Coalescence in a turbulent cloud

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SUMMARY

The results of two experiments show that the collision efficiency of small droplets is significantly increased when the interacting drops fall through a laminar shear flow of about 10 s\(^{-1}\). The structure of microturbulence in clouds is considered and it is shown that shears of this magnitude are likely to occur only within a narrow spectral band. This peak in the shear spectrum occurs at a scale that is much larger than the interaction distance of cloud droplets having radii of about 20 \(\mu\text{m}\). Hence it is a good approximation to describe droplet collisions in a turbulent cloud as though they occurred in a steady laminar shear flow. It is argued that this enhanced droplet coalescence in turbulent clouds may account for the observed development of large cloud droplets (radii < 30 \(\mu\text{m}\)) in cumulus clouds at rates greater than can be explained by condensation and the efficiency of still-air coalescence.

1. INTRODUCTION

It is well established that rain may fall from clouds which are too warm for growth processes involving the ice phase to be operative. Furthermore, it has been pointed out (Levin and Sedunov 1965; Mason 1969) that currently available models of the condensation-coalescence mechanism not only fail to account for this warm rain but also cannot account for the measured spread of cloud droplet size spectra. The problem has been presented in terms of an apparent gap between the effectiveness of the condensation and coalescence mechanisms as they are currently understood. This gap occurs at droplet radii of 15–20 \(\mu\text{m}\). On the one hand, the initial growth rate of droplets by condensation becomes very slow (compared with the lifetime of a typical cumulus cloud cell) as their radii approach 15 \(\mu\text{m}\); while, on the other hand, the growth by coalescence of droplets smaller than 20 \(\mu\text{m}\) is very slow because (a) they fall slowly and therefore encounter each other less frequently (Ludlam 1951) and (b) when they do meet the droplets have low collection efficiencies, as shown theoretically by Hocking (1959), Davis and Sartor (1967), Hocking and Jonas (1970) and confirmed experimentally by Picknett (1960) and Woods and Mason (1964).

Recent unpublished calculations by Dr. J. T. Bartlett indicate that if the collection efficiencies for droplets in the size range 15 to 20 \(\mu\text{m}\) were unity, their low encounter rate would not be a decisive factor preventing the evolution of the observed broad spectrum of droplet sizes. For instance, in such circumstances, a 20 \(\mu\text{m}\) droplet would take about 800 s to grow to 25 \(\mu\text{m}\) by coalescence in a cloud composed of 10 \(\mu\text{m}\) radius droplets with a water content of 1 g m\(^{-3}\), whilst a 15 \(\mu\text{m}\) droplet would take 1,200 s to grow to 20 \(\mu\text{m}\) in a similar cloud. Bartlett and Jonas (1972) have shown also that, even when turbulent mixing is allowed for, growth by condensation cannot alone bridge the gap. The remaining possibility is that the collection efficiencies are very much greater in a cumulus cloud than in laboratory conditions. Two properties of clouds which might increase the collection efficiency have been omitted from the simple condensation-coalescence model, namely electrification and turbulence. The effect that electrical forces have upon small droplet collisions have been studied theoretically by Sartor (1957), Krasnogorskaya (1965) and others; and experimentally by Woods (1965) and Latham (1969). It is concluded that the electrical charges and fields existing in warm (i.e. ice free) clouds whose droplets are still growing principally by condensation will be too small to increase significantly the coalescence rate for drops with radii less than 20 \(\mu\text{m}\).

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The aim of this paper is to show how turbulent motions in a typical cumulus cloud may significantly increase collection efficiencies of collector drops with radii in the range 15–20 μm. We shall show below that the microstructure of turbulence in clouds can be represented so far as small droplet coalescence is concerned by a region of linear sheared air flow and that the collision efficiency of small droplets is enhanced in such flow.

This effect was not revealed by the theoretical models of Gabilly (1949), East and Marshall (1954), Sedunov (1963a, b) and Levin and Sedunov (1965, 1967), who all concluded that turbulent coalescence would not close the gap between droplet growth by condensation and still-air coalescence.

2. TURBULENCE IN CLOUDS

Very little is known about turbulence in clouds. The simplest theoretical model (e.g. Batchelor 1953) assumes that the turbulence is homogeneous and isotropic and that its average properties may be derived by means of similarity theory from observations of the scale and overturning speed of the largest eddies seen by an external observer. This model which forms the basis for the work of Saffman and Turner (1956), Bartlett and Jonas (1972) and others, leads to the conclusion that the average rate of turbulent kinetic energy dissipation is of the order of 1,000 erg g⁻¹s⁻¹ in cumulus clouds whose droplet size spectra have developed widths which hitherto have not been understood because of the apparent gap in the effectiveness of the condensation-coalescence processes. The maximum shear in a field of turbulence occurs on scales close to the Kolmogoroff length scale at which the Reynolds number is unity, \( \lambda_0 = (\nu^3/\epsilon)^{1/4} \). At smaller wavenumbers, in the inertial subrange, where the power spectrum is given by Eq. (1),

\[
E(k) = 0.47 \epsilon^{2/3} k^{-5/3} \tag{1}
\]

the variation of shears is given by similarity theory

\[
S(k) \approx (1/n) (ek^2)^{1/3} \tag{2}
\]

where \( S(k) \) is the average shear over \( \lambda/2 = \pi/k \).

At wavenumbers larger than the Kolmogoroff scale, in the viscous subrange, where the power spectrum is given by Eq. (3),

\[
E(k) \approx 500 \epsilon^2 k^{-7} \tag{3}
\]

the variation of shears is given by similarity theory

\[
S(k) \approx \epsilon/2k^2 \tag{4}
\]

Fig 1 shows the shear spectra for \( \epsilon = 10^2, 10^3, 10^4 \) erg g⁻¹s⁻¹ for which \( \lambda_0 = 0.7, 0.4 \) and 0.2 mm respectively.

This simple model of cloud turbulence is sufficient for the purposes of this paper, but it is relevant to consider here the factors that might be included in a more detailed model, and to see, in a qualitative way, what effects they may have on droplet coalescence.

First, the assumption has been made in the simple model that all the turbulent kinetic energy originates on the scale of the large thermals visible to an observer outside the cloud. In fact, there is release of buoyancy over a wide range of scales down to one metre or smaller. Turbulence, therefore, receives energy over this wide range of scales and the energy at the smallest scales may be larger than one estimates on the basis of an analysis of the dimensions of large thermals.

Second, the microturbulence is not homogeneous, but highly intermittent (Batchelor and Townsend 1949). Several authors, notably Novikov and Stewart (1964) and Pond and Stewart (1965), have developed theoretical models for the spectrum of the fluctuations of the energy dissipation in turbulent flow. It is pointed out later that the availability in a cloud of rare, extra strong shears due to intermittency may impose some limit on the laminar shear assumption made in this paper.
Figure 1. Shear spectra for isotropic homogeneous turbulence for three energy dissipation rates. The wave number, equivalent to the Kolmogoroff scale length, is defined by \( k_s = 2\pi (\epsilon / v^3)^{1/4} \) and the maximum shear occurs when \( \log (k/k_s) = 0.75 \) or \( k = 0.18 \ k_s \).

3. The Motion of Colliding Droplets in Microturbulence

The interaction distance \( l \) that two colliding drops with radii \( R \) and \( r \) fall through the air during their hydrodynamic interaction is given by

\[
I = \frac{U n R}{(U - u)}
\]  

(5)

where \( U \) and \( u \) are the fall speeds of the large and small drops respectively and the hydrodynamic interaction is negligible when they are separated by a vertical distance \( nR \). Hocking (1959) took \( n = 10 \) for small droplets with \( R < 20 \ \mu m \). Similarly the interaction time is given by:

\[
t = \frac{n R}{(U - u)}.
\]  

(6)

Substitution of realistic values in Eqs. (5) and (6) shows that droplet collisions occur within a length and time scale much smaller than the Kolmogoroff scale; turbulent eddies having similar length and time scales lie in the viscous subrange and the turbulent shears on these scales are very much weaker than the peak values in the vicinity of the Kolmogoroff length scale as is apparent in Fig. 1. Thus droplet collisions in a turbulent cloud occur effectively within a constant, uniform, laminar shear.

It was because of this conclusion, that experiments to measure the collision efficiency of droplets falling through a laminar shear flow were designed. i.e. \( E(R, r, S) \) where \( S \) is the shear tensor. The transition from \( E(R, r, S) \) to \( E(R, r, \epsilon) \) is achieved in the simple, uniform \( \epsilon \), model of cloud turbulence by assuming that the mean magnitude of the shear is present everywhere in the cloud and that the two directions characterizing the shear are randomly orientated.

In this simple model, there is no need to consider the spectral distribution of the
turbulent shears since they occur in a wavenumber band well separated from the response band of the drop collisions. However, when more detailed experimental or theoretical values are available for $E(R, r, S)$ it will be necessary to consider the effects of intermittency, which depends strongly on wavenumber. In this more detailed model, it may be argued that a small fraction of the cloud volume will contain shears much larger than average for a given wavenumber. So the laminar shear assumption made above is seen to be only an approximation, though it should not lead to serious errors. The principal effect of intermittency will be to replace the concept of a threshold $\epsilon$, implicit in the simple model, by a gradual increase in $E(R, r, \epsilon)$ starting at very small $\epsilon$.

4. The experimental studies

(a) Coalescence of drops with artificially imposed relative speeds using streak photography

In the first experiment, the collision and coalescence of individual droplets in the upper boundary layer of a wind-tunnel similar to that shown in Fig. 2 but of uniform cross-section, was examined by means of the streak photography method developed by Woods and Mason (1965). The mono-disperse stream of 'collector' drops, generated by a vibrating needle (Mason, Jayaratne and Woods 1963) enter the tunnel through a tiny hole at close to their terminal velocity. As they accelerate to the horizontal speed of the air in the tunnel they encounter a cloud of small droplets (of size distribution shown in Fig. 3) from an MEL 100 kHz ultrasonic atomiser.

![Figure 2. Apparatus to investigate the coalescence of droplets in a linear shear.](image)

The small droplets have sufficient time to accelerate beyond the initial transient response to the horizontal wind, so their horizontal speed lags behind the wind by a calculable amount due to their descent at terminal velocity through the boundary layer shear. The wind velocity profile across the tunnel was derived from the length of the droplet streaks after applying a correction for this small lag.

The two streams of drops intersected within a small area whose centre could be adjusted to lie at various depths below the roof of the tunnel. If the depth was small, then the transient horizontal acceleration of the larger drops was not complete and their horizontal speed lagged the wind by more than the equilibrium lag achieved if the intersection point was deeper in the tunnel. Thus the horizontal acceleration of the small droplets is nearly constant at the equilibrium value, while the horizontal acceleration of the collecting drops could be varied somewhat by adjusting the point of intersection of the two streams.
Figure 4. (a) Streak photograph of interacting droplets showing, at the intersection of the arrows, an interaction not ending in a coalescence; (b) streak photograph of interacting droplets ending in a coalescence.
of drops. The virtue of the streak photography method is that the actual acceleration of each droplet may be measured in any selected event; the disadvantage is that every photograph must be scanned by eye, a slow and laborious task. The streak method is valuable in that it reveals precisely what can happen, but it is not well suited to the statistical analysis of rare events. Consequently, it was decided to start by photographing the collisions of droplets which would have a finite collection efficiency even in still air. The droplet sizes chosen for this preliminary experiment were $R = 40 \mu m$ and $\gamma = 9 \pm 3 \mu m$, for which Shafir and Neiburger (1963) predict a mean collection efficiency of approximately 40 per cent.

Fig. 4(a) shows the encounter of a large and a small drop that does not end in coalescence and Fig. 4(b) shows a collision and coalescence. These events occurred in a shear of approximately $1 \text{ s}^{-1}$, the horizontal acceleration of the large drop is approximately $10^3 \text{ cm s}^{-2}$ and the small drop is approximately $1 \text{ cm s}^{-2}$.

The following statistical analysis of the streak photographs was used to derive a value for the collection efficiency $E(R, \gamma, (du/dt)_R, (du/dt)_\gamma)$ in the above case. The method recognizes that we have time information only for those drops the start of whose streaks are visible within the field of view of the photograph (as in Fig. 4). Nor do we have information concerning the co-ordinates of the drops along the $Y$ direction.* Thus the termination of a small droplet streak at some position along the length of a large drop streak implies only that the two droplets shared common ($X, Z$) co-ordinates, but not necessarily $Y$ and $t$. Some of the small streak terminations will share the full ($X, Y, Z, t$) co-ordinates with the large streak, and this implies coalescence; the remainder, having the same ($X, Z$), but different ($Y, t$) do not necessarily imply an encounter, let alone coalescence. However, statistically we may equate this latter category with the number of occasions in which a small streak starts in coincidence with a large streak, there being no such process as negative coalescence (i.e. drops of radius less than about 0.3 cm do not spontaneously disintegrate). The procedure then is to count the number $N_f$ of small streaks terminating in a large streak and the number $N_s$ of small streaks starting in a large streak.

*Footnote

X is taken in the downwind direction
Y is in the direction of the optical axis of the camera
Z is the upward pointing vertical
Then

\[ E(R, \tau, (du/dt)_R, (du/dt)_\tau) = \frac{(N_1 - N_2)aA}{N_\bar{n}L(R + \tau)^2F} \]  \hspace{1cm} (10)

where
- \( F \) number of frames analysed
- \( a \) mean thickness of cloud, measured in \( Y \) direction
- \( A \) area of camera field of view
- \( N_1, N_\bar{n} \) average number of large and small droplets in each field of view
- \( L \) average length of the long streaks
- \( R, \tau \) radius of large drops and average radius of small droplets.

In the example illustrated in Fig. 4(a) and (b) the collection efficiency \( E(R, \tau, (du/dt)_R, (du/dt)_\tau) = E(40, 9 \pm 3 \mu \text{m}, 10^3, 1 \text{ cm s}^{-2}) = 90 \text{ per cent} \pm 30 \text{ per cent}. \) This value is significantly greater than Shafir and Neiburger's (1963) value for still air \( E(40, 9 \mu \text{m}; 0, 0) = 40 \text{ per cent}. \)

The result of this experiment was considered to be encouraging enough to justify a detailed experimental study with realistic laminar shears and much smaller collector drops.

(b) Coalescence in a linear shear using radioactive tracer methods

(i) Generation of linear shear. The technique of shear inducement used in the experiments described in the remainder of this paper was based on the velocity profile across the boundary of a semi-free jet. Fig. 2 illustrates the wind tunnel together with the associated droplet generators, whilst Fig. 5 gives details of the working section as finally modified for this method. The wind tunnel section was 2\( \frac{1}{2} \) in. \( \times \) 4 in. except in the region immediately following point A. The jet boundary formed by the sudden increase in dimension at A was prevented from free development by the curved constrictor shown. For mean tunnel velocities below approximately 30 cm s\(^{-1}\) no instabilities developed in the jet boundary for a distance of at least 10 cm downstream from point A. The velocity streamlines showed a slight curvature as indicated, and a closed convection loop developed in the enlargement space after A. The velocity in this loop was typically less than 0.02 \( V_m \) where \( V_m \) is the mean velocity in the 6 cm \( \times \) 10 cm tunnel section.

In order to map the velocity profile in the jet boundary, the droplets from the 100 kHz atomiser were 'streak photographed' in the manner described in Section 4(a). This technique only gave knowledge of velocities in the region occupied by the small droplets. However, since interactions between drops would only take place in this region during the main experiment, then, providing the relaxation time for a water drop falling into a region of changing velocity is small, no significant error will be introduced by interpolation of the velocity results for a small distance either side of the mapped area.

![Diagram](image-url)  

Figure 5. The airflow in the wind tunnel in the working section.
It was found that for mean tunnel velocities in the range \(10 \text{ cm s}^{-1} < V_m < 30 \text{ cm s}^{-1}\) the shear developed across the region occupied by the small drops was sensibly constant (i.e. linear increase in velocity with distance from top of mapped area). For distances greater than about 3 cm downstream from A, the magnitude of the shear was sensibly independent of distance from A, and could be represented by a linear increase from zero velocity to \(V_m\) over a depth of 1 cm and independent of the value of \(V_m\) in the stated velocity range. A velocity map constructed from a large number of streak photographs is shown in Fig. 6 for one portion of the jet boundary.

(ii) The radioactive tracer experiment. The experimental apparatus was set up as shown diagrammatically in Fig. 2. A cloud of small water droplets generated by the 100 kHz atomiser was allowed to enter the tunnel at X from the otherwise airtight housing. These droplets were carried into the jet boundary in a thin layer approximately 2 cm wide by 0-5 cm deep. A second stream of larger drops obtained by the vibrating needle technique entered the tunnel through an opening Y and fell vertically towards the shear zone. On entering the tunnel, this stream of droplets had a horizontal dispersion of approximately 0-5 cm. As the larger drops entered the jet they were deflected in the horizontal, forming a curved trajectory that intersected the thin layer of smaller drops.

For the radioactive tracer experiments the wind tunnel was set up with its air intake and exhaust mounted in a fume cupboard. Because of the confined space no satisfactory streak photography of the flow could be made in situ, and so it was assumed that the flow kinetics were unchanged.

A few ml of radio-active solution were prepared by 'milking' a 10 mCi radio-thorium source of thoron gas. An airflow of about 400 ml min\(^{-1}\) was passed over the radio-thorium and through a first filter to avoid any possible carry over of the long-lived radio-thorium. The flow then passed through a delay volume in series with a second millipore (AA) filter. The emanating thoron gas passed through the first filter, but decayed through Th A (polonium 216) to Th B (lead 212) whilst passing through the delay volume. The Th B is effectively captured by the second filter. The decay chain of thoron is

\[
\begin{align*}
\text{Th} & \xrightarrow{\alpha} \text{ThA} \xrightarrow{\alpha} \text{ThB} \xrightarrow{\beta \gamma} \text{ThC} \\
54.5\text{s} & \quad 0.16\text{s} \quad 10.6\text{h} \quad 60.5\text{min} \\
\text{ThC} & \xrightarrow{\beta \gamma} \text{ThC}’ \xrightarrow{\alpha} \text{Pb208}
\end{align*}
\]

After carrying out this milking process overnight, the second filter was removed and the Th B transferred to solution by soaking in 1 to 2 ml of acidified 10 mg 1\(^{-1}\) lead nitrate solution for a few minutes. This soaking procedure was repeated when over 50 per cent of the Th B activity was in solution. These solutions were thoroughly mixed to give a final volume of from 2 to 4 ml with an activity of about 1 mCi of Th B per ml. Dilute control samples were prepared with 10 \(\mu\)g 1\(^{-1}\) carrier solution of Pb(NO\(_3\))\(_2\) and these were counted at intervals throughout the experimental period to check that the activity had the 10-6 hour half life of Th B.

The highly radioactive solution was used as the feed liquid for the ultrasonic atomiser. This atomiser gave a typical size distribution of 9 \(\pm\) 3 \(\mu\)m, as illustrated in Fig. 3, provided the liquid flow rate was below 0.25 ml min\(^{-1}\); this rate was never exceeded during the present experiments.

For given wind tunnel speed, the fraction of drops from the atomiser that entered the tunnel was determined by a weighing technique. With the best possible control obtainable it was found that 20 per cent \(\pm\) 10 per cent of the liquid supplied to the atomiser actually entered the wind tunnel in the form of water drops. This wide margin of doubt accounts for the major quantitative uncertainties in the experimental results.

For a given set of experimental conditions, the landing positions of drops produced from the vibrating needle were calibrated against their radius before entering the wind tunnel. The drops landed on an absorbent paper placed on the floor of the wind tunnel down-
stream from the inlet region. The drops were marked by using a very dilute solution of methyl green, and they left a green stain on the paper. Calculations and measurements showed that no significant evaporation would take place for drops larger than 15 μm radius between the time of entering the tunnel and striking the filter (maximum 4 s). The landing positions had a horizontal spread of ±1 cm for drops of a fixed size, due to dispersion of the droplet stream as it entered the tunnel. The landing position calibration is shown on the deposition distributions later in this paper.

The method of operation was as follows. The tunnel and associated apparatus was thoroughly cleaned to remove radio-active contamination from previous experiments. A foil backed filter paper, grid marked with 1 cm squares, was taped to the bottom to the wind tunnel downstream from the drop inlet positions. The filter paper was covered with a loose mask. The tunnel velocity was set and a smoke test carried out to demonstrate stability. The vibrating needle was adjusted to give a stable drop stream of the appropriate size, determined by the landing position on the filter cover. The ultrasonic atomiser was switched on and allowed to run for a short time until a small quantity of dyed water immediately preceding the arrival of active liquid had almost been discharged. Providing all systems were operating smoothly, the supply of liquid to the atomiser was interrupted whilst the loose filter cover was removed. The active liquid supply was restarted and the experiment run until no further active liquid could be forced through the atomiser.

![Figure 6. Velocity map for part of the working section of the wind tunnel for a mean velocity of 15 cm s⁻¹ showing contours of constant relative streak length. The figures represent the relative streak lengths at the points indicated.](image)

During the run it was sometimes necessary to shut down for short periods when droplet supplies intermittently failed, or the two streams did not interact. The behaviour of both droplet types was closely observed during the entire run, and the landing positions of the larger drops noted. When no further active liquid remained the experiment was stopped. The filter was removed and covered entirely with clear sellotape. The deposition pattern of green dye was noted. The filter paper was divided into 1 cm wide strips across its width and these were each counted for 100 s in a conventional well scintillation counter. Some of the strips were sub-divided into cm squares and counted.

(iii) Calculation of coalescence efficiencies. Consider a drop of radius $R$ falling through a cloud of smaller droplets of radius $r$ having uniform number density $n$. Let the cloud have a depth $\delta$ entirely within the shear-region. Then the interaction time is given by $\delta/(V_1 - V_2)$ where $V_1$ and $V_2$ are the respective terminal velocities.

The number of geometric encounters, $N$, of drops of the two sizes is then given by

$$N = n \pi (r + R)^2 V_{rel} \delta/(V_1 - V_2). \quad (11)$$
where $V_{\text{Rel}}$ is the relative velocity of approach of the two drops. $V_{\text{Rel}}$ is very nearly equal to $(V_1 - V_2)$, the maximum error for the present experimental conditions in assuming this equality being less than 2 per cent. This is negligible compared with other experimental errors.

For an experiment lasting time $t$, having a frequency of large drop generation $f$, the total number of geometric encounters is given by

$$N_{\text{total}} = f t n \pi (r + R)^2 \delta$$

If the encounters are characterized by a coalescence efficiency $E$, and the small drops have a specific radioactive activity $a$, then the total radioactive deposition on a filter designed to collect all coalescing pairs will be given by

$$\text{Disintegration per unit time} = \frac{4}{3} \pi^2 t \tau^3 (r + R)^2 f n \delta E a \quad . \quad (12)$$

Hence, by measuring the radioactivity associated with deposition peaks on the filter, the value of $E$ can be calculated for given experimental conditions.

*(iii) Analysis of results.* The major source of error in determining the quantities on the right-hand side of Eq. (12) lies in estimation of the value of $n$, the number of drops in the active cloud. The atomiser produced a 'sonic wind' that carried the drops away from the vibrating head, and it was necessary to allow them to fall several centimetres in order that they settled into the wind tunnel at approximately terminal velocity. Most of the drop population was lost to the walls and floor of the atomiser housing. The preliminary experiments showed that 20 per cent $\pm$ 10 per cent of the total liquid supplied to the atomizer actually entered the wind tunnel as a cloud.

The depth of the cloud layer in the region of interaction was estimated to be 0.5 $\pm$ 0.1 cm. Calculation shows that the theoretical depth due to the finite length of the inlet orifice, and assuming entry at terminal velocity, is approximately 0.2 cm. The spread due to the difference in terminal velocity of a 8 $\mu$m and 11 $\mu$m radius drop gives an additional 0.3 cm. For this reason, the cloud will not have a uniform drop density over its depth, the majority of the droplets being concentrated in the upper layers of the cloud. The value of $\delta$ used in the calculation below assumes that the cloud could be represented by a uniform distribution of uniform size drops spread over a depth of 0.3 cm, and having a mean velocity appropriate to the drops at the centre of this layer. The errors introduced by these assumptions are estimated to be much smaller than the uncertainty in the amount of active liquid entering the tunnel.

Where more than one drop size was produced by the vibrating needle during the course of an experiment, the value of $t$ appropriate to each size was recorded. The maximum error is $\pm$ 10 per cent. The error in $R$ determined from the landing positions on the filter is $\pm$ 1 $\mu$m, due to dispersion of the drops as they entered the wind tunnel and the slight drift in radius inherent with the vibrating needle when used over long periods.

In each experiment, radioactive deposition on the filter was confined to a central strip in line with the interaction region of the two droplet streams. The deposition pattern was slightly wider than the small droplet cloud, but subsequent tests showed that some spread of liquid took place on the filter. In most cases the peaks in distributions of activity corresponded to the most intense deposition of green dye and also to the noted landing regions of the large drops. The exceptions to this took place when the filter was sufficiently long to capture a significant number of small drops from the atomizer, but the two regions of interest did not overlap.

Several plots of the distribution of radioactivity were made from dummy runs in which no large drops from the vibrating needle were generated. A typical distribution is shown in Fig. 7. The shape of the various backgrounds did not match the shapes of experimental results with large drops present even when all other conditions were apparently identical. Nor were any two background runs similar. However, all the background runs produced reasonably smooth distributions without large peaks. The total counts summed for each background could be accounted for by assuming that the atomiser produced a drop-size
distribution with a long tail, such that one drop in ten thousand has a radius $15 \ \mu m < r < 20 \ \mu m$. The direct sampling technique did not allow detection of drops in this quantity. It is assumed that the production of drops in this tail would vary with each experiment due to slight variations in feed conditions or alignment of the atomiser and that this accounts for the somewhat larger than hoped for magnitude and the variable shape of the background distribution.

Fig. 8 shows the counts for 100 s on each 1 cm strip across the filter as a function of distance from the vibrating needle drop entry point. The counts have been corrected for radio-active decay to a 'zero time'. A scale of drop sizes, for the larger drops from the vibrating needle, is shown at the top of the Figure. Also shown is the green dye deposition region and the noted estimation of the percentage of large drops falling in a given region. The mean tunnel velocity was 26·6 cm s$^{-1}$. During this run some instabilities developed in

![Figure 7](image_url)  
Figure 7. Radioactivity deposited along the floor of the wind tunnel during a 'background' run with radioactively tagged small droplets only.

![Figure 8](image_url)  
Figure 8. Deposited radioactivity along the wind tunnel floor. The shaded portion represents the activity associated with larger drops (23 $\mu m$) due to their interactions with radioactively tagged small droplets.
the jet for a short period, and a large wave motion was set up downstream from the curved flow constrictor. Some of the active small drops were deposited on the filter in the region greater than 49 cm from the origin, and these account for the large peaks in activity in this region. None of the larger drops landed in this region, and no small drops landed outside this region. Consequently, the peak occurring around 40 cm from the origin is considered to be associated with the larger drops due to interaction with the smaller drops.

Fig. 9 shows a distribution for a mean tunnel velocity of 17-5 cm s\(^{-1}\). No instabilities developed during the run for this lower velocity. However, the vibrating needle changed mode several times during the experiment and produced four large drop sizes. The peaks in activity corresponded with the four landing regions indicated by green dye deposition and continuous observation during the experiment. The percentage distribution of large drops in each peak was determined by timing the various vibration modes. There were two regions, where zero or very little green dye deposition took place, and these justify drawing the mean background distribution level through these points.

![Graph showing deposited radioactivity along the wind tunnel floor. The shaded regions represent activity associated with different large drop sizes.](image)

Fig. 10 shows a further distribution of counts for a mean tunnel velocity of 17-5 cm s\(^{-1}\). A much broader peak was produced due to drifting in size of the larger drops from the vibrating needle. Even with continuous manipulation of the controls the drop size could only be kept within a radius range of 3 μm on this occasion.

These figures illustrate the great variability in distribution shape that can arise due to variations in droplet and wind tunnel control. In assigning a radius to the ‘collector’ drop, it is assumed that the actual landing positions in the radioactive peaks are for collector drops making single coalescences with smaller drops. The collector size is then determined from the volume relationship.

Table 1 summarizes the results obtained. Details of the calculation of the first result from Fig. 8 are given below.

A significant peak in radioactive counts occurs at a distance of 41 cm from the larger drop entry position. The landing radius is 23-5 μm. The mean small drop radius is 9-5 μm. Using a volume relationship, this gives a ‘collector’ drop size of 23 μm radius.

The mean rate of supply of active liquid to the atomiser was 0-08 ml min\(^{-1}\), of which 20 per cent entered the tunnel. This gives the number of drops of 9-5 μm mean radius entering the tunnel as 7-4 × 10\(^4\) drops per second.
The wind shear, $S$, on this occasion was 26·6 $s^{-1}$, and the cloud of small droplets had a width of 2·0 cm and effective depth of 0·3 cm in the interaction zone. The mean drop velocity at the centre of the layer is calculated to be $21 \pm 1$ cm $s^{-1}$. These figures give a value $n = 5·9 \times 10^3$ drops per ml.

The experiment ran for a total period of 10 minutes with a 'collector' drop frequency of 1,000 per s. The droplet streams interacted, and the large drops landed in the significant region of the filter corresponding to the peak in activity, for 70 per cent $\pm$ 5 per cent of the total time.

Calculation of $V_{rel}$ for the two drop sizes, together with the difference in terminal velocities, gives $V_{rel} = 5·6$ cm $s^{-1}$. The difference in terminal velocity, $(V_R - V_l) = 5·5$ cm $s^{-1}$.

The specific activity of the small drop supply was $7·28 \times 10^8$ counts per 100 seconds per ml. The counts for 100 s above the background, in the significant peak of Fig. 9 is $10·5 \times 10^5$ counts per 100 s.

Substituting these values into Eq (12) then give $E = 16$ per cent. The extreme limits of error for $E$ calculated from all sources give 5 per cent $< E < 40$ per cent.

(u) *Discussion of results.* Both sets of droplets used in these experiments were prepared from dilute solutions rather than pure water. The calculation performed by Hocking (1959) and Hocking and Jonas (1970) indicate that the collision efficiency for water drops in still air is an insensitive function of surface tension, and the experimentally determined coalescence efficiencies are not likely to be significantly modified by the use of pure water drops.

In Table 2, the results of these experiments are compared with the theoretical findings of Hocking (1959), Davis and Sartor (1967) and Hocking and Jonas (1970) for the collision efficiencies in still air. It is seen that despite the wide margin of error in the experimental coalescence efficiencies in shear flow, they are nevertheless, significantly larger than those for still air interactions for collector drops in the critical 15 to 25 $\mu$m size range.

5. DISCUSSION

The results of the experiments described in this paper show that the collection efficiency of small droplets is significantly increased when they interact within a steady linear shear flow of about 10 $s^{-1}$. A further series of experiments to be published shortly by Jonas and Goldsmith will confirm and extend these results.

The complicated interaction between the colliding droplets and the shear is currently
### TABLE 1. Coalescence efficiencies

<table>
<thead>
<tr>
<th>R (mean) μm</th>
<th>τ (mean) μm</th>
<th>counts per 100 s per ml</th>
<th>n drops per cm$^3$</th>
<th>δ cm</th>
<th>Collector drop frequency per s</th>
<th>Length of experiment t s</th>
<th>counts in peak per 100 s</th>
<th>τ/R</th>
<th>Shear s$^{-1}$</th>
<th>(calculated per cent)</th>
<th>(extreme range per cent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 23·0</td>
<td>9·5</td>
<td>7·28 $\times$ 10$^8$</td>
<td>5·9 $\times$ 10$^3$</td>
<td>0·3</td>
<td>1,000</td>
<td>420</td>
<td>10,550</td>
<td>0·41</td>
<td>26·6</td>
<td>16</td>
<td>5 — 40</td>
</tr>
<tr>
<td>2 20·0</td>
<td>9·5</td>
<td>8·30 $\times$ 10$^8$</td>
<td>3·0 $\times$ 10$^3$</td>
<td>0·3</td>
<td>378</td>
<td>1,130</td>
<td>4,550</td>
<td>0·47</td>
<td>17·5</td>
<td>14</td>
<td>5 — 35</td>
</tr>
<tr>
<td>3 20·0</td>
<td>9·5</td>
<td>3·86 $\times$ 10$^8$</td>
<td>9·8 $\times$ 10$^2$</td>
<td>0·3</td>
<td>475</td>
<td>1,720</td>
<td>813</td>
<td>0·47</td>
<td>17·5</td>
<td>8</td>
<td>3 — 20</td>
</tr>
<tr>
<td>4 18·2</td>
<td>9·5</td>
<td>4·19 $\times$ 10$^8$</td>
<td>3·9 $\times$ 10$^3$</td>
<td>0·3</td>
<td>555</td>
<td>655</td>
<td>2,450</td>
<td>0·52</td>
<td>17·5</td>
<td>16</td>
<td>5 — 40</td>
</tr>
<tr>
<td>5 17·7</td>
<td>9·5</td>
<td>3·21 $\times$ 10$^8$</td>
<td>1·7 $\times$ 10$^5$</td>
<td>0·3</td>
<td>1,020</td>
<td>1,105</td>
<td>34,850</td>
<td>0·54</td>
<td>17·5</td>
<td>23</td>
<td>8 — 57</td>
</tr>
<tr>
<td>6 16·5</td>
<td>9·5</td>
<td>4·19 $\times$ 10$^8$</td>
<td>3·9 $\times$ 10$^3$</td>
<td>0·3</td>
<td>555</td>
<td>1,310</td>
<td>5,050</td>
<td>0·58</td>
<td>17·5</td>
<td>18</td>
<td>6 — 45</td>
</tr>
<tr>
<td>7 14·4</td>
<td>9·5</td>
<td>4·19 $\times$ 10$^8$</td>
<td>3·9 $\times$ 10$^3$</td>
<td>0·3</td>
<td>555</td>
<td>980</td>
<td>3,600</td>
<td>0·66</td>
<td>17·5</td>
<td>21</td>
<td>7 — 50</td>
</tr>
</tbody>
</table>

### TABLE 2. Comparison of the experimental results with the theoretical collision efficiencies for droplets falling in still air

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
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<tr>
<td>R</td>
<td>τ</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 23·0</td>
<td>9·5</td>
<td>16</td>
<td>26·6</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>2 20·0</td>
<td>9·5</td>
<td>14</td>
<td>17·5</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>3 20·0</td>
<td>9·5</td>
<td>8</td>
<td>17·5</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>4 18·2</td>
<td>9·5</td>
<td>16</td>
<td>17·5</td>
<td>14</td>
<td>9</td>
</tr>
<tr>
<td>5 17·7</td>
<td>9·5</td>
<td>23</td>
<td>17·5</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>6 16·5</td>
<td>9·5</td>
<td>18</td>
<td>17·5</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>7 14·4</td>
<td>9·5</td>
<td>21</td>
<td>17·5</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

*These collision efficiencies are calculated assuming a minimum gap between the droplets for coalescence of $10^{-3}$ $\times$ large drop radius
being studied on a computer by Dr. Jonas. The cause of the increase in collision efficiency of small drops is as yet unknown.

Before our results can be applied to the atmosphere in terms of turbulent collection efficiencies \( E(R, r, e) \) it will be necessary to perform further experiments to establish the relationship \( E(R, r, S) \) where \( S \) is the full shear tensor, \( dU/dr \) and to consider the effects of inhomogeneity in the turbulence field in clouds. Nevertheless, we believe that our experiments confirm our initial hypothesis that the micro shears in a turbulent cloud are capable of yielding large collection efficiencies for collector drops with radii in the range 15 to 20\( \mu \text{m} \). The apparent gap between condensation and coalescence may thereby be closed provided the cloud is sufficiently turbulent.

**Acknowledgments**

The generosity of Mr. A. C. Chamberlain in making the radioactive facilities of the Aerosol Laboratory of the Health Physics Division, AERE, Harwell, available for these experiments is gratefully acknowledged.

Thanks are also due to Mr. J. P. Cowley who was involved in the design and building of the linear shear wind tunnel.

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**References**

- Ludlam, F. H. 1951 ‘The production of showers by the coalescence of cloud droplets,’ *Pure Appl. Geophys.*, Basle, 64, pp. 185–196.
COALESCEENCE IN A TURBULENT CLOUD


