Quiz #2. Physical Mechanics, 2000
Total 9 points.

1. The position of an air parcel displaced vertically from its initial level $z=0$ is given by

$$z(t) = Ae^{-\alpha t} \cos(\omega t)$$

where $A$, $\alpha$ and $\omega$ are all positive real numbers.

(3 point) Sketch the solution as a function of time and describe the physical behaviors of the parcel. See Notes for sketch.

The air parcel will undergo periodic oscillations at frequency $\omega$ and the amplitude will decrease exponentially in time.

(1 point) To have such a solution, is the atmosphere stable or unstable?

For the parcel to oscillate around its equilibrium level, the atmosphere must be stable.

(1 point) We have shown in the class that frequency $\omega$ in the solution is proportional to the static stability of the atmosphere. In what situation does the parcel oscillate faster?

The frequency is proportionally to static stability, therefore the stronger the stability is, i.e., the more stable the atmosphere is, the faster are the oscillations (the higher is the frequency).

2. (2 points). Give the physical definitions of $\frac{dF}{dt}$ and $\frac{\partial F}{\partial t}$, as they appear in equation

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + u \frac{\partial F}{\partial x} + v \frac{\partial F}{\partial y} + w \frac{\partial F}{\partial z},$$

where $u$, $v$ and $w$ are velocity components in $x$, $y$ and $z$ directions, respectively.

Physically, $\frac{dF}{dt}$ represents the total rate of change in $F$ following the motion [at the speed of $(u,v,w)]$, $\frac{\partial F}{\partial t}$ represents the rate of change at a fixed spatial location.

3. (2 points) What are the essential elements to have non-zero advection?

(a) Velocity, (b) spatial gradient in the field being advected, and (c) a non-zero component of velocity in the direction of the gradient.