Meteorology 5344, Fall 2003
Computational Fluid Dynamics
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Computer Problem #3 Accuracy of Finite Differences
Due Wednesday, October 15, 2001

1. Consider the analytic function $f(x) = sin(\pi x/2)$. The goal of this exercise is to compare the analytical values of df/dx at a point with those obtained using the following numerical approximations:

- Centered Difference: $\dfrac{f_{i+1} - f_{i-1}}{2\Delta x}$
- Forward Difference: $\dfrac{f_{i+1} - f_i}{\Delta x}$
- Backward difference: $\dfrac{f_i - f_{i-1}}{\Delta x}$
- 3-point Asymmetric: $\dfrac{-1.5 f_i + 2 f_{i+1} - 0.5 f_{i+2}}{\Delta x}$
- 4-point Symmetric $\dfrac{f_{i-2} - 8 f_{i-1} + 8 f_{i+1} - f_{i+2}}{12\Delta x}$

a) Evaluate df/dx at $x = 0.5$ using the above 5 formulae with a step size $\Delta x = 0.1$, and compute the error relative to the analytical solution, i.e.,

$$Error = \frac{\left(\dfrac{df}{dx}\right)_{discrete}}{\left(\dfrac{df}{dx}\right)_{analytical}} - 1.0 \, . \qquad (1)$$

Write a computer program for this.

b). For each of the above five schemes, derive an expression for the truncation error and evaluate analytically the leading term at $x = 0.5$ then calculate the relative error according to (1). Compare this number with the relative error found in part a) and present your results in tabular form. Discuss the results.

c) Repeat part a) using Δx = 0.5, 0.25, 0.125, 0.1, 0.05, 0.025, 0.0125 and compute the error for each Δx and plot the results on a graph using - log_{10}| Error | as the vertical axis and - $log_{10}(\Delta x)$ as the horizontal axis. The slope of this line is the convergence rate or actual order of accuracy of the scheme. How does this rate compare with that predicted by the leading term of the truncation error? (Note: Double precision may be required for the last two schemes if done on a workstation. The default precision of Cray J90 is 8 bytes, which is equivalent to double precision on most workstations).

2. Consider the function $f(x) = tanh[\ k\ (x\text{-}1)\ ]$ where x and k are positive numbers.

a) Plot this function for k = 1.0, 5.0 and 20.0 in a domain $x \in [0.0, 2.0]$ using Δx = 0.01. Note the structure of the function for different values of k.

b). Compute df/dx at x = 1.05 using all five schemes from problem 1 with Δx = 0.32, 0.16, 0.08, 0.04, 0.02, 0.01 for both k=5.0 and k=20.0. Compute the error of the approximation as in part c) of problem 1 and plot it on a graph using - $log_{10}|$ Error $|$ as the vertical axis and - $log_{10}(\ \Delta x\ )$ as the horizontal axis.

c) Discuss the nature of this error with changing Δx and k. What do the results suggest about the influence of a particular function on the truncation error? Do higher order schemes always give better results?