Direct measurement of vapour pressure and its fluctuations using fine thermocouples

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SUMMARY

An instrument was developed to measure directly vapour pressure and temperature and their fluctuations compared to a moving average sampling line of 10 min. Copper–constantan thermocouples of 75 μm in diameter are used for the measurement of fast fluctuations in dry- and wet-bulb temperatures and a thermistor (100 kΩ at 25°C) for the measurement of mean wet-bulb temperature. By applying the thermocouple voltages to a specially designed operational amplifier circuit, the vapour pressure is obtained as output voltage of the operational amplifier. The output voltage varies linearly with vapour pressure in the range 5–40 mb. The values of the input and feedback resistances were theoretically determined by relating the output voltage with the Regnault’s psychrometric equation. Field measurements showed that the power spectra of temperature and water vapour pressure followed $-5/3$ law in the frequency range 0.2–2.0 Hz.

1. INTRODUCTION

The measurement of vertical turbulent flux of water vapour is of great value in the studies of agriculture, forestry, hydrology, meteorology and oceanography. This flux can be assessed directly in terms of the covariance between vapour pressure or specific humidity and vertical velocity. While instruments for the measurement of vertical velocity are now readily available, direct measurement of vapour pressure or specific humidity still remains as a complex problem. Despite such complexity, Swinbank (1951) designed an instrument for measuring vapour pressure fluctuations over short periods of time when the dry- and wet-bulb temperature changes are small. His instrumentation requires pre-setting at the mean wet-bulb temperature as obtained by an Assmann psychrometer and hence it is difficult to adjust during periods of steadily rising or falling wet-bulb temperatures. Sheppard and Elness (1951) also reported an instrument for the measurement of humidity mixing ratio and its fluctuations. The basic principle of their instrument is similar to that of Swinbank and the wet-bulb temperature must also be pre-determined by an Assmann psychrometer.

In the present paper, an instrument is described which does not require either pre-setting or any limitation on the magnitude of the fluctuations. While Swinbank’s instrument measures, for short periods, vapour pressure fluctuations to an accuracy of about 6 per cent, the instrument described here measures continuously both the actual vapour pressure and its fluctuations to an accuracy of about 0.4 per cent. Unlike previous devices, this instrument records also temperature and its fluctuations. In particular, the present instrument was developed for the on-line measurement of sensible and latent heat fluxes by the eddy correlation method.

2. SENSOR SELECTION

In the study of turbulent fluxes of heat and water vapour, temperature and humidity measurements must be made with minimum disturbance of the environment and hence forced ventilation cannot be used. The sensors should be of such small size so as not to hinder the exchange of properties between atmospheric layers nor disturb the radiation exchanges. The sensor should have a very rapid response yet should be sufficiently robust and reliable for field use. A most suitable instrument is the wet- and dry-bulb psychrometer. Thermocouples, thermistors or resistance thermometers could be used as temperature sensors. Thermistors were used earlier (Polavarapu and Munn 1967) for the direct measurement of vapour pressure. With thermistors and resistance thermometers, however, matching of wet- and dry-bulb sensors and electrical insulation of the leads of the wet-bulb sensor involve some practical problems. Hence thermocouples are preferred over the other two, as dry- and wet-bulb sensors are always matched if wires from the same batch of metal are used and the leads of the wet-bulb sensor do not have to be insulated. Also, by using thermocouples frequent recalibration can be avoided.
3. Measurement of vapour pressure

The partial pressure of water vapour in the atmosphere can be determined from dry- and wet-bulb temperature measurements by means of Regnault's psychrometric formula, i.e.,

$$ e = e_w - AP(T_d - T_w) $$  \hspace{1cm} (1)

where $e$ is the vapour pressure, $e_w$ the saturation vapour pressure at the wet-bulb temperature $T_w$, $T_d$ the dry-bulb temperature, $P$ the atmospheric pressure in units consistent with vapour pressure and $A$ the psychrometric constant. In the present instrument, vapour pressure is measured from Eq. (1) as the difference between the saturation vapour pressure and the vapour pressure deficit (Williams and Brochu 1969).

The saturation vapour pressure at any wet-bulb temperature in the range 0°C to 32°C is measured by means of the circuit shown in Fig. 1. Copper-constantan thermocouples are used for the measurement of rapid fluctuations in wet-bulb temperature and a thermistor for the measurement of mean wet-bulb temperature. The reference junction is kept at the melting point of ice by using a commercially available ice bath (0 ± 0.01°C). The thermoelectric voltage, generated between the wet-bulb and the reference junction, is amplified by $A_1$, and fed to an operational amplifier through a resistance network consisting of resistors $R_1$, $R_2$ and thermistor resistance $R_w$. The thermistor is used to simulate the non-linearity in the saturation vapour pressure with wet-bulb temperature. Let the voltage generated between the wet-bulb and the reference junction be denoted as $V_1$. If the gain of the amplifier $A_2$ is $A$, then the output voltage from the operational amplifier is given as

$$ V_1 = AE_wR_2(R_2 + R_w) / (R_1R_w + R_2R_w + R_1R_2) $$  \hspace{1cm} (2)

If the output voltage $V_1$ is to yield saturation vapour pressure $e_w$ at any wet-bulb temperature $T_w$, Eq. (3) must be satisfied, i.e.,

$$ \frac{d e_w}{d T_w} = K \frac{d V_1}{d T_w} $$  \hspace{1cm} (3)

where $K$ is a proportionality constant expressed in mb/mv.

Integrating Eq. (3) with respect to $d T_w$ gives

$$ e_w = KV_1 + C $$  \hspace{1cm} (4)

where $C$ is a constant of integration. When the wet-bulb temperature is equal to the temperature of the reference junction (i.e., 0 ± 0.01°C) the voltage produced between the junctions is zero and hence $V_1$ becomes zero. Substituting $V_1 = 0$ in Eq. (4), $C = e_w$, the saturation vapour pressure at 0°C which is equal to 6.1078 mb, as given by List (1958).

Substituting $V_1$ from Eq. (2) in Eq. (4) and rearranging

$$ e_w - C = KAR_wR_2(R_2 + R_w) / (R_1R_w + R_2R_w + R_1R_2) $$  \hspace{1cm} (5)

By denoting

$$ KAR_w = K_1 $$  \hspace{1cm} (6)

$$ e_w - C = K_1e_wR_2(R_2 + R_w) / (R_1R_w + R_2R_w + R_1R_2) $$  \hspace{1cm} (7)

The three unknowns $R_1$, $R_2$, and $K_1$ could be determined by substituting the values for $e_w$, $E_w$, and $R_w$ in Eq. (7) at three wet-bulb temperatures. But the equations thus formed are not com-
patible with each other as the variation of saturation vapour pressure with wet-bulb temperature is not linear. Hence, the unknowns thus determined do not satisfy Eq. (7) at other wet-bulb temperatures.

Therefore, a technique was developed from the basic method of least squares (Whittaker and Robinson 1924) to find the best possible solution to the governing equation. Expressions for each of the unknowns $R_1, R_2$, and $K_1$ were obtained as functions of the other two unknowns as follows

$$
R_{11} = F(R_{20}K_{10})
$$

$$
R_{21} = G(R_{11}K_{10})
$$

$$
K_{11} = H(R_{11}R_{21})
$$

Starting with initial values $R_{10}, R_{20}, K_{10}$ for $R_1, R_2$, and $K_1$, the process was continued until the magnitudes of $(R_{11} - R_{10}), (R_{21} - R_{20})$, and $(K_{11} - K_{10})$ were within the given tolerances of $dR_1 = 0.01$, $dR_2 = 0.1$, and $dK = 0.1$. The solution was obtained at the end of 790 iterations using initial values of $R_{10} = 11 \text{k}\Omega$, $R_{20} = 450 \text{k}\Omega$, and $K_{10} = 2400 \text{k}\Omega$. The values obtained are $R_1 = 15.15 \text{k}\Omega$, $R_2 = 481.5 \text{k}\Omega$, and $K_1 = 2536.3 \text{k}\Omega$. Thus the three unknown resistances were determined using the values of $E_w$, $e_w$, and $R_w$ at the 13 wet-bulb temperatures given in Table 1. The values of $E_w$ (in millivolts) were obtained by calibrating the thermocouples at the required temperatures. The saturation vapour pressure values $e_w$ (in millibars) were noted from List (1958). The thermistor resistances were computed (Polavarapu and Munn 1967) for a Fenwal thermistor having a $b$ value of $411.5 \pm 150$ and a resistance of $100 \text{k}\Omega$ at $25^\circ \text{C}$. A thermistor with the above resistance value was chosen since it is readily available.

It can be seen from Eq. (6) that $K_1$ represents the product of three terms, i.e., the proportionality constant $K$, amplifier gain $A$ and the feed back resistance $R_f$. By making $K = A = 1$, $R_f$ should be equal to $2.54 \times 10^4 \text{M}\Omega$. When $K = 1$, the changes in output in millivolts will correspond exactly to changes in vapour pressure in millibars. Using the above values of $R_1, R_2$, and $R_f$ and an amplifier gain of unity, the output values corresponding to various wet-bulb temperatures were computed and are given in Table 1. The Table also includes the values of the proportionality constant, which is the ratio of saturation vapour pressure to the output voltage. The proportionality constant varies by about $\pm 0.4$ per cent in the temperature range $0^\circ \text{C}$ to $31^\circ \text{C}$.

The vapour pressure deficit $d_e$ is measured by means of the circuit given in Fig. 2. The voltage generated between the dry-bulb and reference junction temperatures, $E_d$, is opposite in polarity to that generated between wet-bulb and reference junction temperatures, $E_w$. If we assume, for simplicity, the gains of the amplifiers $A_1$ and $A_2$ are unity as before, the output voltage $V_2$ is given as

$$
V_2 = (E_w - E_d)R_f/R_3 = -(E_d - E_w)R_f/R_3.
$$

Since $|E_d > E_w|$, $V_2$ is opposite in polarity to that of $V_1$. For $V_2$ to represent vapour pressure deficit, Eq. (10) has to be satisfied, i.e.,

$$
AP(T_d - T_w) = K(E_d - E_w)R_f/R_3.
$$

Here the proportionality constant $K$ should be the same as before, i.e., $K = 1 \text{mb/mv}$.

The only unknown $R_3$ in Eq. (10) can be determined for any given values of $A$ and $P$. In the present instrument the psychrometric constant, $A = 6.60 \times 10^{-4}(1 + 1.15 \times 10^{-3} T_w)$, given by List (1958), was used since the size of the psychrometer is small (see Huovila 1958). For atmospheric pressure $p$ a value of 1,000 mb was used. Using these values of $A$ and $P$, the $d_e$ value was obtained for a depression of 1°C at a wet-bulb temperature of 13°C and the corresponding values of $E_d$ and $E_w$ were noted from thermocouple calibrations. Substituting the values of $d_e$,

![Figure 2. Circuit for the measurement of vapour pressure deficit. DB, WB, RJ = Dry-bulb, Wet-bulb and Reference junction thermocouples; A1, A2 = Instrumentation amplifiers, Op = Operational amplifier, R3 = 152 k\Omega, and Rf = 2.54 M\Omega.](image-url)
<table>
<thead>
<tr>
<th>Temperature in degrees Celsius</th>
<th>Thermistor resistance $R_w$ in kΩ</th>
<th>e.m.f. generated by wet-bulb thermo-couple $E_w$ in mv</th>
<th>Output voltage $V_1$ in mv</th>
<th>Saturation vapour pressure $e_w$ in mb</th>
<th>$e_w - C^*$</th>
<th>Proportionality constant $K$ in mb/mv</th>
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<tr>
<td>1</td>
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* $C = 6.1078$ mb, the saturation vapour pressure corresponding to the reference junction temperature, 0°C.
$E_a$, $E_w$, and $R_f$ in Eq. (10) the value of $R_3$ was determined as 152 kΩ. The value of $R_3$ must be re-evaluated for use with other atmospheric pressures or psychrometric constants such as those given by Bindon (1965). It can be seen from Eq. (10) that the slight increase in the psychrometric constant at higher wet-bulb temperatures is mostly accounted for as the thermocouple voltage per degree Celsius also increases at higher temperatures. As such, the output $V_x$ in millivolts represents the vapour pressure deficit in millibars for any depression if the atmospheric pressure variations from that used (i.e. 1,000 mb) are negligible.

To measure the atmospheric water vapour pressure, Figs. 1 and 2 are combined as given in Fig. 3. The output voltage $V_o$ of Fig. 3, measured in millivolts, directly gives a measure of vapour pressure in millibars. The output is positive when the vapour pressure is higher than 6-11 mb and negative if lower than 6-11 mb. By the use of a voltage follower, the necessity of a second wet-bulb sensor was eliminated in Fig. 3. The voltage follower effectively isolates the input resistance $R_1$ from the other input resistance network consisting of $R_1$, $R_2$ and the thermistor.

Figure 3. Circuit for the measurement of vapour pressure. Symbols as in legends of Fig. 1 and 2.

4. Measurement of temperature

The ambient temperature is also measured from the same sensors which are used for the measurement of vapour pressure. The amplified voltage of the dry-bulb temperature (form $A_2$ in Fig. 3) is again amplified so as to produce an output of approximately 80 mv/°C. The exact temperature corresponding to the output voltage is obtained from the predetermined thermocouple calibration diagram. The output voltage is negative for temperatures higher than 0°C and positive for temperatures lower than 0°C.

5. Measurement of fluctuations

Temperature and vapour pressure fluctuations are determined by an analog method as described by Brock and Provine (1962). The fluctuating component is the difference between the moving average and the instantaneous value, and is defined as

\[ x'(t) = \{ x(t) - x(t - \tau / 2) \} . \tag{11} \]

The moving average of the variable with the sampling time $\tau$ is given as

\[ x(t - \tau / 2) = \frac{1}{\tau} \int_{t-\tau}^{t} x(t)dt . \tag{12} \]

The fluctuating component and the moving average over a sampling time of 10 min were obtained following Brock and Provine (1962) using the circuit given in Fig. 4.

6. Field measurements

Temperature and vapour pressure measurements were made during the summer months of 1968 and 1969 at Perch Lake (see Barry 1967), AECL, Chalk River in connection with the determination of surface fluxes of heat and water vapour. The sensor assembly was mounted on a tower which was installed in the lake about 100 m from the shore as shown in Fig. 5. The sensor consists
Figure 5. Field installation.

Figure 6. A record of temperature, vapour pressure and their fluctuations over a moving average of 10 min.
Figure 4. Circuit for the measurement of fluctuations over a moving average having a sampling time of 10 min. \( x \) = Instantaneous value, \( \bar{x} \) = Moving average, \( x - \bar{x} \) = Fluctuating component, \( R_1 = 1 \, \text{M} \Omega, R_2 = 17.3 \, \text{M} \Omega, R_3 = 10 \, \text{M} \Omega, \) and \( C_1 = C_2 = 10 \, \mu \text{F} \).

mainly of a thermistor and two fine thermocouple junctions made from copper constantan wires of diameter 0·0075 cm. The fine junctions were connected at the end of two pairs of 0·075 cm diameter copper-constantan wires, 5 cm apart, with junctions formed at the mid-points one behind the other. The inner junction serves as the wet-bulb and its time constant is almost matched to the dry-bulb (0·2 s in a wind of 1 m s\(^{-1}\)) by constructing the wick (thin tissue paper) as a porous cage of tubular cross-section, 3 mm in diameter, surrounding the wet-bulb as described by McBean (1968). Several other ways of covering the wet-bulb with wick were tried but the one adopted showed equally fast responses for the wet and dry thermocouples while the mean wet-bulb depression accurately followed that of a standard Assman psychrometer. A Fenwal thermistor, GA51J11, was potted in a thin-wall stainless steel tube, 6 mm in diameter, filled with epoxy resin to increase the thermistor time constant. This unit measures the mean wet-bulb temperature.

A one-minute record of temperature, vapour pressure and their fluctuations, made at a height of 1 m over Perch Lake, is reproduced in Fig. 6. This record was selected from a 10-min trial extending from 1400-1410 on 16 October 1969. The sky was partly clear and the mean wind speed, at that height, was about 1·0 m s\(^{-1}\). The mean dry- and wet-bulb temperatures, measured with an Assman psychrometer, were 15·5°C and 11°C. It can be seen from the Figure that the vapour pressure obtained directly, using the present instrument, agrees very well with that computed (10·11 mb) from the Assman psychrometer measurements.

The vapour pressure scale is marked with zero equal to 6·11 mb as this corresponds to zero output from the vapour pressure circuit. The temperature scale is slightly non-linear as the rate of change of thermocouple voltage increases slightly with increase in temperature. The temperature and vapour pressure signals are normally opposite in phase when the vapour pressure is higher than 6·11 mb. For a better understanding, however, both the signals were made to be in phase in Fig. 6 by inverting the temperature signal. The fluctuating components (over a moving average of 10 min) of temperature and vapour pressure signals are opposite in phase to those of temperature and vapour pressure as can be understood from Fig. 4. In Fig. 6, the traces of temperature and vapour pressure fluctuations lag those of temperature and vapour pressure by a small division due to the mechanical separation of pens in the recorder.

The temperature and vapour pressure signals are highly correlated as observed by Miyake and McBean (1970). The vapour pressure signal shows more high frequency information than the temperature signal. During the one-minute period, the temperature was increasing from the beginning while the vapour pressure was decreasing due to the increasing rate of advection of warm, dry air from the shore.

The power spectrum of the ten-minute trial was computed by an analog method using ISAC Statistical Analyser manufactured by NORATOM, Norway. Fig. 7 shows a schematic comparison of the spectra of vapour pressure and temperature with a line having the \(-5/3\) slope. The slopes of the spectra followed \(-5/3\) law in the frequency range 0·2 - 2·0 Hz as reported by Miyake and McBean (1970). The gradual fall-off above 2 Hz is due to the attenuation of the signal due to the limited response of the sensor. The fall-off of power at frequencies below 0·2 Hz may be characteristic of both temperature and vapour pressure spectra.

The instrument design was later improved to measure temperature and vapour pressure fluctuations up to 10 Hz. Thermocouple junctions were formed by electroplating constantan wire of 12 \( \mu \) diameter with copper. The response times of the wet- and dry-bulb junctions are almost matched either by wrapping the wet-bulb junction with a single layer of thin tissue paper or by winding the wet-bulb with fine cotton thread (about 10 turns/cm) and controlling the flow of water from the reservoir. Further, electronic circuitry is used to filter the dry- and wet-bulb temperature fluctuations over 10 Hz.
Figure 7. Power spectra of temperature and vapour pressure.
7. Conclusions

The instrument described in this paper enables the fine structure of vapour pressure and temperature changes to be studied. It provides a direct measurement of both vapour pressure and temperature and also of their fluctuations compared to a moving average period of 10 minutes. In particular, this instrument was developed to measure directly the vertical turbulent flux of water vapour (and heat) in terms of the covariance between vapour pressure (and temperature) and the vertical velocity component.

Field measurements over Perch Lake showed that the temperature and vapour pressure signals are highly correlated. The slopes of the power spectra of temperature and vapour pressure followed — 5/3 law in the frequency range 0.2 — 2.0 Hz.

The instrument was later improved to measure temperature and vapour pressure fluctuations up to 10 Hz.

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