Structure of the atmosphere in the vicinity of large-amplitude
Kelvin-Helmholtz billows

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SUMMARY

Data are presented for 17 separate occurrences of Kelvin-Helmholtz billows in the upper troposphere for
which the crest-to-trough amplitude exceeded 200 m. The billows were detected in clear air by means of
high-power radar. The associated mesoscale patterns of static stability, vertical wind shear, and Richardson
number have been derived from sequences of hourly radiosondes tracked by precision radar. Every occurrence
of billows was found to be closely associated with well-defined, local, wind-shear maxima. In some cases the
shear was as strong as 9 m s\(^{-1}\) per 200 m. The minimum value of the Richardson number when evaluated
over layers 200 m deep was usually found to be in the range 0.15-0.3.

1. INTRODUCTION

Theoretical considerations and observations in the laboratory, ocean, and the free
atmosphere suggest that Kelvin-Helmholtz instability is probably responsible for most
occurrences of clear air turbulence (CAT). Kelvin-Helmholtz instability (KHI) is a form of
dynamic instability which occurs within a hydrostatically stable flow wherever there is both
an inflexion in the velocity profile and a sufficiently strong vertical shear. It appears as
amplifying waves ('billows') oriented perpendicular to the shear vector, into which the
vorticity is concentrated, and which eventually 'break' into turbulent flow on a range of
smaller scales. For a sheared layer of finite thickness \(\Delta Z\), the fastest growing wavelength is
given by Miles and Howard (1964) as

\[
\lambda = 7.5 \Delta Z.
\]  

(1)

Some of the most convincing evidence of KH billows in the atmosphere comes from visual
observations of clouds (Ludlam 1967) and high-power radar observations of the clear
atmosphere (e.g. Hicks and Angell 1968, Browning and Watkins 1970).

The onset of KHI over the entire depth of an (unsaturated) atmospheric layer of
thickness \(\Delta Z\) is determined by the value of the corresponding layer Richardson number

\[
Ri (\Delta Z) = \frac{g}{\bar{\vartheta}} \left( \frac{\Delta \vartheta}{\Delta Z} \right) / \left( \frac{\Delta V}{\Delta Z} \right)^2,
\]

(2)

where \(\Delta V\) is the magnitude of the vector wind shear over the depth \(\Delta Z\) and \(\bar{\vartheta}\) is the mean
potential temperature. The quantity \(\frac{1}{\bar{\vartheta}} \left( \frac{\Delta \vartheta}{\Delta Z} \right)\) represents the static stability of the layer.

Theoretical studies indicate that \(Ri < 0.25\) is a necessary but not sufficient condition for the
onset of KHI (Miles and Howard 1964). A critical value \(Ri_c\) of 0.25 has also been obtained
in laboratory tank experiments by Thorpe (1968). However, because of observational
difficulties, there is not yet any definite evidence to confirm this value in the free atmosphere.
The observations reported by Ludlam (1967) were suggested to be consistent with
\(Ri_c = 0.25\) but the representativeness of the soundings was not always adequate to
establish this. According to Vinnichenko and Dutton (1969) the most that can be said with
confidence at the present time is that \(Ri_c < 1\). The purpose of this paper is to obtain
improved estimates of \(Ri_c\) for the free atmosphere using measurements of wind shear and
static stability representative of the vicinity of reliably identified major occurrences of KHI.

Difficulties in determining $R_i$ in the atmosphere arise because of the dependence of
$R_i$ on the thickness of the layer over which it is measured. Thus $R_i$ is often near-critical
over shallow layers when its bulk value over deeper layers is much above critical. The
vertical extent $\Delta Z$ of KHI in the free atmosphere varies over wide limits, from hundreds
of metres probably down to metres. The problem, therefore, is

(a) to identify the time and vertical extent $\Delta Z$ of the dynamically unstable layer
responsible for a particular occurrence of KHI,

(b) to obtain vertical profiles of stability and shear with a resolution comparable
with $\Delta Z$, and

(c) in view of the strong mesoscale variability of $R_i$, to make these soundings as close
as possible to the occurrence of KHI.

In the present study high-power radar observations have been used to identify the
times of occurrence, wavelength and amplitude, of KH billows in the upper troposphere.
The study has been restricted to billows of large amplitude, first, because such billows are
easy to resolve with the high-power radar and, second, because it is only in these cases that
it is possible to obtain profiles of stability and shear with the necessary resolution. A fairly
large number of occurrences of large-amplitude billows has been studied, most of them
having a crest-to-trough amplitude of between 300 and 400 m. The wavelength of the
billows was between 0.8 and 4 km, with a mean value of 1.8 km. According to Eq. (1) the
depth of the dynamically unstable layer responsible for such billows can be expected to have
been between 100 and 530 m, with a mean value of 240 m. The vertical structure of the
atmosphere in the vicinity of these billows has been determined by means of sequences of
hourly radiosondes tracked by precision radar. The vertical resolution of the soundings was
adequate to permit static stability, vertical wind shear and Richardson number to be
evaluated over layers 200 m deep, thereby providing a resolution comparable with the
depth of the dynamically unstable layer.

2. The radar observations of Kelvin-Helmholtz billows

Echoes having the appearance of KH billows were observed in the clear atmosphere
using the high-power 107 mm wavelength radar at the Royal Radar Establishment site at
Deford, Worcestershire (2°09'W, 52°06'N). The radar has been described by Watkins
(1971). The normal mode of operation in this study was for the radar to be scanned
continuously in a vertical section into the direction of the upper tropospheric winds at the
rate of 1 scan every 3 minutes. The received echoes were displayed and photographed on a
conventional Range-Height-Indicator (RHI) display. An example of large-amplitude
billows observed by this radar is shown in Fig. 1. Several clear air echo layers can be seen
below 4 km; however, the echoes of interest are centred at 5-6 km within a layer of
relatively strong wind shear. These echoes depict billows with wavelength 1.5 km and
crest-to-trough amplitude 400 m. That these, and the other similar echoes discussed here,
were indeed due to KHI is substantiated later by detailed analyses of the vertical profiles of
wind and stability.

In order to understand what the radar is detecting in Fig. 1 it is important to realise
that KHI was occurring simultaneously on quite different scales (Hicks 1969). The largest
scale corresponds to the 400 m amplitude of the observed billow pattern; the smallest
scales are orders of magnitude smaller and occur within; and are responsible for, the echo
itself. Layers of the atmosphere within which such echoes are situated are characterized by
large refractive index gradients associated with vertical gradients of potential temperature
and/or humidity (in the upper troposphere the effect of potential temperature gradients
usually predominates, whereas in the lower troposphere the effect of humidity gradients
predominates). Turbulence due to small-scale KHI within such regions gives rise to layers
containing irregularly disposed inhomogeneities of refractive index and this leads to
Figure 1. Photograph of the Range-Height-Indicator display of the Defford radar, showing large-amplitude Kelvin-Helmholtz billows (Event G) centred at 5-6 km altitude. The vertical section scanned by the radar is oriented along 320°, into the direction of both the wind and wind shear at the level of the billows. The two range rings are at 5 and 10 km. (Side-lobe echoes from the ground, usually a major problem with high-power radars, have been largely avoided by using a Moving-Target-Indicator system.)
incoherent backscatter in which the radar detects echoes from the inhomogeneities whose size is equal to half the radar wavelength. Existing scattering theory accounting for the intensity of the radar echo (Atlas, Hardy and Naito 1966) assumes that the turbulence on this very small scale is within an inertial subrange and is locally homogeneous, isotropic and stationary. However, the importance of such a layer of small-scale turbulence in the present

Figure 2. Time-height pattern of (a) wind shear $\Delta V/\Delta Z$ and (b) Richardson number ($R_i$) over layers 200 m deep in the vicinity of billow event $D$ on 6 February 1970. The duration and vertical extent of the billow event is indicated by the small rectangular frame. Times of radiosonde soundings are indicated by arrows. In (a) widely spaced, closely spaced, and cross hatching, represent wind shears of 4–6, 6–8 and >8 m s$^{-1}$ per 200 m, respectively. In (b), widely spaced, and closely spaced hatching represent values of $R_i$ of 0.3–0.5, and 0.2–0.3, respectively.
context is that it serves as a tracer which reveals the presence of anisotropic and non-stationary large-amplitude KH billows of the kind depicted in Fig. 1. As the large-amplitude billows grow the wind shear and the small-scale turbulence is locally increased; at the same time gradients of refractive index are also sharpened in a manner similar to that described by Thorpe (1969; see his Fig. 3). The combined result is that the intensity of the radar echo increases significantly in the presence of the large-amplitude billows.

The limit of resolution of the Deford radar is 100-200 m (the pulse length is 187 m, and the half-power beam width is 0.33° in the horizontal and vertical) and so it is of course only possible to resolve clearly the structure of billows with amplitudes in excess of 200 m.

Figure 3. Time-height pattern of (a) wind shear and (b) Richardson number over layers 200 m deep in the vicinity of billow events F and G on 13 April 1970. Format is similar to that in Fig. 2 except that there are two additional categories of Ri; viz: 0.1 < Ri < 0.2 (cross-hatched), Ri < 0 (stippled).
Horizontal trains of billows with much smaller amplitudes are detected as almost featureless layer echoes. In all cases of large-amplitude billows studied so far, the large billows were preceded by one or more relatively undistorted layer echoes at roughly the same altitude, thereby indicating the presence of smaller-amplitude KHI at these times. As is shown later, the initiation of the large-amplitude billows is generally preceded by an increase in wind shear $\Delta V/\Delta Z$. Thus, as suggested by Ottersten (1970), it appears that the large-amplitude billows develop when the small billows are no longer able to accomplish completely the rearrangement of the shear flow.

Observations with the Defford radar have been obtained for a total of 176 hours during 30 days from January to July 1970. Large-amplitude billows were detected at radar ranges up to 50 km (most commonly to 15 km) for a total of 8 hours during 11 of these days. Although large billows were thus observed for as much as 5 per cent of the time, it must be borne in mind that the periods of observation were biased towards occasions of strong shear. The total duration of large-amplitude billows on any given day was usually less than 30 min, although on one rather exceptional day (3 February) they were observed continually for about 4 hours. On most occasions large-amplitude billows occurred in well-defined rows of from 2 to 8 billows, each of which will be referred to as a 'billow event'. However, on 3 February, billows were observed repeatedly and more extensively; in this case the billows occurring close to two separate radiosondes are arbitrarily classified as separate events. In Section 3 the vertical structure of the atmosphere is analysed in the vicinity of 17 large-amplitude billow events all of which were well documented by means of sequential radiosondes. The main characteristics of these events as determined by the Defford radar are summarized in Table 1; the last column, giving the location of the events, has been derived from the special radiosondes combined with routine synoptic data. The mean wavelength appropriate to each billow event in Table 1 was estimated from the radar data assuming that the billows were oriented perpendicular to the wind shear. (The radar sections were within 30° of the wind shear vector for 13 out of the 17 events.) The ratio between the crest-to-trough amplitude and wavelength varied appreciably from one event to another, from 0.1 to 0.5, with a mean value of 0.25. The maximum value of this ratio for billows forming on a surface of discontinuity was predicted theoretically by Rosenhead (1931) to be rather less than 0.5.

3. Vertical structure of the atmosphere in the vicinity of the large-amplitude KH billows

The structure of the atmosphere has been determined from balloon-borne radiosondes which were released hourly over long periods from Pershore, 8 km north-east of the Defford radar. Temperature data were evaluated with the maximum resolution obtainable at height intervals of about 100 m, giving an r.m.s. error in $\Delta \theta$ of about 0.2°C. Height was evaluated from the pressure and temperature data. Corner reflectors accompanying each radiosonde were tracked with a precision (monopulse) radar to give horizontal position at 1-second intervals. The accuracy of the tracking data has been assessed by Wiley and Starr (1971), who found three principal sources of error. First, a r.m.s. range error of 6 m. Second, a r.m.s. error in azimuth of 1 minute of arc (worse at close range and occasionally also at longer ranges owing to radar maladjustment). And, third, an error due to the self-induced motion of the balloons, which were found to ascend in a helix of radius 8 m with a periodicity of 8 seconds. Taking all of these errors into account, it was found that the use of data at 20-second intervals permitted $\Delta V$ to be evaluated over height intervals of 200 or 400 m with an r.m.s. error usually about 0.5 m s$^{-1}$ at the ranges appropriate to this study.

The radiosondes reached the altitude of the billows (as given in Table 1), 15 to 27 min

* It is possible that some such layer echoes are associated with inhomogeneities of refractive index which remain after the turbulence has decayed (i.e. the so-called 'fossil' turbulence of Woods 1969), but evidence from joint radar-aircraft studies (Clover Boucher, Ottersten and Hardy 1969) indicates that clear air layer echoes in the middle and upper troposphere are almost always associated with a perceptible degree of turbulence, presumably originating from KHI.
### TABLE 1. SUMMARY OF BILLOW EVENTS

<table>
<thead>
<tr>
<th>Billow event</th>
<th>Date (1970)</th>
<th>Time of start (GMT)</th>
<th>Duration (minutes)</th>
<th>Max. amplitude crest-to-trough (m)</th>
<th>Wave-length (km)</th>
<th>Altitude (km)</th>
<th>Location (see note under Table)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A and B</td>
<td>3 Feb</td>
<td>1140</td>
<td>&gt; 240</td>
<td>450</td>
<td>1.5</td>
<td>10.7</td>
<td>Trj</td>
</tr>
<tr>
<td>C</td>
<td>6 Feb</td>
<td>1030</td>
<td>9</td>
<td>240</td>
<td>2</td>
<td>7.5</td>
<td>Fsl</td>
</tr>
<tr>
<td>D</td>
<td>6 Feb</td>
<td>1243</td>
<td>10</td>
<td>400</td>
<td>4</td>
<td>6.1</td>
<td>Fsl</td>
</tr>
<tr>
<td>E</td>
<td>6 Feb</td>
<td>1211</td>
<td>&lt; 2</td>
<td>300</td>
<td>0.8</td>
<td>9.5</td>
<td>Tsj</td>
</tr>
<tr>
<td>F</td>
<td>13 April</td>
<td>1205</td>
<td>13</td>
<td>220</td>
<td>2</td>
<td>6.6</td>
<td>Osj</td>
</tr>
<tr>
<td>G</td>
<td>13 April</td>
<td>1236</td>
<td>16</td>
<td>400</td>
<td>1.5</td>
<td>5.6</td>
<td>Osj</td>
</tr>
<tr>
<td>H</td>
<td>17 April</td>
<td>1001</td>
<td>11</td>
<td>380</td>
<td>1</td>
<td>5.8</td>
<td>Osj</td>
</tr>
<tr>
<td>I</td>
<td>17 April</td>
<td>1033</td>
<td>12</td>
<td>400</td>
<td>1.5</td>
<td>6.4</td>
<td>Osj</td>
</tr>
<tr>
<td>J</td>
<td>20 April</td>
<td>1528</td>
<td>&lt; 2</td>
<td>430</td>
<td>2.5</td>
<td>10.0</td>
<td>Tsj</td>
</tr>
<tr>
<td>K</td>
<td>20 April</td>
<td>1546</td>
<td>8</td>
<td>380</td>
<td>0.8</td>
<td>11.0</td>
<td>Tsj</td>
</tr>
<tr>
<td>L</td>
<td>20 April</td>
<td>1714</td>
<td>4</td>
<td>300</td>
<td>2</td>
<td>10.4</td>
<td>Tsj</td>
</tr>
<tr>
<td>M</td>
<td>29 June</td>
<td>2027</td>
<td>&lt; 2</td>
<td>220</td>
<td>1</td>
<td>6.5</td>
<td>Osj</td>
</tr>
<tr>
<td>N</td>
<td>29 June</td>
<td>2036</td>
<td>&lt; 2</td>
<td>400</td>
<td>1.5</td>
<td>8.0</td>
<td>Tsj</td>
</tr>
<tr>
<td>O</td>
<td>3 July</td>
<td>1041</td>
<td>5</td>
<td>340</td>
<td>3.5</td>
<td>7.7</td>
<td>Osj</td>
</tr>
<tr>
<td>P</td>
<td>3 July</td>
<td>1147</td>
<td>18</td>
<td>430</td>
<td>2</td>
<td>7.1</td>
<td>Osj</td>
</tr>
<tr>
<td>Q</td>
<td>10 July</td>
<td>1435</td>
<td>3</td>
<td>400</td>
<td>0.8</td>
<td>8.1</td>
<td>Osj</td>
</tr>
</tbody>
</table>

The location code is as follows:
(a) Position of billows with respect to features of the vertical structure: T = near tropopause, F = within major frontal zone, O = away from major stable layers.
(b) Position of billows with respect to pressure pattern at their altitudes: t = near trough, r = near ridge, s = within region of relatively straight flow between trough and ridge.
(c) Position of billows with respect to axis of main upper-tropospheric jet stream: = within ± 100 km of axis, l = on low-pressure side of axis, h = on high-pressure side of axis.

After release during which time they travelled 10 to 75 km downwind. During the same period, however, the billows also travelled downwind (at about the velocity of the wind at their mid-level). In the case of most of the billow events in this study, a radiosonde released when the billows were observed would have risen to within 20 km of the same billows (a rather larger separation of 40-50 km is applicable to events C, E, J and L). Accordingly, radiosonde data have been related to the radar observations of billows at times which are uncorrected for the time taken for sondes to ascend to a given level. An assumption implicit in the analysis is, of course, that the structure of the atmosphere in the vicinity of the moving patch of billows was not changing appreciably over the period of ascent of the radiosondes (typically 20 min).

Radiosonde data obtained in the vicinity of each of the major billow events listed in Table 1 have been evaluated to give vertical profiles of $\Delta \theta/\Delta Z$, $\Delta V/\Delta Z$ and Ri over layers 200 m deep. Time-height sections showing patterns of $\Delta V/\Delta Z$ and Ri in relation to 6 of these events are presented in Figs. 2, 3 and 4; patterns of $\Delta \theta/\Delta Z$ showed a rather less well-defined relationship to the billow events and the corresponding Figures are not reproduced here. The most obvious features illustrated by these Figures are that each large-amplitude billow event was associated with a separate and discrete region of maximum $\Delta V/\Delta Z$ (i.e. an inflexion in the velocity profile) and also with a minimum value of Ri. Apart from event K, which is discussed later, the minimum value of Ri in the vicinity of the events in these figures was close to 0.2. The maximum depth of strongly sheared regions with Ri < 0.5 varied from about 400 to 700 m, and their persistence in one locality varied from 2 to 4 hours. The time interval between the first appearance of
Ri < 0.5 and the initial detection of large-amplitude billows varied between 1 and 2 hours. Evidently, therefore, radiosonde ascents at roughly 1 hour intervals were usually adequate to detect the regions of low Ri associated with each event.

Data from the radiosondes nearest to each of the 17 billow events in Table 1 are summarized in Fig. 5 (i) to (iv). All radiosondes, the data from which are plotted in Fig. 5, were released within ±40 min of the corresponding event; for 10 of the events the sondes were released as close to the event as ±15 min. The layer occupied by the billows is
Figure 5. Caption on page 293.
Figure 5 continued. Caption on page 293.
Figure 5 continued. Caption on facing page.
Figure 5. Profiles of the vertical gradient of potential temperature $\Delta \theta/\Delta Z$, the magnitude of the vertical wind shear vector $\Delta V/\Delta Z$, and the layer Richardson number $R_i$, each evaluated over layers 200 m deep, for the 17 events of large-amplitude Kelvin-Helmholtz billows labelled A to Q in Table 1. (The event labelled ‘e’ had an amplitude of only 150 m and is not considered elsewhere in the analysis.) The layers occupied by the KHI billows are indicated by hatching, the thickness of the hatched regions being a measure of the crest-to-trough amplitude of the billows. The time of occurrence of billow events and the time of release of the radiosondes are indicated at the right of each diagram.

indicated in Fig. 5 by hatching. On a few occasions when the altitude of a layer of low $R_i$ was changing rapidly with time, the indicated height of the billow event has been adjusted slightly ($\leq 250$ m) to take into account the non-simultaneity of the radiosonde and radar observations. Any vertical displacements of the flow by gravity waves will also change $R_i$ locally, by a factor

$$f = \left(1 + \frac{\delta \eta}{\eta_0}\right)^{-2} \quad (3)$$

where $\eta_0$ is the initial component of vorticity due to the mean wind shear and

$$\delta \eta = \left(\frac{\partial \theta}{\partial z}\right) \frac{\delta z}{V}$$
is the change in vorticity due to a vertical displacement $\delta z$, $V$ being the phase velocity of the wave with respect to the local air motion (Scorer 1969). Typical values for the occasions studied are

$$\eta_0 = 3 \times 10^{-2} \text{s}^{-1} \quad \text{and} \quad \left( \frac{g \partial \theta}{\partial z} \right) = 3 \times 10^{-4} \text{s}^{-1}.$$ 

Assuming the waves to be standing (lee or mountain) waves, a representative value of $V$ for the occasions studied would be 40 m s$^{-1}$. Taking 100 m as a likely upper limit to $\delta z$, (based partly upon radar observations of the slopes of clear air echo layers) this gives a value of 0.95. Thus the values of $R_i$ in Fig. 5 are unlikely to have been significantly modified by standing gravity waves. However, travelling gravity waves would have had a more important effect on $R_i$ since the appropriate value of $V$ may be less than that for standing waves.

The results in Fig. 5 fail to show any very clearly defined relationship between the billows and static stability. In 4 cases the billow events occurred near a maximum of stability; in 8 cases they could be described as occurring on the edge of a stability maximum (stability increasing upwards in 6 cases and downwards in 2 cases); in the remaining 5 cases they occurred in regions of relatively uniform stability. On the other hand, as might be expected in the light of Figs. 2-4, Fig. 5 does show a generally rather clear relationship between the altitude of the billows and layers of maximum wind shear corresponding to inflexions in the velocity profile. Since $R_i$ depends on the inverse square of the shear and on the first power of the stability, it is therefore not surprising that there should also have been a generally clear relationship between the altitude of the billows and of layers of minimum $R_i$.

The magnitude of the wind shear maximum associated with the billow events varied considerably, from as little as 4 m s$^{-1}$ in 200 m in the case of billow event $I$ to 9 m s$^{-1}$ in 200 m in the case of events $A$ and $D$. However, the magnitude of the shear maxima were

![Figure 6](image-url)  

**Figure 6.** The maximum value of $\Delta V/\Delta Z$ over layers 200 m deep within the height interval occupied by the KH billows, plotted against the corresponding value of $\Delta \theta/\Delta Z$ for the 17 KH events in Fig. 5. Solid lines are isopleths of $R_i$ over the corresponding layers.
related to the magnitude of the stability in such a way that the minimum Ri at the level of the billows fell within relatively narrow limits, as indeed one might expect if one has more or less succeeded in identifying the critical Richardson number Ri. This point is brought out more clearly in Fig. 6 where data are plotted from all 17 billow events. Stability and shear, as plotted in Fig. 6, are the values occurring at the level of the smallest value of Ri within the height range occupied by the billows. In all cases the smallest Ri was produced by a maximum, or near-maximum, value of shear. Fig. 6 has been derived on the assumption that ΔZ was equal to 200 m for all of the billows events. Although the thickness of the dynamically unstable layer associated with the billow events was usually within a factor of 2 of this value, even a factor of 2 could be enough to lead to a significant error in estimating Ri from Fig. 6. Nevertheless, Fig. 6 shows that most of the billow events were consistent with Ri over 200 m layers being between 0.15 and 0.3, i.e. broadly consistent with the theoretical critical value of 0.25. The most notable exceptions were events E and K, for which Ri was significantly greater than 0.5.

An explanation for the anomalously large value of Ri measured for event E is suggested by the wind hodograph in Fig. 7. Events D and E as depicted in this Figure may be taken as representative of two extreme situations. Event D was characterized by a shear vector of almost constant orientation over a substantial height interval thereby indicating that a large-scale dynamical system was mainly responsible for the strong shear; event E, on the other hand, was characterized by a looped hodograph in which the orientation of the

Figure 7. Partial wind hodograph showing winds in the vicinity of billow events D and E (and e) on 6 February 1970. Heights are labelled at intervals of 0.2 km; levels of the billow events are shown by double lines.
shear vector was changing rapidly with height, thereby suggesting that the enhanced shear was due to a relatively small-scale feature. The small size of this feature is borne out in the case of event $E$ by the fact that wind profiles obtained about an hour before and after that depicted in Fig. 7 showed almost no evidence of the looped hodograph pattern, whereas major shear zones such as that responsible for event $D$ persisted for several hours (Fig. 2(a)). Thus events $E$ (and $K$) can probably be attributed to small-scale systems whose persistence in one locality was too short to permit them to be adequately observed using hourly radiosondes. Loop ed hodographs above the tropopause similar to that associated with event $E$ have been reported elsewhere by Sawyer (1961), Danielsen and Duquet (1967), and Axford (1968); Sawyer attributed them to inertia waves.

The data summarized in Fig. 6 have also been evaluated over layers 400 m deep, with the results shown in Fig. 8. One advantage of making measurements over larger depths is that their accuracy is enhanced; at the same time, however, as soon as the depth begins to significantly exceed the thickness of the dynamically unstable layer, Ri can be expected to increase to above-critical values. It is interesting, however, that the increase in Ri was usually less than a factor of 2; indeed Ri for 9 of the billow events was still consistent with a value between 0.15 and 0.3. This was a result of the billows usually being embedded within relatively deep shear layers in which the orientation of the shear vector was fairly constant with height (e.g. event $D$ in Fig. 7). Events $A$ and $B$, are particularly noteworthy for the fact that Ri over 400 m remained significantly less than 0.2. Both of these events were associated with the persistent large-amplitude billows on 3 February, and it is suggested that Ri on this occasion was being maintained at sub-critical values by a strong deformation field associated with the large-scale flow (Roach 1970). A strong deformation field has in fact been measured within an intense low-level warm-frontal zone by Browning, Harrold and Starr (1970); on that occasion Ri over 200 m layers was found to be maintained persistently in the range 0.2 to 0.4. These values of Ri are rather larger than those measured.

Figure 8. The data of Fig. 6 evaluated over layers 400 m deep instead of 200 m.
in the 3 February study, possibly because the KHI was occurring on smaller vertical scales. Unfortunately, Browning et al. (1970) were unable to establish reliably the vertical scale of individual occurrences of KHI; however, the strong variation with height in the orientation of the shear vector within the frontal zone as it neared the ground was a factor tending to inhibit deep KHI on that occasion.

4. Conclusions

The analysis in this paper has dealt with large-amplitude Kelvin-Helmholtz billows observed by radar in clear air at heights between 5·6 and 10·7 km. Seventeen separate occurrences (events) have been studied, with crest-to-trough amplitudes between 220 and 450 m and wavelengths between 0·8 and 4 km. The mean depth of the associated dynamically unstable layers implied by Eq. (1) was 240 m. Radiosonde measurements in the vicinity of these events have been evaluated to give static stability and shear over 200 m layers, corresponding roughly to the probable depth of the dynamically unstable layers. The resulting Richardson numbers (Ri) over layers 200 m deep have been taken as an approximate measure of the critical Richardson number (Ri_c). All of the KH events were found to be clearly associated with well-defined local maxima in the vertical wind shear, with a vertical scale of about 600 m. The dynamically unstable layers were embedded within, and formed only a fraction of, the overall layers of strong shear.

The persistence in one locality of the mesoscale wind shear maxima associated with the KH events was usually (but not invariably) long enough to permit them to be measured satisfactorily by radiosondes released at intervals of 1 hour. The magnitude of the wind shear maxima varied considerably from one event to another, from as little as 4 m s^{-1} in 200 m to 9 m s^{-1} in 200 m. The largest shears were associated with large static stabilities in such a way that the minimum value of Ri over 200 m layers for these events fell within a comparatively narrow range. Apart from events associated with shear zones of very small scale, the minimum Ri fell within the range 0·15 - 0·3, broadly consistent with the theoretically predicted Ri_c = 0·25. When evaluated over layers 400 m deep instead of 200 m, Ri often increased only slightly, remaining in the range of 0·15 - 0·3 for 9 out of the 17 events. On most of the occasions studied the large-amplitude billows were isolated events believed to have been associated with local transitions from smooth to turbulent flow on the scale of the billows. However, on the occasion of events A and B, billows were observed persistently for 4 hours with a value of Ri as small as 0·15. Since the tendency is for billows to increase Ri to above-critical values, the implication in the case of events A and B is that turbulence was being maintained in the presence of a sub-critical value of Ri by means of the large-scale wind field.

The intensity of the turbulence which results from the breaking of KH billows depends on the amplitude of the billows and on the vertical wind shear across the depth of the billows. Aircraft measurements in the vicinity of one of the events studied (event A) indicated moderate, almost severe, CAT (Browning, Watkins, Starr, and McPherson 1970); whereas most of the events were associated with rather weaker shear, four other events in this study (B, D, J and L) probably fell in the same turbulence category. Although there appeared to be a well-defined lower limit to the wind shear associated with the events in this study, this does not imply that large-amplitude billows do not occur in the presence of smaller shear; rather, it merely implies that the refractivity fluctuations associated with billows in weakly sheared situations are too weak to permit them to be detected by the present radar.

Finally, a distinction should be made between the data in the present paper and those in a recent paper by Atlas, Metcalf, Richter and Gossard (1970). Each of the KH events reported here occurred in association with a minimum value of Ri embedded within a strongly sheared stable layer; the wave-induced turbulence reported by Atlas et al. on the other hand, occurred at the top of the planetary boundary layer on the edge of a region of
near-neutral static stability. The observations described in the present paper are considered to be more representative of the kind of event responsible for intense CAT.

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