

Chapter 2. Mountain forced flows

2.1. Mountain waves

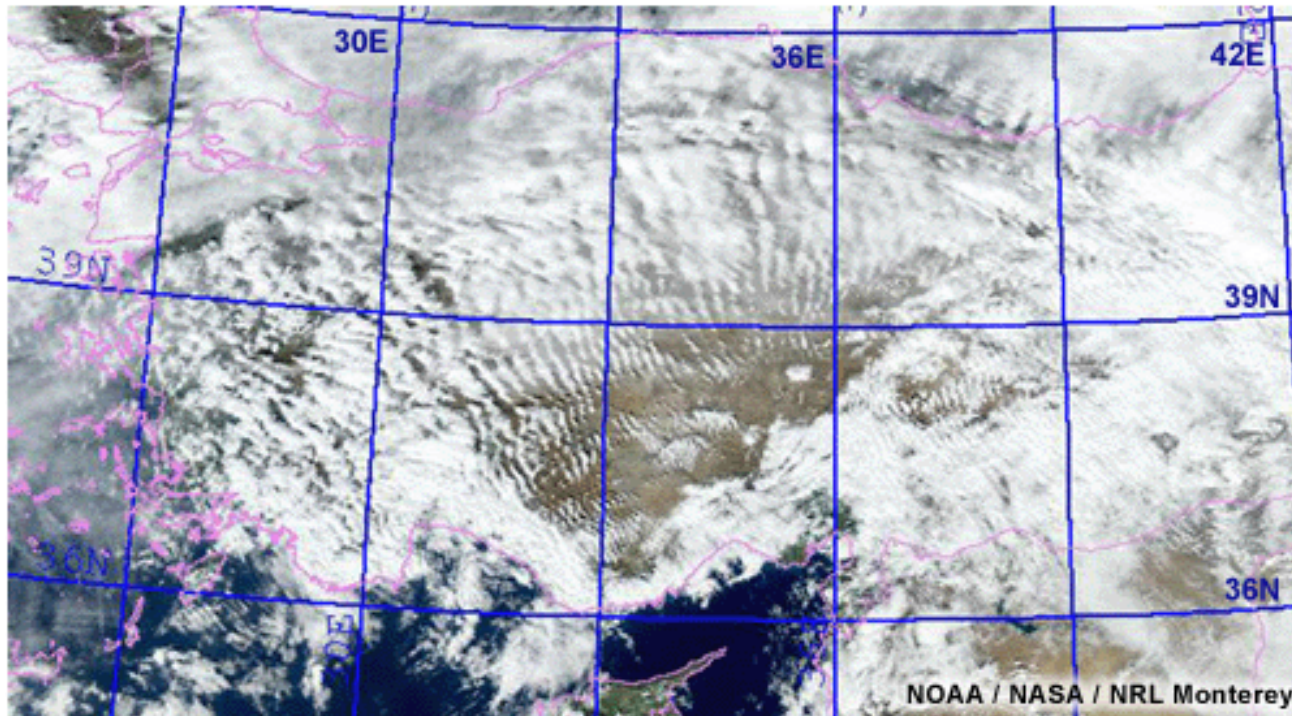
Well-known weather phenomena directly related to flow over orography include

- mountain waves
- lee waves and clouds
- rotors and rotor clouds
- severe downslope windstorms
- lee vortices
- lee cyclogenesis
- frontal distortion across mountains
- cold-air damming
- track deflection of midlatitude and tropical cyclones
- coastally trapped disturbances
- orographically induced rain and flash flooding
- orographically influenced storm tracks.

A majority of these phenomena are mesoscale and are induced by stably stratified flow over orography.

Mountain Waves

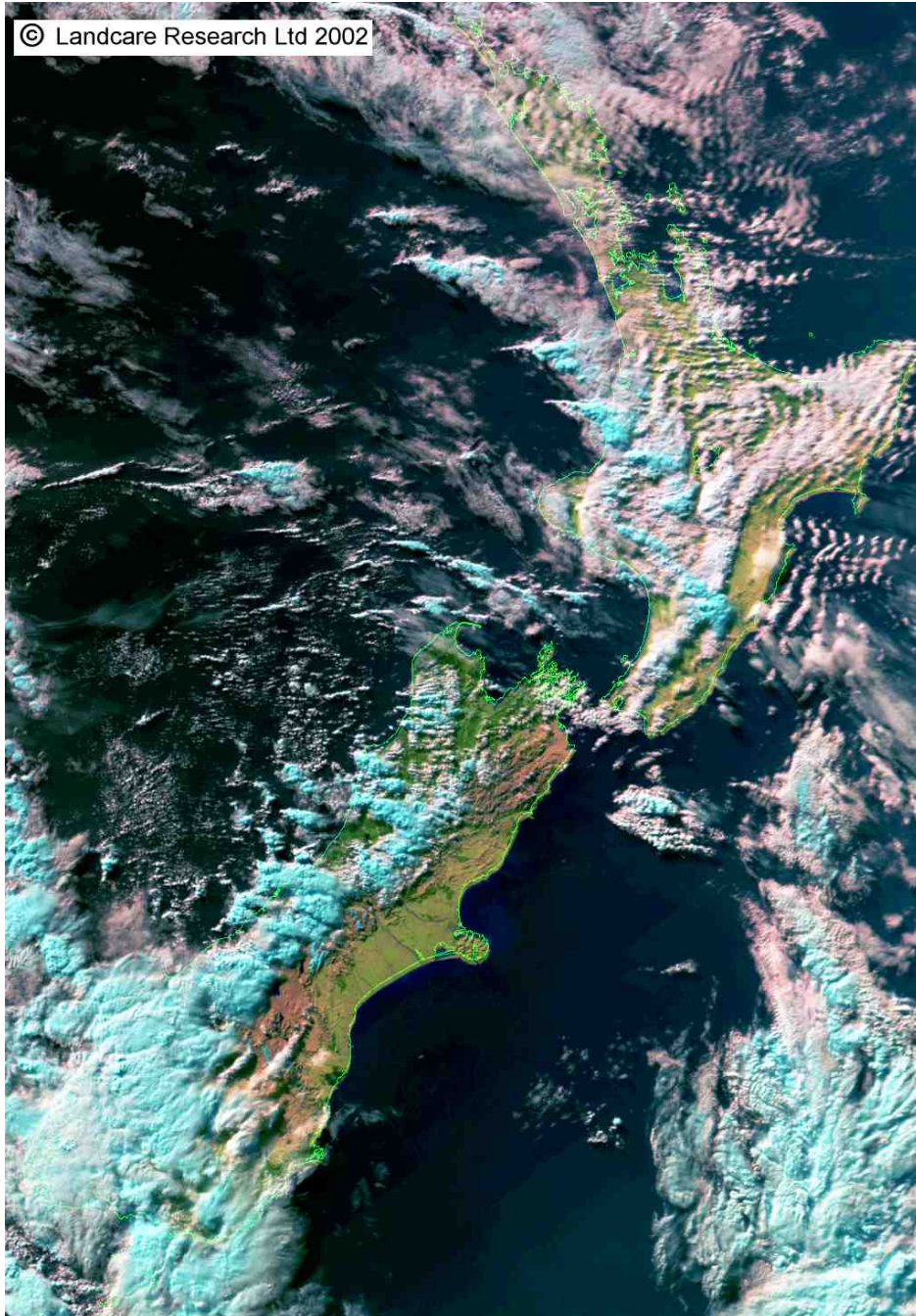
MODIS True Color Image 1050 UTC 30 Jan 2003

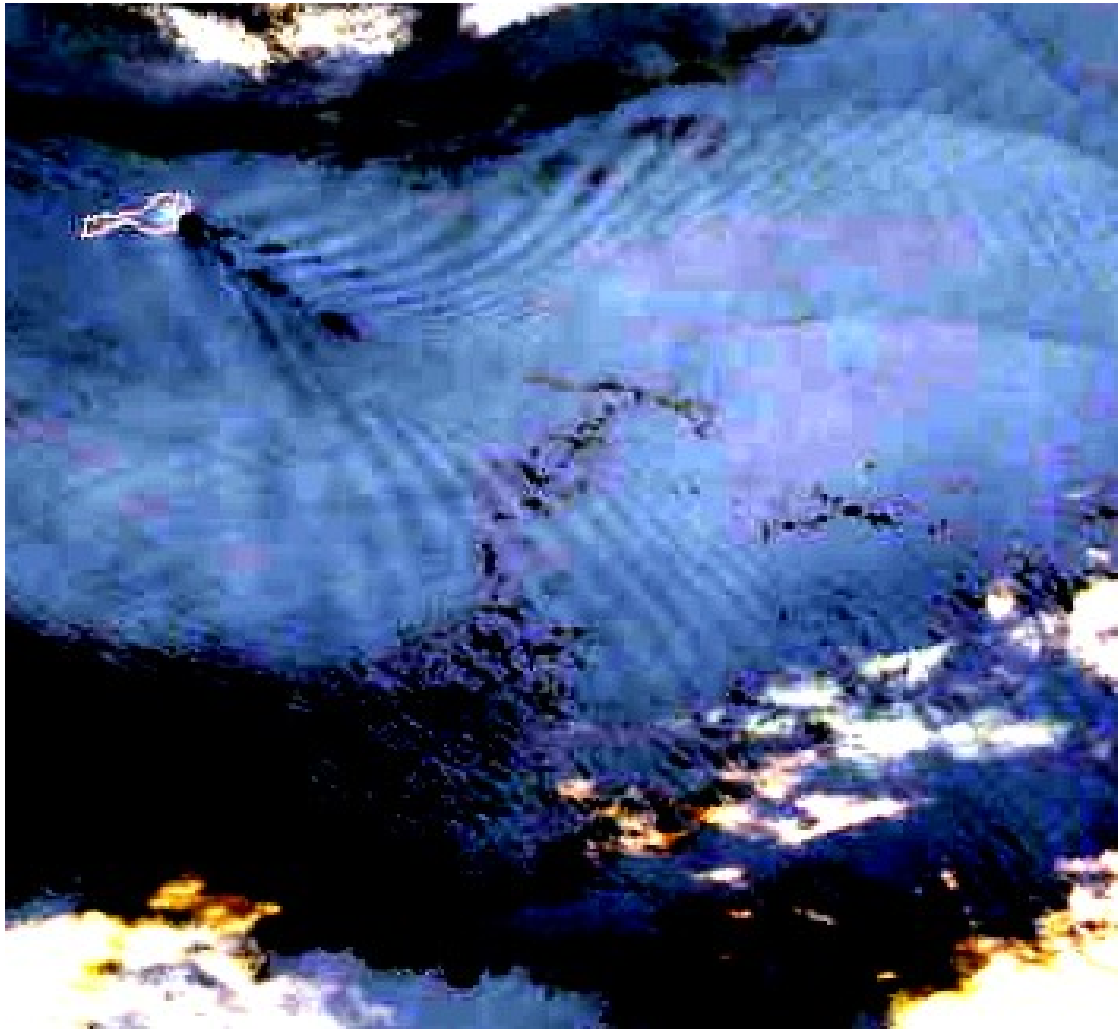


MODIS image of mountain wave clouds over Turkey.

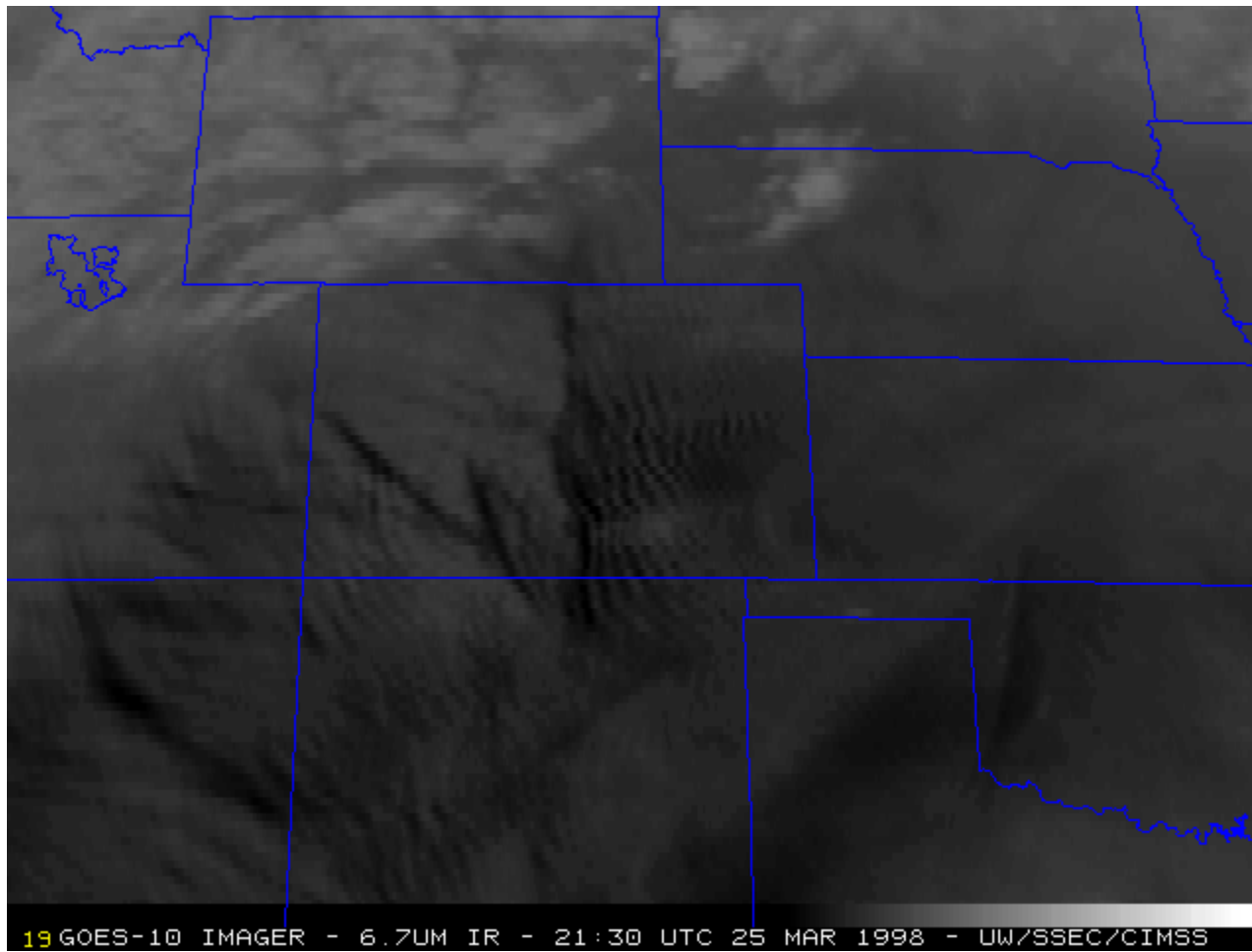
Mountain waves form above and downwind of topographic barriers when strong winds blow with a significant vector component perpendicular to the barrier in a stable environment.

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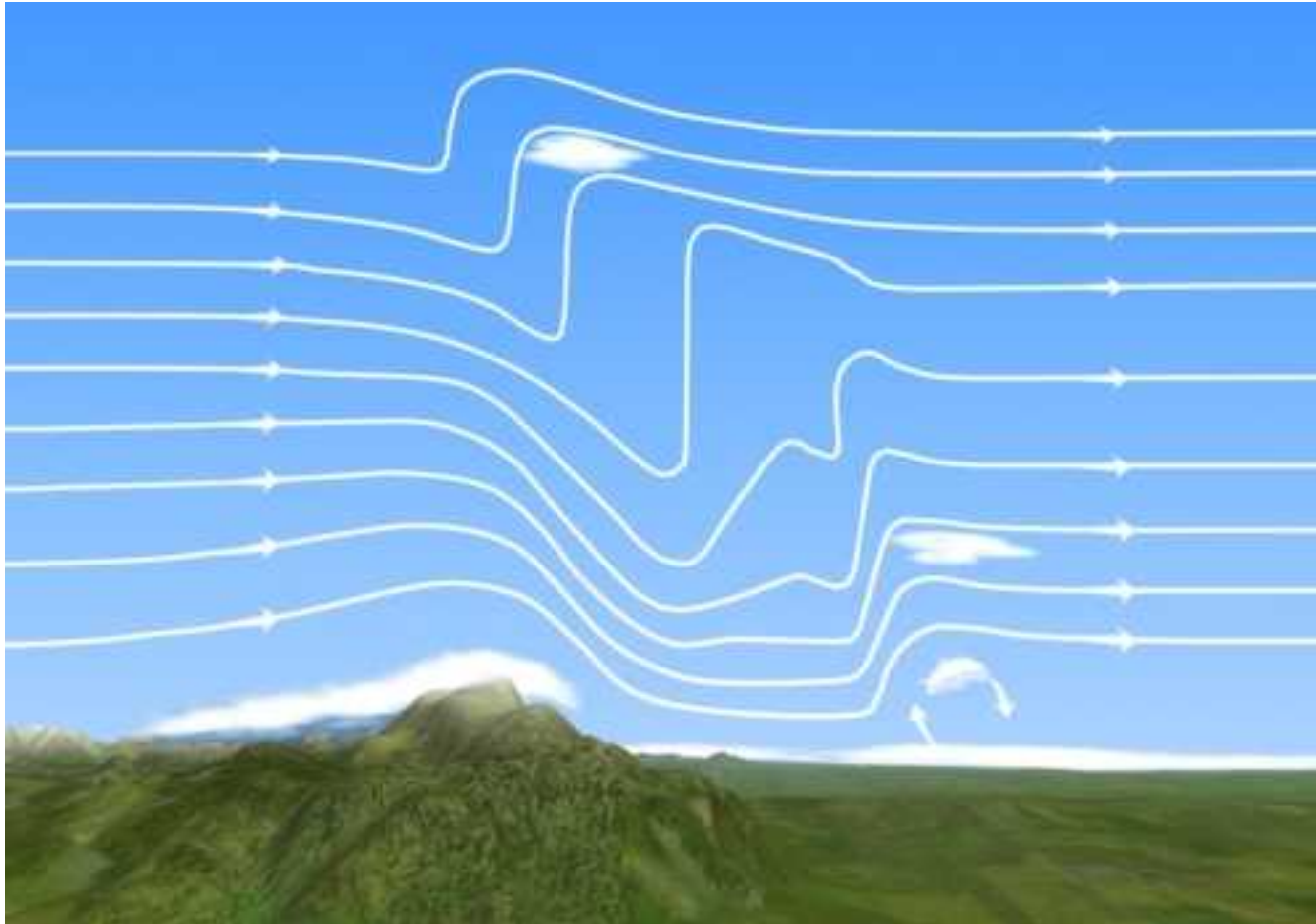




False color image of mountain wave cloud patterns emanating from Jan Mayen (a small Arctic island chain ~550 km north-east of Iceland) on 25th. January, 2000 at 1627Z. The orange-white regions are polar stratospheric cloud decks formed by cold temperatures on this day.

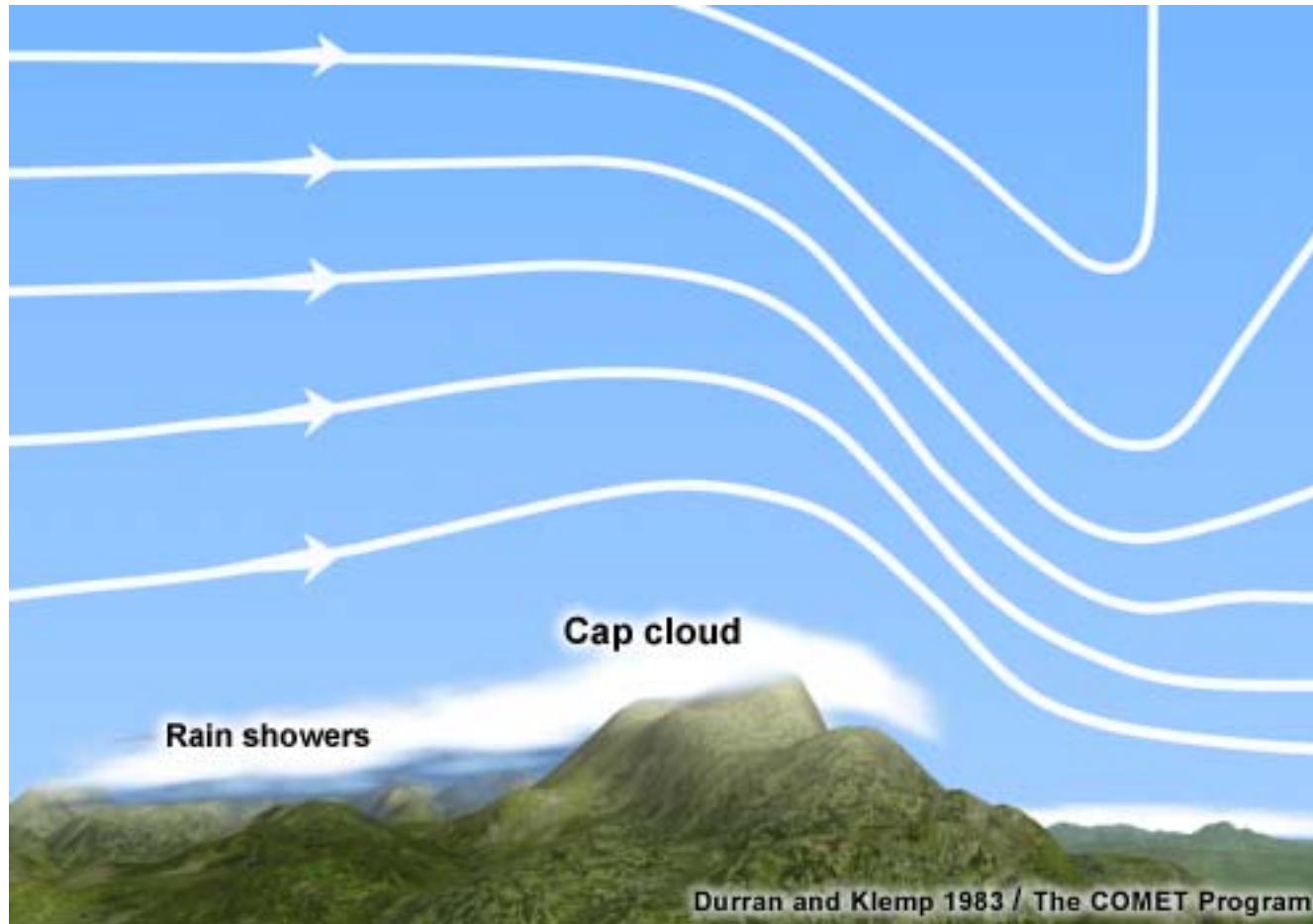


Mountain wave clouds forming east of the Rockies.



This figure shows the development of the typical features often associated with a mountain wave system. Notice the wind flow, with a strong component perpendicular to the primary ridge line. This is a typical condition for mountain wave development, as is a stable atmosphere. If air is being forced over the terrain, it will move downward along the lee slopes, then oscillate in a series of waves as it moves downstream. Sometimes these waves can propagate long distances in "lee wave trains."

Cap Clouds

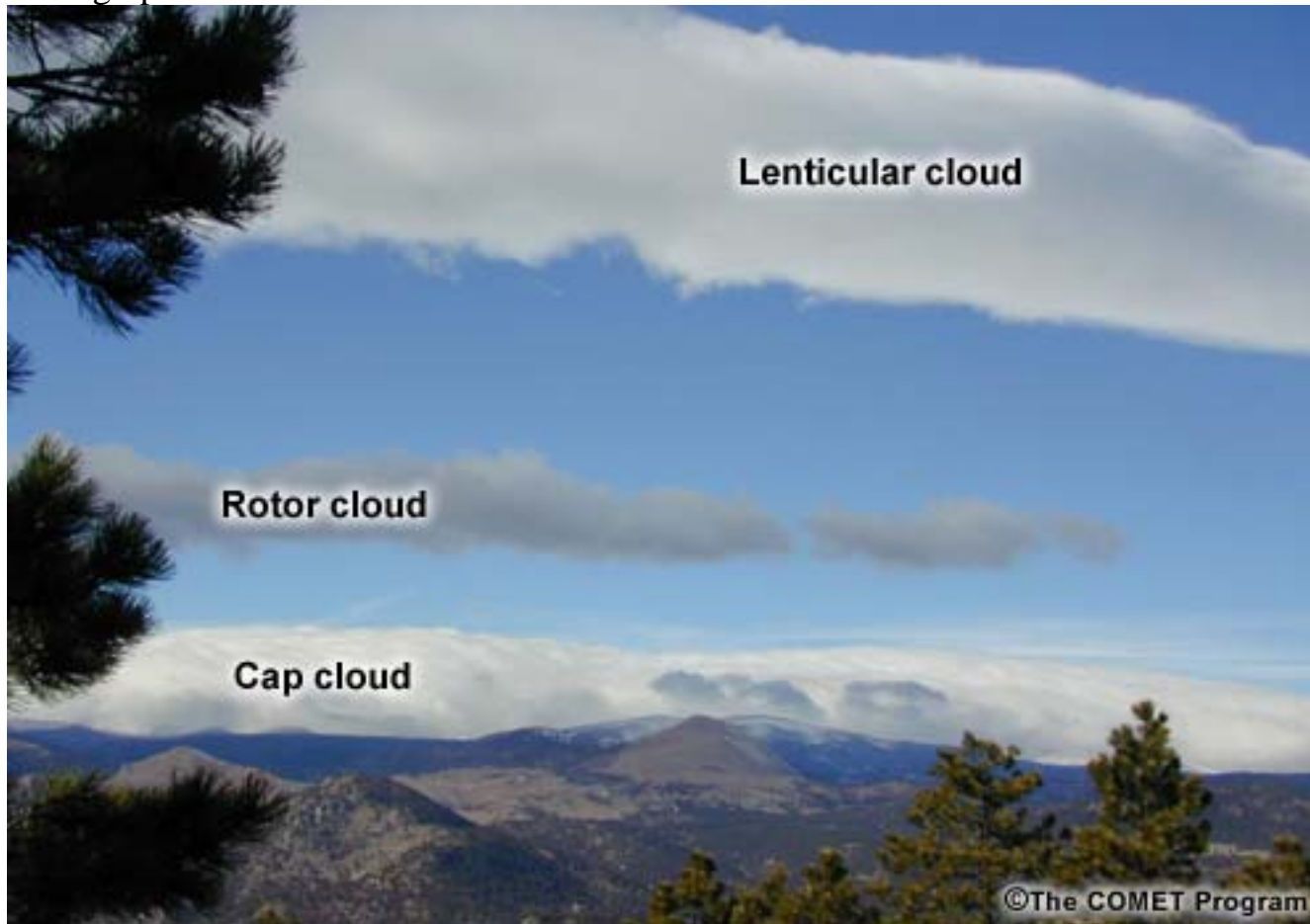


Cap clouds indicate likely wave activity downstream. They often appear along mountain ridges as air is forced up the windward side. If the flow is sufficiently humid, the moisture will condense into a cloud bank that follows mountain contours. Quite often, heavy orographic precipitation occurs on the upwind side of the barrier,

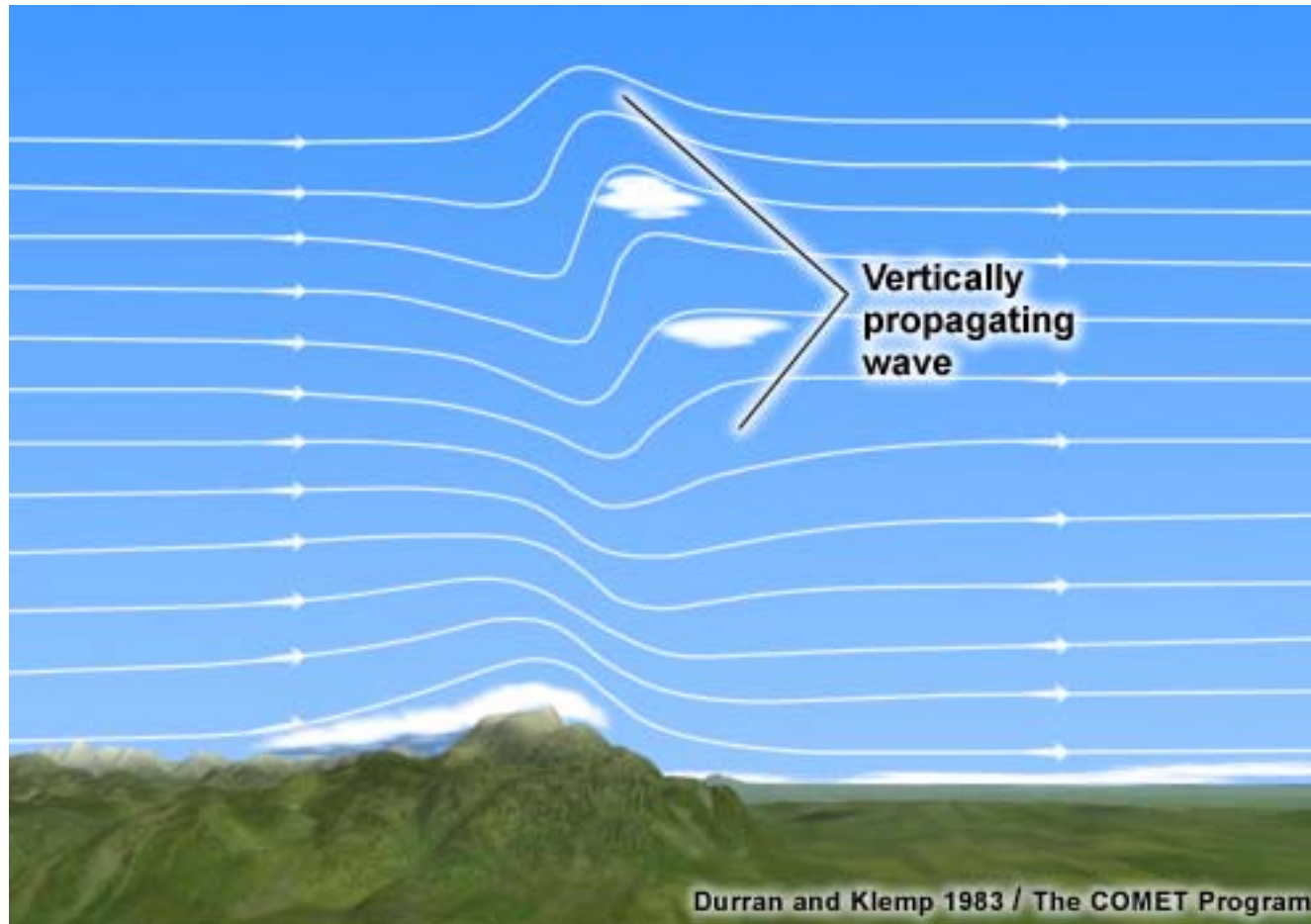
particularly for barriers located near the sea. As the flow descends in the lee of the mountain ridge, the cloud evaporates. Viewed from downstream, cap clouds frequently appear as a wall of clouds hanging over the ridge top.

It is important to remember that while cap clouds indicate likely wave activity, their absence does not mean that waves are absent. Under drier conditions, waves may be present without cap clouds.

Photograph of Wave Clouds

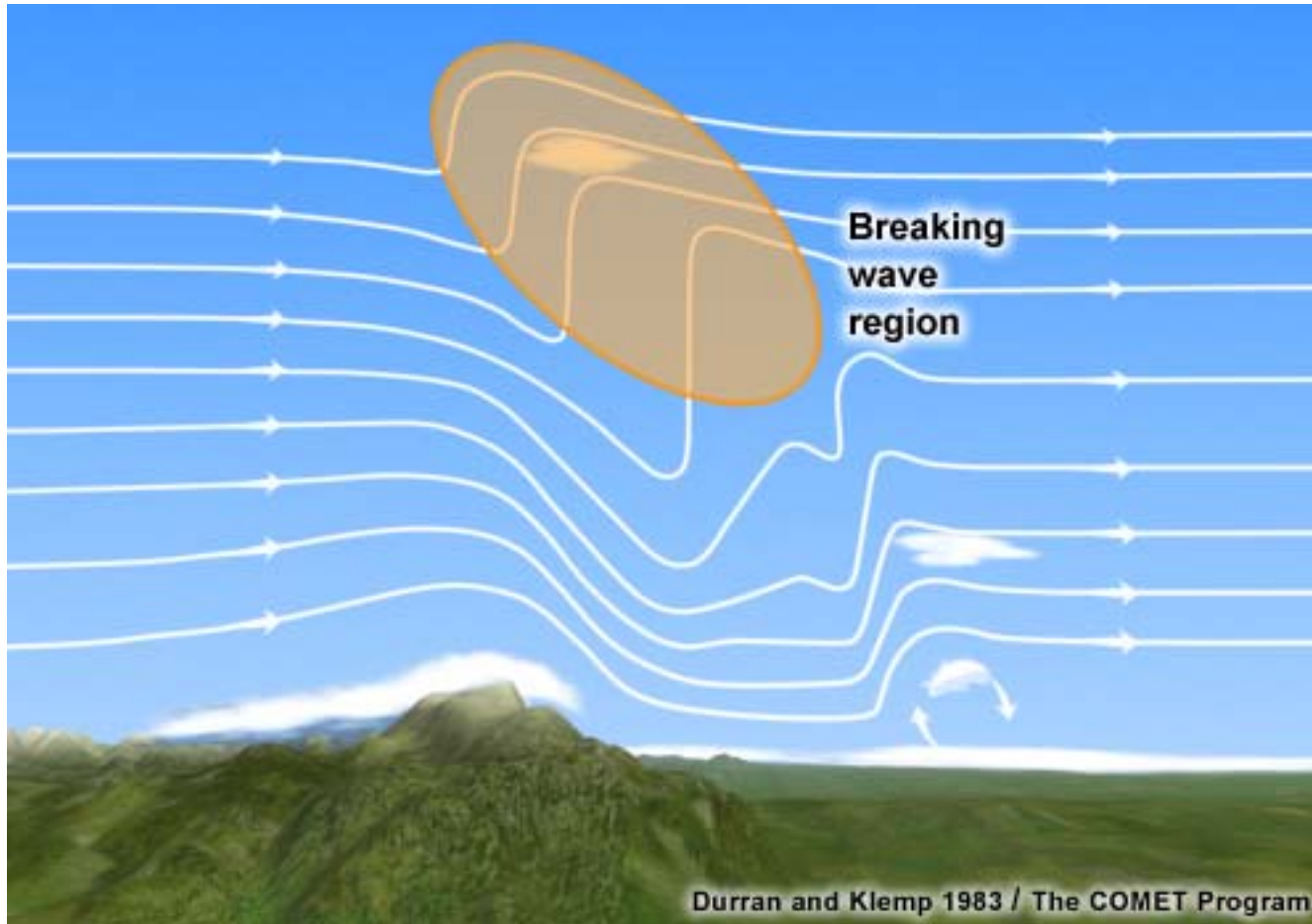


The Vertically-Propagating Wave



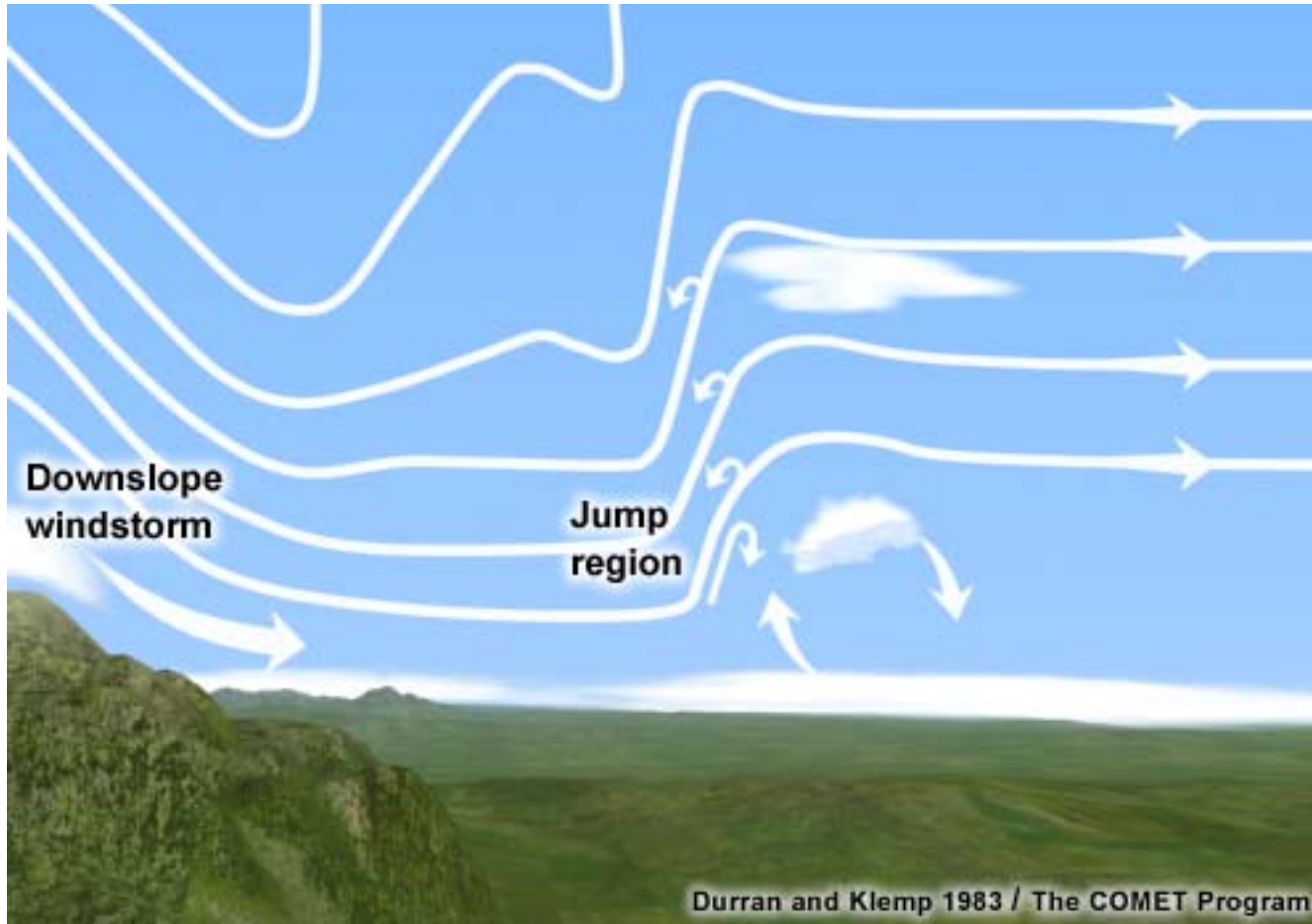
The vertically-propagating wave is often most severe within the first wavelength downwind of the mountain barrier. These waves frequently become more amplified and tilt upwind with height. Tilting, amplified waves can cause aircraft to experience turbulence at very high altitudes. Clear air turbulence often occurs near the tropopause due to vertically-propagating waves. Incredibly, these waves have been documented up to 70 km and higher.

Breaking Waves



Vertically-propagating waves with sufficient amplitude may break in the troposphere or lower stratosphere. Wave-breaking can result in severe to extreme turbulence within the wave-breaking region and nearby, typically between 7 and 14 km. If a vertically-propagating wave doesn't break, an aircraft would likely experience considerable wave action, but little turbulence.

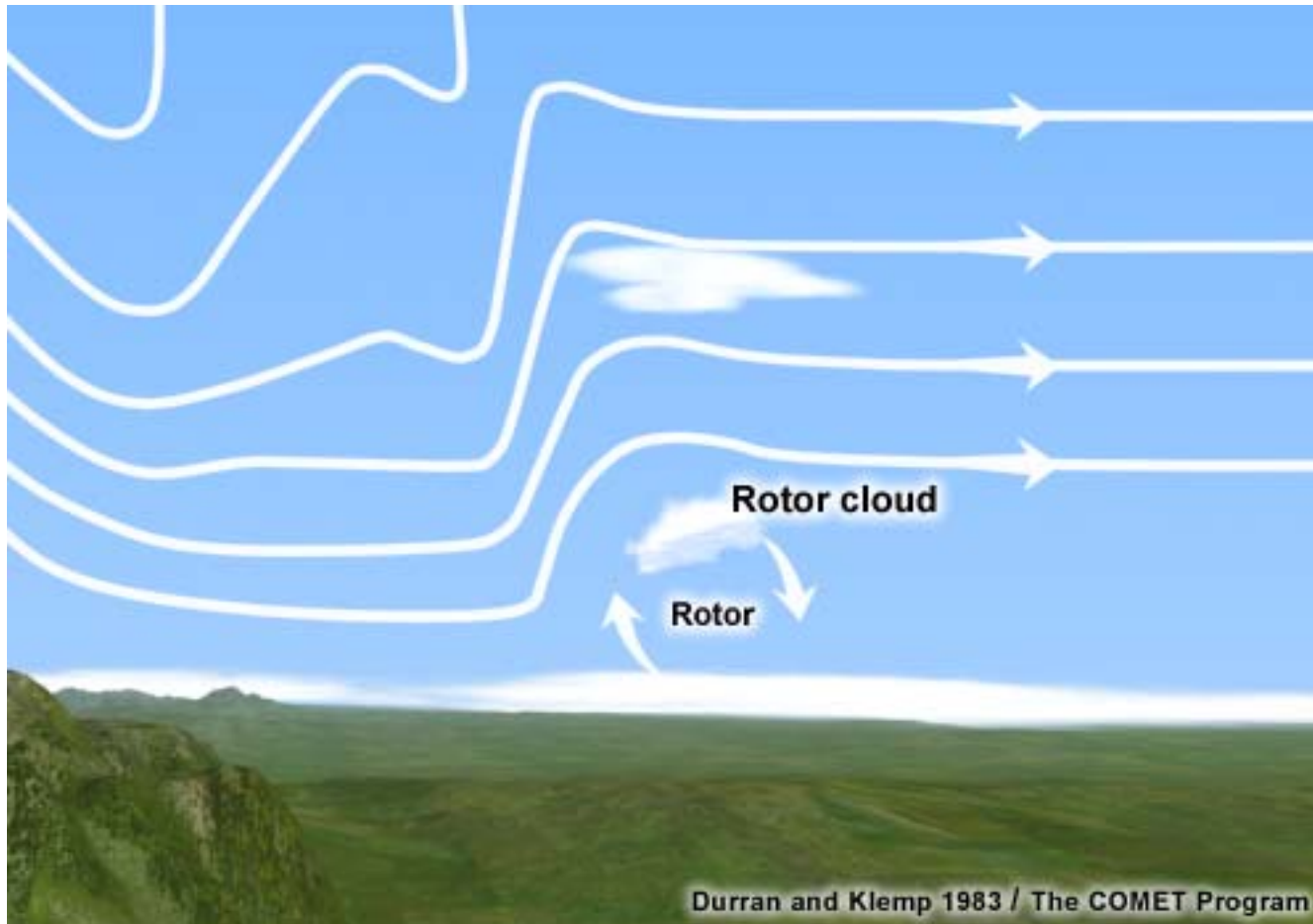
Downslope Winds



At times, strong downslope winds accompany mountain wave systems. Strong downslope wind cases are usually associated with strong cross-barrier flow, waves breaking aloft, and an inversion near the barrier top. In extreme cases, such as in our Alps scenario, winds can exceed 100 knots. This may be double or triple the wind speed at mountaintop level. These high winds frequently lead to turbulence and wind shear at the surface, causing

significant danger to aircraft and damage at the surface. Downslope windstorms often abruptly end at the "jump region," although more moderate turbulence can exist downstream. The jump region is an extremely turbulent area that can extend up to 3 km.

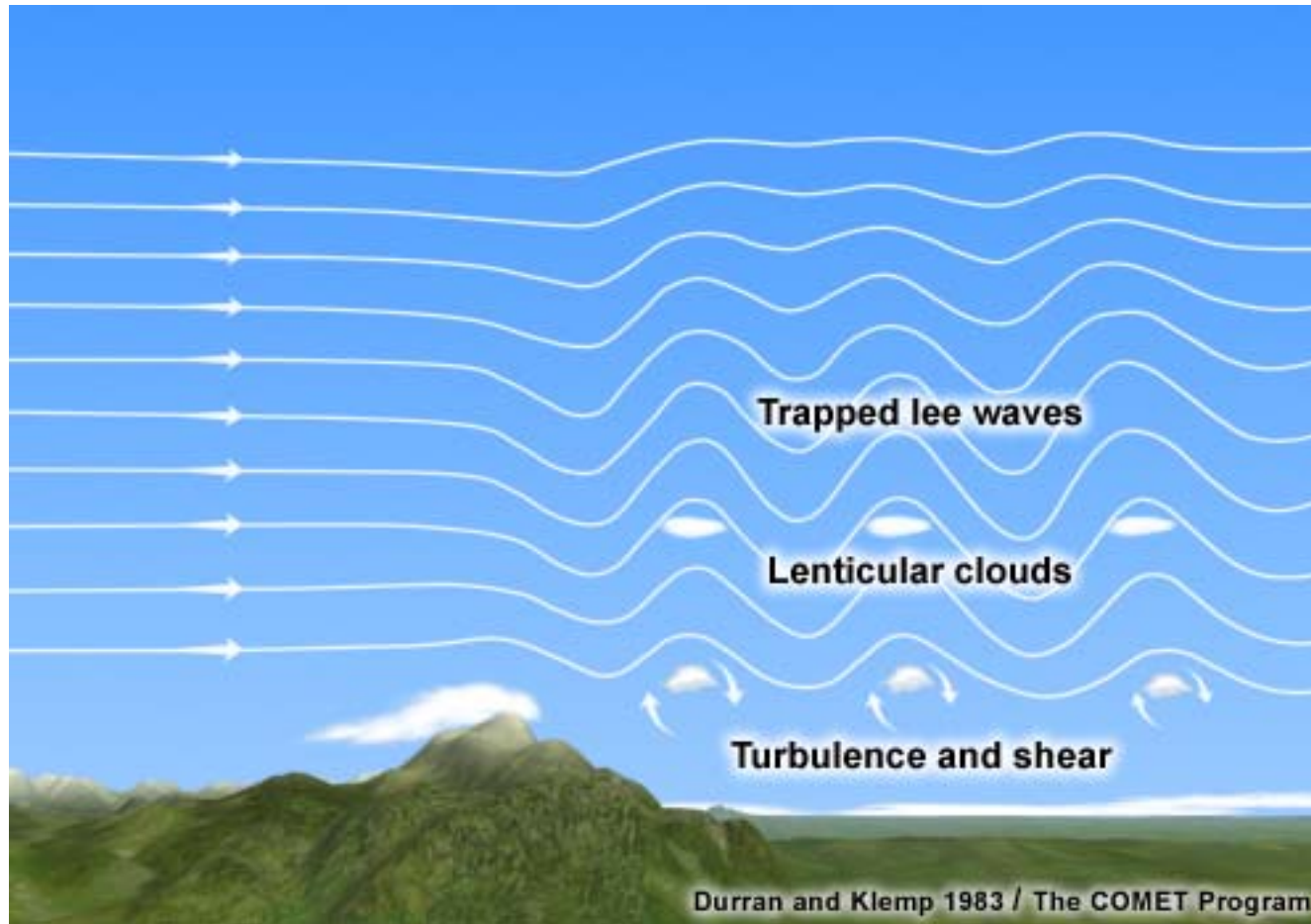
Rotor Clouds



Rotor location can often be identified if sufficient moisture is available to form an associated rotor cloud. Rotor clouds are found near the top of the rotor circulation and under higher lenticular clouds. Immediately above the rotor cloud, smooth, wavy air is likely.

The rotor cloud can look innocuous, but does contain strong turbulence and should be avoided by pilots. Eventually, we can expect operational NWP models to resolve rotors so that they can be identified in the absence of rotor clouds.

Trapped Lee Waves and clouds



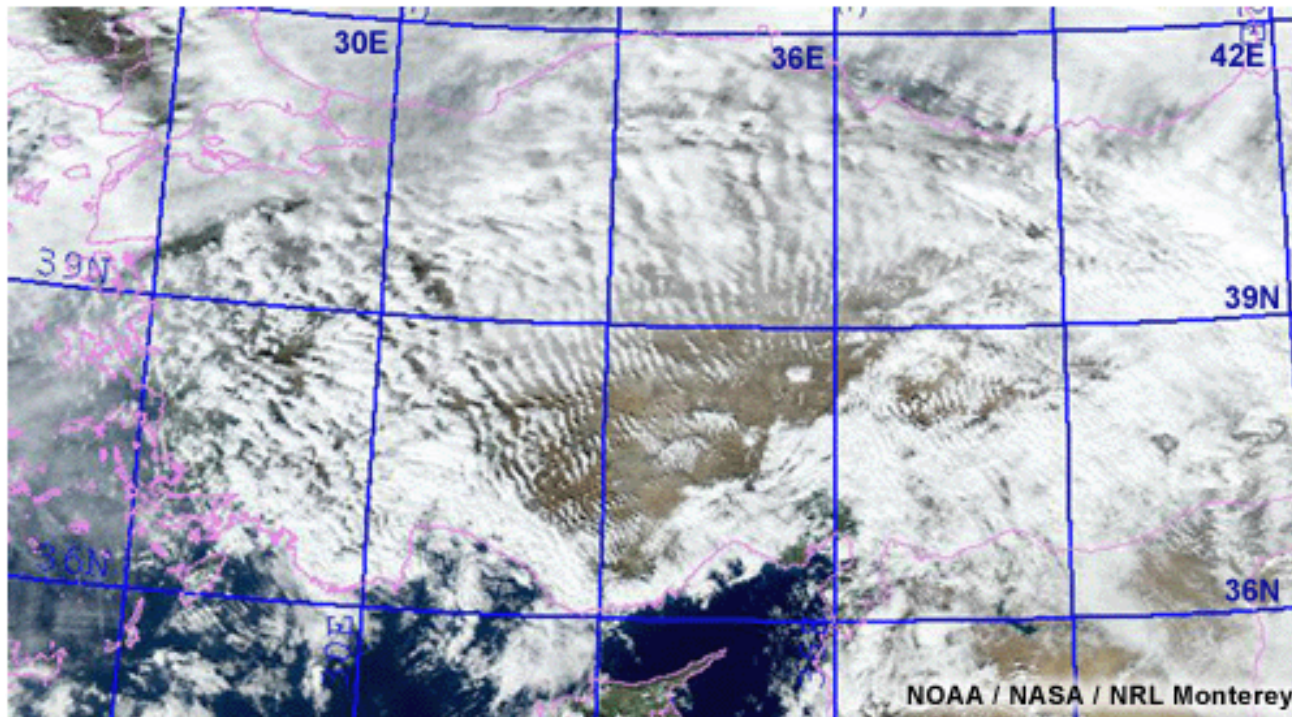
Lee waves whose energy does not propagate vertically because of strong wind shear or low stability above are said to be "trapped." Trapped lee waves are often found downstream of the rotor zone, although a weak rotor may exist under each lee wave. These waves are typically at an altitude within a few thousand feet of the mountain ridge

crest and turbulence is generally restricted to altitudes below 25,000 feet. Strong turbulence can develop between the bases of associated lenticular clouds and the ground.

Lenticular clouds form near the crests of mountain waves. As air ascends and cools, moisture condenses, forming the cloud. As that air descends in the lee of the wave crest, the cloud evaporates. Because air flows through the cloud while the cloud itself is relatively stationary, many people refer to these clouds as standing

Areal Extent of Mountain Waves

MODIS True Color Image 1050 UTC 30 Jan 2003



Mountain wave activity can occur over broad regions. This MODIS true color satellite image shows wave clouds covering most of Turkey, a region spanning about 1000 km! However, despite their occasionally broad extent, regions of strong or severe turbulence within mountain wave systems are often limited horizontally and vertically.

Equations for two-dimensional, steady-state, adiabatic, inviscid, nonrotating, Boussinesq fluid flow over a small-amplitude mountain.

The linear equations can be simplified from the original governing equations:

$$U \frac{\partial u'}{\partial x} + U_z w' + \frac{1}{\rho_o} \frac{\partial p'}{\partial x} = 0, \quad (1)$$

$$U \frac{\partial w'}{\partial x} - g \frac{\theta'}{\theta_o} + \frac{1}{\rho_o} \frac{\partial p'}{\partial z} = 0, \quad (2)$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0, \quad (3)$$

$$U \frac{\partial \theta'}{\partial x} + \frac{N^2 \theta_o}{g} w' = 0. \quad (4)$$

The above set of equations can be further reduced to *Scorer's equation* (1954),

$$\nabla^2 w' + l^2(z) w' = 0, \quad (5)$$

where $\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial z^2$

is the two-dimensional Laplacian operator, and

l is the *Scorer parameter* (Scorer 1949), which is defined by:

$$l^2(z) = \frac{N^2}{U^2} - \frac{U_{zz}}{U}. \quad (6)$$

Equation (5) serves as a central tool for numerous theoretical studies of small-amplitude, two-dimensional mountain waves.

Equation (5), when multiplied by U as

$$U\nabla^2 w' + \frac{N^2 w'}{U} - U_{zz} w' = 0, \quad (5a)$$

may also be interpreted as a vorticity equation, because the y-component of vorticity is

$$\eta = \frac{\partial u'}{\partial z} - \frac{\partial w'}{\partial x},$$

and making use of the mass continuity equation (3)

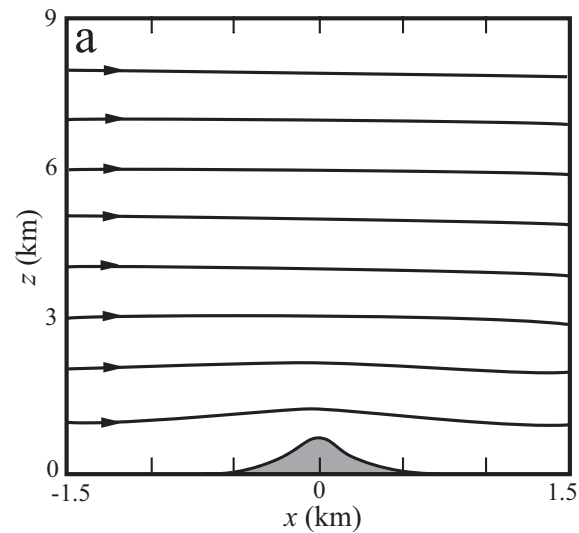
$$U \frac{\partial}{\partial x} \left[\frac{\partial w'}{\partial x} - \frac{\partial u'}{\partial z} \right] + \frac{N^2 w'}{U} - w' \frac{\partial U_z}{\partial z} = 0. \quad (5b)$$

- The first term, $U(w'_{xx} + w'_{zz})$, is the rate of change of vorticity following a fluid particle.
- The second term, $N^2 w'/U$, is the rate of vorticity production by buoyancy forces.
- The last term, $-U_{zz} w'$, is the rate of vorticity production by the vertical redistribution of the background vorticity (U_z).

In the extreme case of very small Scorer parameter, e.g., when $N=0$ and vertical shear is zero, (5) reduces to irrotational or potential flow,

$$\nabla^2 w' = 0. \quad (7)$$

If the forcing is symmetric in the basic flow direction, such as a cylinder in an unbounded fluid or a bell-shaped mountain in a half-plane, then the flow is symmetric (see below).



For this particular case, there is no drag produced on the mountain since the fluid is inviscid.

Flows over two-dimensional sinusoidal mountains

Assuming constant $U(z)$ and $N(z)$ with height and a sinusoidal terrain

$$h(x) = h_m \sin kx, \quad (8)$$

with mountain height h_m . k is the wave number of the terrain.

For an inviscid fluid flow, the lower boundary condition requires the fluid particles to follow the terrain, so that the streamline slope equals the terrain slope locally,

$$\frac{w}{u} = \frac{w'}{U + u'} = \frac{dh}{dx} \quad \text{at} \quad z = h(x). \quad (9)$$

For a small-amplitude mountain, this is linearized to

$$w' = U \frac{dh}{dx} \quad \text{at} \quad z = 0, \quad (10)$$

or

$$w'(x,0) = U h_m k \cos kx \quad \text{at} \quad z = 0, \quad (11)$$

for $h(x)$ given in (8). It is the linearized lower boundary condition!

Because of the sinusoidal nature of the forcing, we can expect solutions in the following form,

$$w'(x, z) = W_1(z) \cos kx + W_2(z) \sin kx. \quad (12)$$

Substituting the above solution into (5) with a constant Scorer parameter leads to

$$W_{izz} + (l^2 - k^2)W_i = 0, \quad i = 1, 2. \quad (13)$$

Two cases are possible:

(a) $l^2 < k^2$, (vertically decaying solution)

(b) $l^2 > k^2$. (oscillation solution)

The first case requires $N/U < k$ or $Na/U < 2\pi$, where a is the terrain wavelength.

Physically, this means that the basic flow has relatively weaker stability and stronger wind, or that the mountain is narrower than a certain threshold.

For example, to satisfy the criterion for a flow with $U = 10 \text{ ms}^{-1}$ and $N = 0.01 \text{ s}^{-1}$, the wavelength of the mountain should be smaller than 6.3 km. In fact, this criterion can be rewritten as $(a/U)/(2\pi/N) < 1$.

The numerator, a/U , represents the advection time of an air parcel passing over one wavelength of the terrain, while the denominator, $2\pi/N$, represents the period of buoyancy oscillation due to stratification.

This means that the time an air parcel takes to pass over the terrain is less than it takes for vertical oscillation due to buoyancy force. In other words, buoyancy force plays a smaller role than the horizontal advection.

In this situation, (13) can be rewritten as

$$W_{izz} - (k^2 - l^2)W_i = 0, \quad i = 1, 2. \quad (14)$$

The solutions of the above second-order differential equation with constant coefficient may be obtained

$$W_i = A_i e^{\lambda z} + B_i e^{-\lambda z}, \quad i = 1, 2, \quad (15)$$

where

$$\lambda = \sqrt{k^2 - l^2}. \quad (16)$$

The upper boundedness condition requires $A_i = 0$ because the energy source is located at $z = 0$.

Applying the lower boundary condition, (11), and the upper boundary condition ($A_i = 0$) to (15) yields

$$B_1 = U h_m k; \quad B_2 = 0. \quad (17)$$

This yields the solution,

$$w'(x, z) = W_1(z) \cos kx = U h_m k e^{-\sqrt{k^2 - l^2} z} \cos kx, \quad (18)$$

The vertical displacement (η) is defined as $w' = D\eta / Dt$ which reduces to

$$w' = \frac{D\eta}{Dt} = U \frac{\partial \eta}{\partial x} \quad (19)$$

for a steady-state flow.

Equation (18) can then be expressed in terms of η ,

$$\eta = \frac{1}{U} \int_0^x w' dx = h_m \sin kx e^{-\sqrt{k^2 - l^2} z}. \quad (20)$$

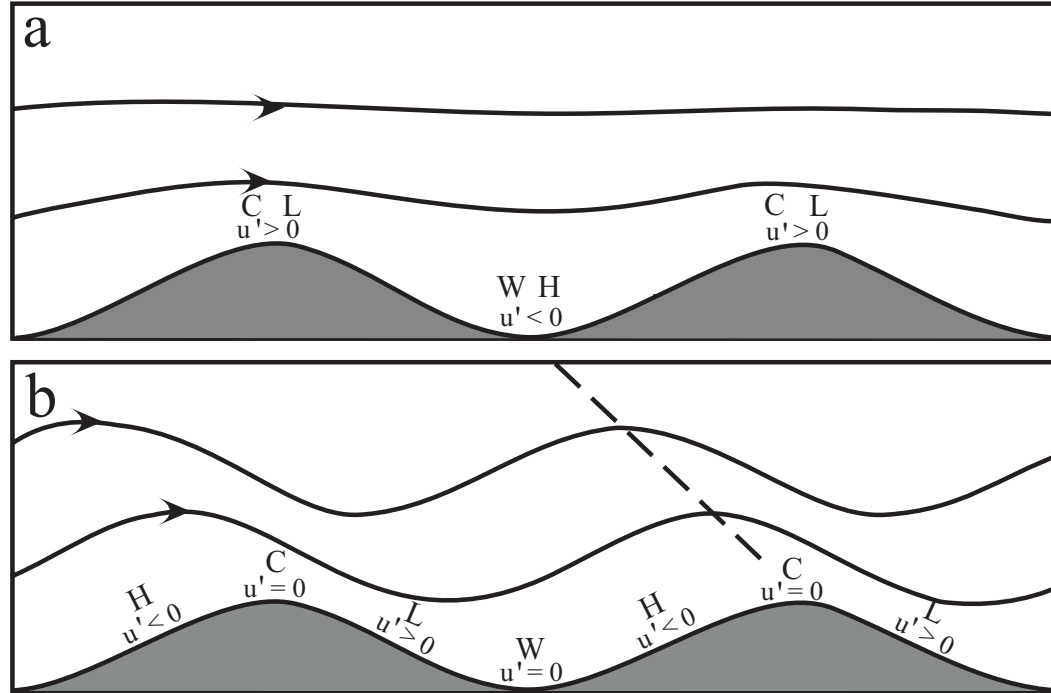


Fig. 5.1: The steady-state, inviscid flow over a two-dimensional sinusoidal mountain when (a) $l^2 < k^2$ (or $N < kU$), where k is the terrain wavenumber ($=2\pi/a$, where a is the terrain wavelength), or (b) $l^2 > k^2$ (or $N > kU$). The dashed line in (b) denotes the constant phase line which tilts upstream with height. The maxima and minima of u' , p' (H and L), and θ' (W and C) are also denoted in the figures.

The above solution is sketched in Fig. 5.1a. The disturbance is symmetric with respect to the vertical axis and decays exponentially with height. Thus, the flow belongs to the evanescent flow regime. The buoyancy force plays a minor role compared to that of the advection effect.

The other variables can also be obtained by using the governing equations and (18),

$$u' = U h_m \sqrt{k^2 - l^2} \sin kx e^{-\sqrt{k^2 - l^2} z}, \quad (21)$$

$$p' = -\rho_o U^2 h_m \sqrt{k^2 - l^2} \sin kx e^{-\sqrt{k^2 - l^2} z}, \quad (22)$$

$$\theta' = -(\theta_o N^2 / g) h_m \sin kx e^{-\sqrt{k^2 - l^2} z}. \quad (23)$$

The maxima and minima of u' , p' , and θ' are also denoted in Fig. 5.1a.

The coldest (warmest) air is produced at the mountain peak (valley) due to adiabatic cooling (warming). The flow accelerates over the mountain peaks and decelerates over the valleys. From the horizontal momentum equation, (1) with $U_z = 0$, or (22), a low (high) pressure is produced over the mountain peak (valley) where maximum (minimum) wind is produced.

Note that (1) is also equivalent to the *Bernoulli equation*, which states that the pressure perturbation is out of phase with the horizontal velocity perturbation. Since no pressure difference exists between the upslope and downslope, this flow produces no net wave drag on the mountain (mountain drag).

In the second case, $l^2 > k^2$, the flow response is completely different.

This case requires $N/U > k$ or $Na/U > 2\pi$. This means that the basic flow has relatively stronger stability and weaker wind or that the mountain is wider.

For example, and as mentioned earlier, to satisfy the criterion for a flow with $U = 10 \text{ ms}^{-1}$ and $N = 0.01 \text{ s}^{-1}$, the terrain wavelength should be larger than 6.3 km.

Since $(a/U)/(2\pi/N) > 1$, the advection time is larger than the period of the vertical oscillation. In other words, buoyancy force plays a more dominant role than the horizontal advection.

In this case, (13) can be written as

$$W_{izz} + m^2 W_i = 0, \quad m^2 = l^2 - k^2, \quad i = 1, 2. \quad (26)$$

We look for solutions in the form

$$W_i(z) = A_i \sin mz + B_i \cos mz, \quad i = 1, 2. \quad (27)$$

Substituting (27) into (12) leads to

$$w'(x, z) = C \cos(kx + mz) + D \sin(kx + mz) + E \cos(kx - mz) + F \sin(kx - mz). \quad (28)$$

Terms of $(kx + mz)$ have an upstream phase tilt with height, while terms of $(kx - mz)$ have a downstream phase tilt.

It can be shown that terms of $(kx + mz)$ have a positive *vertical energy flux* and should be retained since the energy source in this case is located at the mountain surface. Thus, the solution requires $E = F = 0$. This flow regime is characterized as the upward propagating wave regime.

As in the first case, the lower boundary condition requires

$$C = Uh_m k, \quad D = 0. \quad (29)$$

This leads to

$$w'(x, z) = Uh_m k \cos(kx + mz). \quad (30)$$

Other variables can be obtained through definitions or the governing equations,

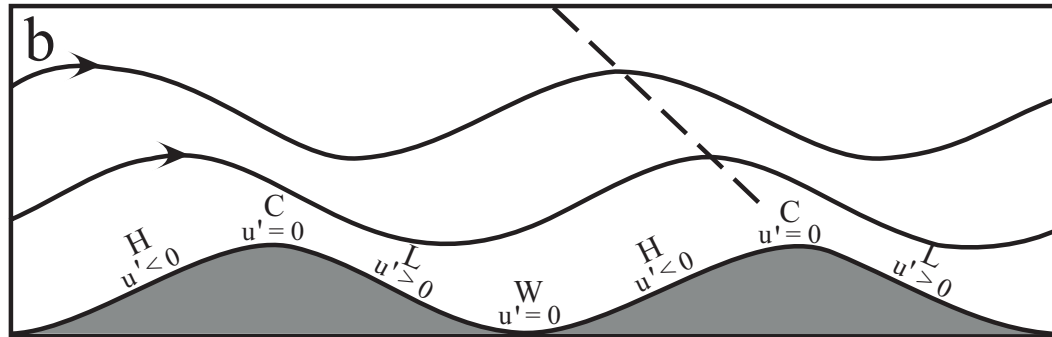
$$\eta(x, z) = h_m \sin(kx + mz), \quad (31)$$

$$u'(x, z) = -Uh_m m \cos(kx + mz), \quad (32)$$

$$p'(x, z) = \rho_0 U^2 h_m m \cos(kx + mz), \text{ and} \quad (33)$$

$$\theta'(x, z) = -\frac{N^2 \theta_0 h_m}{g} \sin(kx + mz). \quad (34)$$

The vertical displacement of the flow, and the maxima and minima of u' , p' , and θ' are depicted in Fig. 5.1b (below).



Note that the flow pattern is no longer symmetric. The constant phase lines are tilted upstream (to the left) with height, thus producing a high pressure on the windward slope and a low pressure on the lee slope.

Based on (32) or the Bernoulli equation (1), the flow decelerates over the windward slope and accelerates over the lee slope. The coldest and warmest spots are still located over the mountain peaks and valleys, respectively.

Positive wave drag on the mountain is produced by the high pressure on the windward slope and the low pressure on the lee slope.

When $l^2 \gg k^2$, the flow approaches a limiting case in which the buoyancy effect dominates and the advection effect is totally negligible. In other words, the vertical pressure gradient force and the buoyancy force are roughly in balance and the vertical acceleration can be ignored. Thus, the mountain waves become hydrostatic. In this limiting case, the governing equation becomes

$$w'_{zz} + l^2 w' = 0. \quad (36)$$

The flow pattern repeats itself in the vertical with a wavelength of $\lambda_z = 2\pi / l = 2\pi U / N$, which is also referred to as the hydrostatic vertical wavelength.

The regime boundary between the regimes of vertically propagating waves and evanescent waves can be found by letting $l = k$, which leads to $a = 2\pi U / N$.

The relation among the mountain waves discussed in this subsection is sketched in Fig. 5.2.

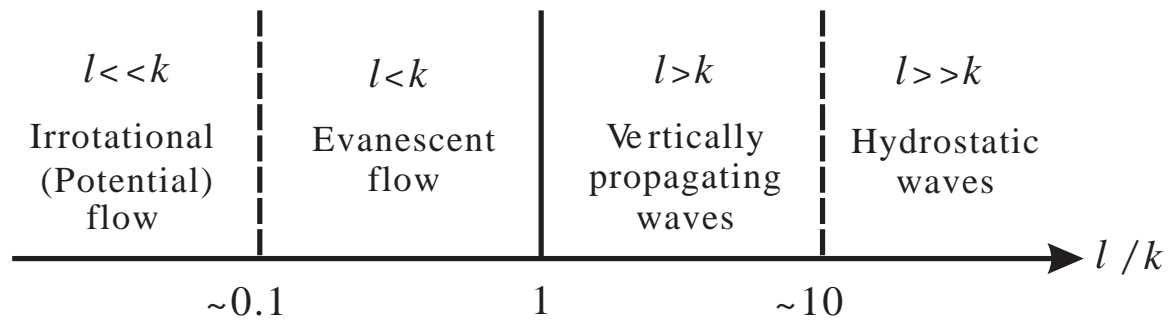


Fig. 5.2: Relations among different mountain waves as determined by l/k , where l is the Scorer parameter and k is the wave number.