The spatial variability of eddy fluxes in the constant flux layer

By A. J. DYER and B. B. HICKS
Division of Atmospheric Physics, C.S.I.R.O., Aspendale, Victoria, Australia

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SUMMARY

At the 1970 International Comparison of Turbulence Instruments held at Tsimlyansk, U.S.S.R., a large number of momentum and heat flux measurements were obtained simultaneously with two Fluxatrons. The data are analysed from the point of view of spatial variation of eddy fluxes, bearing in mind the limitations of the sensors, the variability of the eddy fluxes themselves, and the possibility of errors due to tilt. It is concluded that, at the Tsimlyansk site, the variation of heat and momentum fluxes both in the vertical (4–14 m) and the horizontal (1–150 m) is of the order 10 per cent or less. This result is in accord with traditional opinion, and provides a satisfactory experimental basis for the concept of a 'constant flux layer'.

1. INTRODUCTION

Much basic research in micrometeorology rests on the premise that the various vertical eddy fluxes, such as of heat and of momentum, are relatively constant in the first few tens of metres of the lower atmosphere — hence the common usage of the term 'constant flux layer'. Monin and Obukhov (1954), for example, taking a variation of 20 per cent in the momentum flux as 'permissible', estimated that this would occur at a height of 50 m.

In a similar way only a small variation in the horizontal is anticipated for the vertical eddy fluxes, provided that measurements are made at sites selected for good uniformity of surface and free from upwind obstruction for considerable distances.

We are thus led to expect that in the first few tens of metres of the atmosphere the spatial variation of the vertical eddy fluxes of heat and momentum will be typically of the order of 10 per cent or less.

The correctness of this has been questioned in two papers, both involving direct measurement of the eddy fluxes. Mordukhovich and Tevang (1966) found differences as large as a factor of 2 in both heat and momentum fluxes between 1 and 4 m height. Businger, Miyake, Dyer and Bradley (1967) also reported differences of similar magnitude between two heat flux measurements at 4 m height, separated by a horizontal distance of 5 m. These results are particularly disturbing in that they raise serious doubts as to the usefulness and validity of eddy flux measurements, and of the 'constant flux layer' concept.

By contrast, an analysis of wind-profile data obtained with two masts separated downwind by 200 m, whilst demonstrating an increased variability of the eddy fluxes with increasing height, indicated that, on the average, the momentum flux up to 16 m height differed from the flux at the surface to the extent of about 1 per metre (Dyer 1968). Because of the enforced neglect of certain terms in the analysis, this value could be an underestimate.

During the International Comparison of Turbulence Instruments held at Tsimlyansk in June and July 1970, under the auspices of the Academy of Sciences of the U.S.S.R., the authors obtained a considerable body of data bearing on this question. Whilst a full report by all participants will appear in due course, it was considered desirable, in view of the importance of this matter, to present a brief survey of the results.

2. INSTRUMENTATION

Two identical Fluxatrons (Dyer, Hicks and King 1967) capable of measuring eddy fluxes of heat and momentum, were employed on the expedition. The circuit format was that of the most recent design (Hicks 1970). Propeller anemometers were used for the measurement of $u$ and $w$ velocity fluctuations, and temperature fluctuations were detected by a small bead thermistor.

In the Fluxatron long time-constant C-R filters are used to remove both the mean levels and the long period fluctuations. These are normally set at 80 s, although for some exploratory investigations these were sometimes set at 160 s. Precise electrical calibrations can be carried out in
the field, and at no time were the calibration factors found to differ from the values determined in the laboratory in Australia by more than 2 per cent.

The Tsimlyansk site is described by Mordukhovich and Tsvang (1966). It is fairly uniform in plant characteristics but possesses a significant slope (~1°). Most of the eddy flux measurements were 30 min in duration, and made during the day-time, mainly between the hours of 0600 to 1900.

3. Preliminary considerations

Before discussing actual results some comments are necessary on the performance of the Fluxatrons. These concern the frequency response of the sensors and filtering system, the effect of tilt, and the variability of eddy flux measurements generally.

(a) High frequency response

It had always been recognized that the response time of the propeller anemometers was marginally sufficient for the measurement of eddy fluxes, but previous experience suggested that the loss of flux involved at 4 m height would be less than 10 per cent. At the Tsimlyansk site, however, the flux spectra obtained by the fast response sensors of other participants revealed a significant component beyond a normalized frequency \( f = n \omega / u \) of unity, the value previously regarded as the cut-off point of the flux spectrum over a uniform site (Blackadar and Panofsky 1970). There is some evidence that the cut-off frequency over sloping or non-uniform sites extends to higher frequencies.

The results of a typical comparison between the Australian and U.S.S.R. spectra at 4 m height is shown in Figs. 1 and 2, where the ordinates have been adjusted to provide equality of the integral flux at the low frequency end of the spectrum, i.e. for \( f < 0.1 \). This adjustment is necessary because the preliminary results indicated a slight discrepancy even at the low frequency end of the spectrum, presumably a consequence of minor errors in the various calibration factors involved.

![Figure 1. Comparison of U.S.S.R. and Australian momentum flux spectra. Ordinate adjusted to provide equality of the integral flux for \( f < 0.1 \).](image)

The deficiency in the Fluxatron flux measurements can thereby be estimated as 11 per cent for the heat flux and 16 per cent for the momentum flux. These values are consistent with the known response of the \( w \)-propeller, the slowest element in the system. Since the \( w \)-response time is approximately proportional to \( 1/u \), we can thus regard all Fluxatron measurements at 4 m height to be deficient by about 14 per cent. When the Fluxatron is operated at 14 m height, it is estimated from the data of Figs. 1 and 2 that this deficiency is reduced to 3 per cent.

(b) Low frequency response

The 80 s time constant filters used in the Fluxatron attenuate the low frequency eddies to the extent that, for eddies of 500 s period, the covariance response is reduced to 50 per cent. The critical question here is whether there is any significant flux in this region of the spectrum.
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Figure 2. Comparison of U.S.S.R. and Australian sensible heat flux spectra. Ordinate adjusted to provide equality of the integral flux for $f < 0.1$.

A feature of the Fluxatron performances has been that the variability of reported fluxes from run to run has been considerably less than with a conventional eddy flux measurement, defined, for example, as $\bar{w' T'}$ and $\bar{w' u'}$, at the same time retaining reasonable long-term equality with the latter. It was the high variability of the conventional type of measurement from run to run which lead to the idea of low frequency filtering. The essential assumption is that a well defined gap exists between the micro- and meso- portions of the flux spectrum (Fiedler and Panofsky 1970). Whilst there is presumably only a small amount of net flux in the gap, this part of the spectrum will be poorly sampled in any one run, leading to either positive or negative contributions in a conventional eddy flux measurement.

A similar philosophy is expressed by Kukharets and Tsvang (1969), who argue that, whilst there may be fluxes, either positive or negative, in this region of the spectrum, they should not be regarded as true turbulent fluxes, but as meso- or macro-scale fluxes.

The validity of the low-frequency filtering approach is supported by some tests in which the time constant for one instrument was set at 160 s. No significant differences were found between the long period averages of two simultaneous flux measurements at a height of 4 m.

(c) The effect of tilt on a flux measurement

Where two instruments are separated by a considerable distance, either in the horizontal or vertical, there is the possibility of relative tilt between the two sensors causing an undesirable error. Opinions differ slightly as to the magnitude of the tilt effect (Kaimal and Haugen 1969; Dyer, Hicks and Sitaraman 1970; Tsvang private communication), but it is generally agreed that momentum fluxes will be the more vulnerable to error due to the direct correlation of spurious $u$ and $w$ signals when the instrument is tilted.

At the Tsimlyansk expedition, this problem was examined empirically by retaining one instrument in a vertical position, and tilting the other one at a variety of angles in the range $\pm 10^\circ$. Both sensors were operated at a height of 4 m, and separated horizontally across-wind by approximately 1 m.

The results are presented in Fig. 3 and Table 1, where $H$ and $\tau$ represent the heat and momentum fluxes reported by the tilted instruments, and $H_0$ and $\tau_0$ are those for the vertical instrument.

Although tilt errors as large as $10^\circ$ would never occur in practice, this wide range of angle was used in order to assist in defining the magnitude of the effect of small angles. Ideally, both instruments should report identical fluxes in the vertical position, but differences of the order of 5 – 10 per cent were not untypical.

The results of Fig. 3 suggest that an error of approximately 14 per cent per degree will occur in the case of a momentum flux measurement, and 4 per cent per degree in the case of a heat flux measurement. Similar values have been found over the ocean (McBean, private communication).

(d) Variability of flux measurements

It is well recognized that turbulent flux measurements are subject to statistical variation, and
Figure 3. Experimental assessment of the effect of tilt on momentum and heat flux measurements.

TABLE 1. Effect of tilt on flux measurements $H_\theta$ and $\tau_\theta$ refer to the vertical instrument and $H$ and $\tau$ to the tilted instrument

<table>
<thead>
<tr>
<th>Angle of tilt (degrees)</th>
<th>Heat fluxes (mW cm$^{-2}$)</th>
<th>Momentum flux (dyne cm$^{-2}$)</th>
<th>Duration (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H$</td>
<td>$H_\theta$</td>
<td>$\tau$</td>
</tr>
<tr>
<td>-10</td>
<td>8.6</td>
<td>17.9</td>
<td>-0.40</td>
</tr>
<tr>
<td>-5</td>
<td>10.9</td>
<td>14.1</td>
<td>-0.02</td>
</tr>
<tr>
<td>-2</td>
<td>13.0</td>
<td>14.6</td>
<td>0.41</td>
</tr>
<tr>
<td>0</td>
<td>12.7</td>
<td>13.7</td>
<td>1.11</td>
</tr>
<tr>
<td>1</td>
<td>9.5</td>
<td>10.7</td>
<td>1.22</td>
</tr>
<tr>
<td>5</td>
<td>7.9</td>
<td>7.1</td>
<td>1.93</td>
</tr>
<tr>
<td>10</td>
<td>4.6</td>
<td>3.6</td>
<td>2.38</td>
</tr>
</tbody>
</table>

in any study, such as the one under consideration here, a significant body of data must be obtained to obviate the sampling problem.

In Fig. 3, and again later in Fig. 5 and Table 2, it is seen that differences as much as 5 per cent are usual, even in a long series of measurements.

A similar expression of variability is seen in Fig. 4, where the friction coefficient $C_\tau$ (defined as $u_\phi = C_\tau u_\tau$) is plotted as a function of the windspeed at 1 m height ($u_\tau$) for all the data obtained. The variability evident in Fig. 4 is clearly a consequence of statistical sampling both in $u'w'$ (i.e. in friction velocity $u_\phi$) and $u_\tau$, and of possible alignment errors from day to day. The increased scatter at low speeds is typical of sampling variability.

Summarizing the discussion of this Section, the following features emerge:

Figure 4. Friction coefficient (defined by $u_\phi = C_\tau u_\tau$) as a function of the wind-speed at 1 m.
(i) The variability in flux measurements is such that a confidence of about 5 per cent is typical for a long-term average.

(ii) An error due to tilt of approximately 14 per cent per degree may occur in the case of a momentum measurement, but only 4 per cent per degree in the case of a heat flux measurement. The instruments were therefore always carefully aligned with the aid of a spirit level. This was the levelling criterion used by all participants in the Tsimlyansk expedition.

(iii) The limited response of the w-propeller is such that eddy fluxes measured at a height of 4 m at Tsimlyansk will be low by approximately 14 per cent. This does not prevent valid comparisons of eddy fluxes measured at different separations, since both instruments will be subject to a similar deficiency when operated at the same height. At the 14 m level the deficiency is estimated to be 3 per cent.

(iv) The low frequency filtering technique is considered not to affect the accuracy of an eddy flux measurement, and in any case, valid comparisons can be made for identical instruments.

4. Horizontal Separation Measurements

During the course of the expedition, a series of flux measurements were made at a nominal height of 4 m, with horizontal crosswind separation ranging from 1 to 150 m. On one day (19 July), the instruments were operated at 150 m downwind separation.

The results for a typical day are presented in Fig. 5, and a summary of all data is given in Table 2 in the form of ratios of fluxes over the total running period.

![Figure 5. Comparison of heat and momentum flux measurements at 4 m height for a horizontal separation of 30 m.](image)

The results for the 1 m separation, when the sensors are virtually in the same airstream, establish that the two instruments agree to within a few per cent. We can thus use the data at larger separations to investigate the spatial variation of the eddy fluxes.

Proceeding towards greater separations, the sensors experience increasingly different airstreams but presumably having the same turbulent structure in a statistical sense. This, together with the difficulty of accurate vertical alignment, might be expected to lead to increased variability. This is evident in the summarized form of Table 2. The quantities \( \Delta H/H \) and \( \Delta \tau/\tau \), where \( \Delta H \) and \( \Delta \tau \) are the r.m.s. of the differences for each 30 min run, are presented in Table 2 to indicate the degree of variability.

It is clear from Table 2 that the horizontal variation of the heat fluxes is probably less than 10 per cent. The momentum fluxes show greater variation, but much of this can be attributed to possible errors in alignment.

A broad statement of the overall results is that the horizontal variation of eddy fluxes over the Tsimlyansk site is no more than about 10 per cent, and may possibly be less, considering the limitations of the measurements.
TABLE 2. COMPARISON OF AVERAGE FLUXES \((H_1, \tau_1)\) AND \((H_2, \tau_2)\) TAKEN AT A NOMINAL HEIGHT OF 4 m AT VARIOUS CROSS-WIND SEPARATIONS (EXCEPT FOR 19 JULY)

<table>
<thead>
<tr>
<th>Separation (m)</th>
<th>(H_2/H_1)</th>
<th>(\Delta H/H_1)</th>
<th>(\tau_2/\tau_1)</th>
<th>(\Delta \tau/\tau_1)</th>
<th>Duration (min)</th>
<th>Date</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.05</td>
<td>0.12</td>
<td>1.04</td>
<td>0.17</td>
<td>570</td>
<td>21 June</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.98</td>
<td>0.09</td>
<td>1.06</td>
<td>0.12</td>
<td>313</td>
<td>25 June</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.13</td>
<td>0.11</td>
<td>0.88</td>
<td>0.20</td>
<td>420</td>
<td>12 July</td>
<td>TC_1 = 160 s</td>
</tr>
<tr>
<td>10</td>
<td>0.94</td>
<td>0.10</td>
<td>1.09</td>
<td>0.15</td>
<td>360</td>
<td>14 July</td>
<td>TC_1 = 160 s</td>
</tr>
<tr>
<td>30</td>
<td>1.03</td>
<td>0.14</td>
<td>0.96</td>
<td>0.18</td>
<td>622</td>
<td>3 July</td>
<td>TC_1 = 160 s</td>
</tr>
<tr>
<td>30</td>
<td>1.12</td>
<td>0.13</td>
<td>1.05</td>
<td>0.22</td>
<td>513</td>
<td>5 July</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>1.08</td>
<td>0.16</td>
<td>1.28</td>
<td>0.37</td>
<td>207</td>
<td>29 June</td>
<td>TC_1 = 160 s</td>
</tr>
<tr>
<td>60</td>
<td>1.14</td>
<td>0.21</td>
<td>1.23</td>
<td>0.53</td>
<td>268</td>
<td>9 July</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>1.07</td>
<td>0.16</td>
<td>—</td>
<td>—</td>
<td>470</td>
<td>18 July</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>1.10</td>
<td>0.12</td>
<td>1.15</td>
<td>0.10</td>
<td>320</td>
<td>19 July</td>
<td>Downwind</td>
</tr>
</tbody>
</table>

Filter time constant (TC) of both instruments set at 80 s, unless otherwise indicated for \(H_1\) and \(\tau_1\) measurements. \(\Delta H\) and \(\Delta \tau\) are the r.m.s. of the differences for each 30 min run.

5. VERTICAL SEPARATION MEASUREMENTS

The variation of fluxes in the vertical was examined on two days only. One instrument was mounted on a telescopic mast (initial height 5 m) which was raised to 14 m height; the second instrument was operated at 4 m height. Whilst the upper level instrument was carefully aligned in the vertical when the telescopic mast was lowered, no convenient method was available for accurate alignment at the 14 m level. A time constant of 160 s was used at the 14 m level, and 80 s at the 4 m level. Instrument 1 (in the notation of Table 2) was used at the 14 m level and Instrument 2 at the 4 m level.

For the first day (16 July) there was a horizontal separation of 10 m between the two instruments which was increased to 50 m in the second day (17 July). The results for 17 July are presented on a run-to-run basis in Fig. 6, and for both days in consolidated form in Table 3.

![Figure 6. Comparison of heat and momentum flux measurements at heights of 4 and 14 m, respectively. The dotted line refers to a net correction for high frequency loss of 11 per cent applied to the 4 m data.](image)

On the basis of the spectrum information mentioned previously, the flux data at 4 m height were considered to be deficient by 14 per cent, and the 14 m measurements by 3 per cent due to losses at the higher frequencies. In Fig. 6 a net correction of 11 per cent has been applied to the 4 m data to allow a meaningful comparison.

It is clear that some uncertainty must be attached to the assessment of these corrections, but the overall impression from Fig. 6 and Table 3 is that the variation of fluxes in the vertical height range 4 – 14 m is certainly less than 10 per cent.
TABLE 3. COMPARISON OF AVERAGE FLUXES \( (H_{14}, \tau_{14}) \) AT 14 m HEIGHT (TIME CONSTANT 160 s) WITH AVERAGE FLUXES \( (H_{4}, \tau_{4}) \) AT 4 m HEIGHT (TIME CONSTANT 80 s)

<table>
<thead>
<tr>
<th>Date</th>
<th>Uncorrected data</th>
<th>Corrected data</th>
<th>Duration (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{H_{4}}{H_{14}} )</td>
<td>( \frac{H_{4}}{H_{14}} )</td>
<td>( \frac{\tau_{4}}{\tau_{14}} )</td>
</tr>
<tr>
<td>16 July</td>
<td>0.83</td>
<td>0.19</td>
<td>0.88</td>
</tr>
<tr>
<td>17 July</td>
<td>0.90</td>
<td>0.23</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Corrected values obtained by applying a 3 per cent correction to 14 m data and a 14 per cent correction to 4 m data for high frequency loss. \( \Delta H \) and \( \Delta \tau \) are the r.m.s. of the differences for each 30 min run.

6. CONCLUDING REMARKS

The foregoing analysis indicates that the variation of eddy fluxes at the Tsimlyansk site, both in the vertical \((4 - 14 \text{ m})\) and the horizontal \((1 - 150 \text{ m})\) is less than 10 per cent. This conclusion is in accord with traditional opinion, and provides a satisfactory experimental basis for the concept of a 'constant-flux layer.'

At the same time it must be noted that considerable variability of the various eddy fluxes occurs and a large body of data is required to provide a valid basis for discussions of this kind. This variability may have been a contributing factor to the discordant results referred to earlier when only a relatively small body of data was obtained. The earlier results may also have been a reflection of the state of the art of eddy-flux measurements at that time.

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