Meteorology 5344, Fall 2003
Computational Fluid Dynamics
Dr. Ming Xue

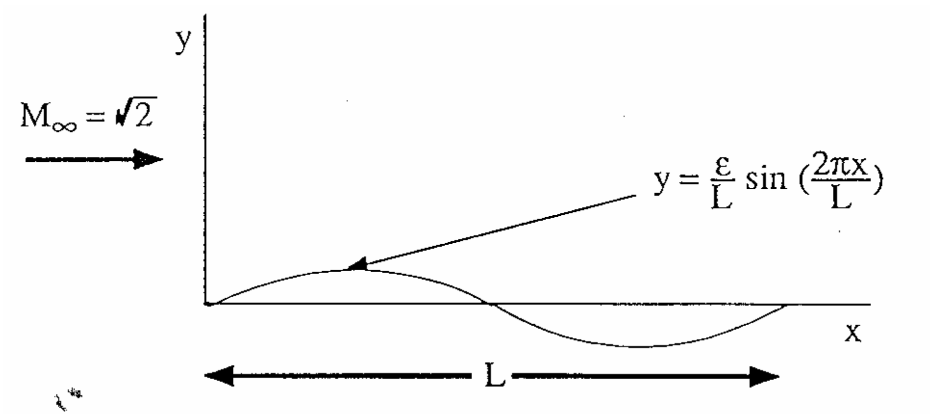
Computer Problem #2: PDE and Method of Characteristics

Distributed Monday September 15, 2003
Due Wednesday October 1, 2001

1. Classify the following system of equations using matrix method as well as the auxiliary equation method.

$$\frac{\partial u}{\partial t} + 8\frac{\partial v}{\partial x} = 0$$
$$\frac{\partial v}{\partial t} + 2\frac{\partial u}{\partial x} = 0$$

2. Consider the situation in which a uniform inviscid supersonic flow with free-stream Mach number $\sqrt{2}$ encounters a sine wave wrinkle in the floor of a wind tunnel, as shown in the sketch below:



The this type of steady flow is governed by the linear equations:

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0$$
$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

where u and v are <u>perturbation</u> velocity components in the x- and y-directions, respectively. Let us assume that, prior to encountering the sine-wave from the left, no perturbations are introduced into the flow so that u=0 and v=0 along x=0, y>0. Further, let the velocity normal to the free stream at the lower boundary be given by:

$$v(x, y=0) = \frac{2\pi\varepsilon}{L^2}\cos(\frac{2\pi x}{L}) \qquad 0 \le x \le L$$

(Note that the above condition may be applied at y = 0 because the perturbation velocities are assumed to be small.) Determine the solution for the perturbation velocities using the method of characteristics, assuming that ε = 1, L = 10Δx, and Δx = Δy = 1.0. Write a computer code to calculate the values of u and v at all grid points and plot the fields using contours.

Strategy: Derive the characteristic and compatibility equations and associated conservative quantities (called Riemann invariants), and then make a sketch of the net of characteristic curves. Knowing what is conserved along the characteristics, apply the boundary conditions to determine the constants of integration and then determine u and v at each point in the mesh, beginning at the left edge and proceeding in the x direction (be cautious at the point 0,0). Your computer code should solve for the entire net of points, and you should make a 2-D contour plot of the final solution. Pay attention to the coding style and code performance, and hand in your computer code.