Meteorology 5344, Fall 2003
Computational Fluid Dynamics
Hour Exam #2
November 7, 2003

Make sure you read and answer all questions! Many questions require only brief answers.

1. (15%) Discuss one advantage and one disadvantage of implicit schemes for solving hyperbolic equations.

2. (15%) Define nonlinear instability. Give two commonly used methods for controlling nonlinear instability and state the rational behind them.

3. (70% total, 10% for each sub question) For a 1-D advection equation $\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$ with constant advection speed *c*,

a) Using a suitable 1st-order finite difference scheme to discretize the equation in both space and time, write out the finite difference equation (FDE). Make sure you consider both $c > 0$ and $c < 0$ cases.

b) Sketch out the numerical domain of dependence of the FDE, again for both $c > 0$ and $c < 0$ cases.

c) Based on the domain of dependence concept, what is the necessary condition of stability for such an advection equation? What does that tell you about the time step size Δt that you can use for your scheme? For this question and later ones, you need to consider only $c > 0$ case.

d) Using von Neumann method, determine the amplification factor λ. Show that the von Neumann method gives you the same stability condition as the domain-of-dependence concept.

e) What is the amplitude change per time step for the shortest wave that can be represented on your grid? Considered the cases of Δt = maximum allowable Δt, and Δt = half of the maximum allowable Δt. What do the results tell you about the amplitude error of your scheme.

f) If you derive a modified equation for your FDE equation (you are not required to derive it), what do you expect of the form of the leading/dominant term in the truncation error and why?

g) If you have a 2-D advection equation $\frac{\partial u}{\partial t} + c_x\frac{\partial u}{\partial x} + c_y\frac{\partial u}{\partial x} = 0$ instead, write out the finite difference equation as a direct extension of your 1-D FDE.

You may find the following formula useful.

$u_j^n = U_k e^{i(kj\Delta x - \omega n\Delta t)} = U_k \lambda^n e^{i(kj\Delta x)}$, $e^{iz} = \cos(z) + i\sin(z)$,

$1-\cos(x) = 2\sin^2(x/2)$, $1+\cos(x) = 2\cos^2(x/2)$, $\sin(2x) = 2\sin(x)\cos(x)$.