0 \quad (1)$$

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} = 0 \quad (2)$$

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0 \quad (3)$$

a) Classify these equations in terms of the canonical types;

Test $B^2 - 4AC$ and determine the types.

b) Why is the number of characteristics equations important in the classification of PDEs?

The number of real characteristics that exist, if any, tells us if the equation support propagation mode(s), therefore the fundamental properties of the equation.

c) Discuss their associated domain of dependence with the help of schematics.

See notes.

3. (40%) For first-order spatial derivative $\frac{\partial u}{\partial x}$,

a) Construct a consistent finite difference approximation by using three equally spaced grid points, one at the current location, and two on the left (that's right, both on the left side).

$$\text{Let } \frac{\partial u}{\partial x} = au_{i-1} + bu_i + cu_{i+1} + O(\Delta x)^m$$

Using our Taylor series for u_{i-1} , u_i and u_{i-2} , we can write

$$\begin{aligned} au_{i-2} + bu_{i-1} + cu_i &= \\ a(u_i - 2\Delta x \frac{\partial u}{\partial x} + \frac{(2\Delta x)^2}{2!} \frac{\partial^2 u}{\partial x^2} - \frac{(2\Delta x)^3}{3!} \frac{\partial^3 u}{\partial x^3} + \dots) &+ \\ + b(u_i - \Delta x \frac{\partial u}{\partial x} + \frac{\partial^2 u}{2! \partial x^2} - \frac{(\Delta x)^3}{3!} \frac{\partial^3 u}{\partial x^3} + \dots) &+ \\ + cu_i &= \\ = (a+b+c)u_i - (2a+b)\Delta x \frac{\partial u}{\partial x} + \frac{4a+b}{2} (\Delta x)^2 \frac{\partial^2 u}{\partial x^2} - \frac{(8a+b)(\Delta x)^3}{3!} \frac{\partial^3 u}{\partial x^3} + \dots \end{aligned}$$

Setting

$$a+b+c = 0$$

$$-(2a+b)\Delta x = 1$$

$$\frac{4a+b}{2} (\Delta x)^2 = 0$$

Solve for a, b and c, we obtain

$$a = \frac{1}{2\Delta x}, b = -\frac{4}{2\Delta x}, c = \frac{3}{2\Delta x}, \text{ therefore}$$

$$\frac{\partial u}{\partial x} = \frac{u_{i-2} - 4u_{i-1} + 3u_i}{2\Delta x} + \frac{(\Delta x)^2}{3} \frac{\partial^3 u}{\partial x^3} + \dots$$

b) Show that the scheme is consistent and determine the order of accuracy of the scheme.

And the scheme is second-order accurate.

The scheme is consistent because the truncation error goes to zero when Δx goes to zero.

c) If you are to use this spatial difference scheme to solve simple advection equation

$$\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0, \text{ in what situation this scheme is not suitable? When it is not, is there any}$$

alternative to the difference formulation that would work?

When $c > 0$, the scheme is upstream biased, which is physically reasonable because the signals described by the equation propagates from left to right, and the numerical domain of dependence can cover the true domain of dependence. When $c < 0$, the scheme becomes downstream biased, which is physically unreasonable and the scheme will be unstable. In this situation, one can use 2 points to the right instead of those on the left.

Let $\frac{\partial u}{\partial x} = au_{i-1} + bu_i + cu_{i+1} + O(\Delta x)^m$

Using our Taylor series for u_{i-1} , u_i and u_{i+1} , we can write

$$\begin{aligned} au_{i-2} + bu_{i-1} + cu_i &= \\ & a(u_i - 2\Delta x \frac{\partial u}{\partial x} + \frac{(2\Delta x)^2}{2!} \frac{\partial^2 u}{\partial x^2} - \frac{(2\Delta x)^3}{3!} \frac{\partial^3 u}{\partial x^3} + \dots) \\ & + b(u_i - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2!} \frac{\partial^2 u}{\partial x^2} - \frac{(\Delta x)^3}{3!} \frac{\partial^3 u}{\partial x^3} + \dots) \\ & + cu_i \\ & = (a+b+c)u_i - (2a+b)\Delta x \frac{\partial u}{\partial x} + \frac{4a+b}{2}(\Delta x)^2 \frac{\partial^2 u}{\partial x^2} - \frac{(8a+b)(\Delta x)^3}{3!} \frac{\partial^3 u}{\partial x^3} + \dots \end{aligned}$$

Setting

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b)