Terminal velocities of ice crystals

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SUMMARY

The terminal velocities of small plate-like and columnar ice crystals have been measured experimentally. Good agreement was found between theory and experiment.

SYMBOLS

Re  Reynolds number
X  Best number (product of the square of the Reynolds number and the drag coefficient)
V  terminal velocity
d  length of a-axis for plate-like and columnar ice crystals
L  length of columnar ice crystals
ρs  bulk density of ice crystals
ρa  density of air
ν  kinematic viscosity
g  acceleration due to gravity
m  mass of plate-like ice crystals
f  ratio of Reynolds number to Best number

1. INTRODUCTION

Experimental determinations of the fall velocities of individual ice crystals are rare and those that exist have been made only for large ice crystals or snow flakes (see e.g. Nakaya and Terada 1935; Langleben 1954). These values have been used extensively and empirical equations to fit them have been derived by Magono (1954) and Cornford (1965). The application of these empirical equations is limited to the size range of the ice crystals on which the equations were based, and hence their application to small ice crystals of the size frequently found in clouds is questionable. Fukuta (1969) has measured the growth rate and velocity of freely-falling small ice crystals as a function of time. It is difficult, however, to see how to interpret his results in terms of a relationship between falling velocity and ice crystal size.

Recently Jayaweera and Cottis (1969) derived the fall velocities of ice crystals using the results of model experiments. They showed that the terminal velocity of any crystal could be calculated, provided that the relationship between the Reynolds number and the Best number was known. Using the results of the model experiments they established the Reynolds number-Best number relationships for the two basic types of ice crystals, namely columns and plates. However, since the model experiments were conducted using solid cylinders and discs the results are only strictly applicable to solid columns and plates. Jayaweera and Cottis postulated that those relationships would also hold for other crystal habits such as hollow columns.

The experiment described below tests the application of the model experiments to calculation of the terminal velocities of small columnar and plate-like ice crystals.

2. EXPERIMENT

The experiment was conducted in a cold room. After seeding the supercooled fog in an upper chamber by means of a rod chilled with liquid air a copious supply of ice crystals was formed. The air inside the box was relatively turbulent and would sustain ice crystals in the chamber from five to ten minutes after seeding. The base of the upper chamber was connected to a double-walled observation chamber (4.5 × 15 × 15 cm) by a tube, and the number of ice crystals entering the

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observation chamber was controlled by means of a shutter. The observation chamber rested on a refrigerated base some 2°C colder than the ambient temperature; this procedure was necessary to prevent convection within the observation chamber.

The terminal velocities of the ice crystals were measured from streak photographs obtained as the crystals fell through a collimated beam of mercury light pulsating at 100 Hz. The crystals were collected on glass slides coated with silicone oil. They were subsequently photographed and then melted so that their mass could be determined. It was found that unless the oil was saturated with cetyl alcohol the ice crystals rapidly evaporated or dissolved.

When two crystals were observed to fall, the results were only used if two streaks appeared on the film and both crystals were found. It was assumed that the fastest measured streak corresponded to the fastest calculated terminal velocity. On two occasions it was possible to melt only one crystal (see Table 1). In these instances the density of the unmelted crystal was estimated by comparing it with similar crystals that had been melted.

3. Results

The terminal velocities of columnar crystals in air have been calculated using the Best number-Reynolds number relationship as found by Jayaweera and Cottis (1969) in their model experiments.

For columns the Best number is given by

$$X = \frac{3\sqrt{3}}{4} \frac{d^3 \rho_s}{\nu \rho_a g},$$  \hspace{1cm} (1)

and the terminal velocity is given by

$$V = \left(\frac{3\sqrt{3}}{4} \frac{g}{\nu \rho_a}\right) f d^3 \rho_s,$$  \hspace{1cm} (2)

where $f$ is the ratio of the Reynolds number to the Best number for the appropriate value of the parameter $d/L$.

The measured and calculated values for the columnar crystals are shown in Table 1, together with the parameter $d/L$, the density of the crystal, the temperature, and the habit of the crystal. In Fig. 1, the measured velocities are plotted against the parameter $fd^3 \rho_s$; these values are in good agreement with the theoretical relationship shown by the solid line.

![Figure 1. Terminal velocity of columns when plotted as a function of the parameter $fd^3 \rho_s$.](image)

* Dow Corning DC-330. Fukuta (1960) has discussed the reasons for using this oil rather than a less viscous one.
<table>
<thead>
<tr>
<th>Crystal type</th>
<th>d/L</th>
<th>Density (kg m⁻³ × 10⁴)</th>
<th>L (μm)</th>
<th>Temp. (°C)</th>
<th>V_exp (m s⁻¹)</th>
<th>V_calc (m s⁻¹)</th>
<th>'Apparent' density (kg m⁻³ × 10⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid column</td>
<td>0.55</td>
<td>0.7</td>
<td>90</td>
<td>-7</td>
<td>0.055</td>
<td>0.053</td>
<td>0.9</td>
</tr>
<tr>
<td>Hollow/solid column</td>
<td>0.87</td>
<td>0.64</td>
<td>50</td>
<td>-8.5</td>
<td>0.050</td>
<td>0.057</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>0.80</td>
<td>0.56</td>
<td>44</td>
<td>-</td>
<td>0.043</td>
<td>0.039</td>
<td>0.85</td>
</tr>
<tr>
<td>Two parallel solid columns</td>
<td>0.57</td>
<td>0.72</td>
<td>55</td>
<td>-7</td>
<td>0.037</td>
<td>0.038</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>0.40</td>
<td>0.65</td>
<td>56</td>
<td>-8</td>
<td>0.030</td>
<td>0.022</td>
<td>0.94</td>
</tr>
<tr>
<td>Hollow/solid column</td>
<td>0.23</td>
<td>0.58</td>
<td>90</td>
<td>-8.5</td>
<td>0.030</td>
<td>0.025</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>0.65</td>
<td>87</td>
<td>-7</td>
<td>0.029</td>
<td>0.031</td>
<td>0.85</td>
</tr>
<tr>
<td>Solid column</td>
<td>0.9</td>
<td>0.7</td>
<td>31</td>
<td>-8</td>
<td>0.020</td>
<td>0.024</td>
<td>0.88</td>
</tr>
<tr>
<td>Hollow column</td>
<td>0.64</td>
<td>0.36</td>
<td>44</td>
<td>-8</td>
<td>0.013</td>
<td>0.014</td>
<td>0.79</td>
</tr>
</tbody>
</table>
The bulk density is calculated from the mass of the ice crystal and from the volume of the column as deduced from the values of \( d \) and \( L \) (i.e. volume \( = \frac{3 \sqrt{3}}{8} d^3 L \)). It should be noted that the measured values of the bulk densities of the columnar ice crystals are less than that of solid ice. Further, these values are consistent with similar measurements made by Fukuta (1969). Ono (1970) has examined replicas of columns formed in natural clouds and has estimated their bulk densities using the geometric configuration of the replicas. In this way he attempted to allow for the hollowness of the crystal. The 'apparent' bulk densities shown in Table 1 are those calculated using a mass derived from the geometry of the crystals in a manner similar to that used by Ono. In all but one case the 'apparent' bulk density was greater than the measured density. While these crystals were formed in conditions of high supersaturation, and therefore may be less dense than those found in natural clouds, the results of the experiment suggest that care should be taken in estimating the bulk densities of ice crystals from their external dimensions. Clearly, field experiments need to be designed that will permit a direct measurement of the bulk density of natural ice crystals.

The Best number for solid hexagonal plates is given by

\[
X = \frac{16}{3 \sqrt{3}} \frac{mg}{\nu_R a},
\]

and the velocity is given by

\[
V = \left( \frac{16}{3 \sqrt{3}} \frac{g}{\nu_R a} \right) \frac{f m}{d},
\]

where \( f \) is the ratio of the Reynolds number to the Best number for discs. For small ice crystals Jayaweera and Cottis (1969) have shown that \( f \) has the value 1/20. In Fig. 2 the measured velocities for plate-like crystals are plotted against the parameter \( fm/d \), and again these values are in good agreement with the theoretical relationship shown by the solid line.

![Graph showing terminal velocity of plates plotted as a function of the parameter \( fm/d \).](image)

**Figure 2.** Terminal velocity of plates plotted as a function of the parameter \( fm/d \).

### 4. Conclusions

The present experiments have shown that the terminal velocities of small columnar and plate-like ice crystals can be calculated using the results obtained from model experiments and that these results are valid for habits such as hollow columns where the bulk densities are less than that of solid ice. Further work is required to confirm whether the results can be applied to dendritic crystals.
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REFERENCES


