Problem Set #2

For problem 1, you are encouraged to write a computer program to do the calculations and plotting - it will make your life easier!

1. (50%) A vertical wind profile is given by the following table:

<table>
<thead>
<tr>
<th>z (height, km)</th>
<th>θ (direction, deg)</th>
<th>V(speed, m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>110</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>150</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>180</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>190</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>250</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>270</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>310</td>
<td>40</td>
</tr>
</tbody>
</table>

Assume that the storm motion vector is from 225 degrees (from SW) and the speed is 12 m/s.

a). Plot the hodograph and the storm-relative velocity vectors at each level

b). Calculate the horizontal vorticity (vector, in terms of the vorticity components or in magnitude and direction) in each of the six layers between the levels of observations

c). Determine the mean (storm-relative) wind vector in each of the six layers

d). Using the layer-mean wind obtained above, calculate the storm-relative helicity in each of the six layers, and determine the vertically integrated environmental helicity in the lowest three kilometers

e). Calculate the (storm-relative) streamwise vorticity and (storm-relative) relative helicity in each of the six layers

f). Discuss your results and their significance in terms of their effect on the behavior and type of storms that occur in such an environment

g). For this wind profile, what kind of CAPE values will give you a BRN that suggests a high probability of multicell and supercell storms, respectively?
2. (50%) The storm-relative environment helicity in the lowest 3 km layer is given as

\[ SREH = \int_{0}^{3km} [(\vec{V} - \vec{\bar{C}}) \cdot \vec{\omega}_H] dz \]

which, based on definition \( \vec{\omega}_H = \hat{k} \times \frac{d\vec{V}}{dz} = -\frac{dv}{dz} \hat{i} + \frac{du}{dz} \hat{j} \), can be rewritten as

\[ SREH = -\int_{0}^{3km} \hat{k} \left[(\vec{V} - \vec{\bar{C}}) \times \frac{d\vec{V}}{dz}\right] dz = -\int_{0}^{3km} \frac{1}{k} \left[(\vec{V} - \vec{\bar{C}}) \times d\vec{V}\right] = -\int_{0}^{3km} \frac{1}{k} \left[\vec{V}_r \times d\vec{V}\right] \]

where \( \vec{V}_r \equiv (\vec{V} - \vec{\bar{C}}) \) is the storm-relative velocity.

a). Using the above information and your knowledge of analytic geometry, show that the SREH is equal to minus twice the signed (i.e., positive or negative) area swept out by the storm-relative wind vector between 0 and 3 km on a hodograph. Note that, by convention, an area is positive (negative) if it is swept out counterclockwise (clockwise).

To keep the problem simple, let's assume that wind observations are available at the 0 and 3 km levels only.

b). If the storm-relative velocity at 0 and 3 km levels are \( (u_{r1}, v_{r1}) \) and \( (u_{r2}, v_{r2}) \), respectively, show that SREH can be calculated from

\[ SERH = u_{r2}v_{r1} - u_{r1}v_{r2} \]

Hint: Think of how you calculated the storm-relative helicity in Problem 1, for each of those 6 layers. Also, \( d\vec{V} = d\vec{V}_r \) because the storm motion vector is constant with height.

c). Verify that for the following hodograph and a zero storm-motion vector, the above two methods give the same results.

\[ V(3km) = 10 \text{ m/s} \]
\[ V(0km) = 15 \text{ m/s} \]

\[ \text{V(3km)} = 10 \text{ m/s} \]
\[ \text{V(0km)} = 15 \text{ m/s} \]

d) Explain why larger SERH tends to promote longer lasting supercell storms?