Humidity and temperature microstructure near the ground

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SUMMARY

Day-time records of the microstructure of humidity and temperature at 2 and 4 m are analysed. Supporting data include the temperature and wind gradients, the net radiation, the sensible heat flux, and a derived value for the latent heat flux.

The two variables are remarkably similar and share the following properties: The characteristic scale of turbulence increases with height and decreases with increasing instability beyond near-neutral conditions. The logarithmic spectra follow a $-5/3$ slope for wave numbers greater than $0.15 \text{ m}^{-1}$. In this range, the spectral estimates increase with increases in the appropriate flux (sensible or latent heat flux for temperature or humidity spectra), and decrease with height.

The humidity variance contains large-scale components which are of lesser importance in the temperature variance. This difference in the two variables has tentatively been attributed to properties of the site.

1. INTRODUCTION

Measurements of the fine structure of atmospheric variables have been confined largely to overland studies of temperature and wind. These measurements have yielded insight into the processes by which heat and momentum are transferred.

The statistical properties of these two variables have been reviewed in Radio Science (1969) and by Obukhov and Yaglom (1967). Elagina (1963) reported results from one of the few studies of the fine structure of humidity.

The purpose of the present study was to obtain statistical information on the humidity field for scale sizes from about 50 cm to 100 m at two heights, 2 and 4 m. A refractometer (Hay, Martin and Turner 1961) was employed to obtain the humidity measurements. In addition, a fast-response thermometer adjacent to the refractometer yielded the temperature field.

2. INSTRUMENTATION AND DATA REDUCTION

The site at Edithvale, Victoria (described by Swinbank 1955), is a flat alluvial plain. Apart from small trees, occasionally in groups, the fetch is unobstructed within about one mile and the elevation does not change by more than 10-15 ft.

The recordings were made in the day-time in the last two weeks of December 1966 and the first week of January 1967. December had been slightly wetter than average; 2.9 in. of rain fell in the three weeks prior to the recording period and an additional 0.75 in. fell during the recording period.

Mean wind speeds and temperatures were measured at 1, 2 and 4 m with conventional cup anemometers and nickel resistance thermometers, respectively. The temperatures were in the form of differences (intervals 1-2 and 2-4 m) and were accurate to about $\pm 0.05^\circ \text{C}$. A fluxatron (Dyer, Hicks and King 1967) at 2 m provided a measure of the sensible heat flux. Net radiation was measured with a radiometer at 1.5 m. These variables were averaged over 15 or 30 min intervals.

The refractometer was a transistorized version of the instrument described by Hay et al. (1961). The fast-response thermometer was made of fine nickel wire and was mounted 5 cm from the refractometer sensor. Both sensors had a time constant of about 0.03 s. A refractometer-thermometer pair was operated simultaneously at 2 and 4 m.

Forty-two samples covering a wide range of ambient conditions were selected for digital analysis. The records were digitized with a sampling interval of 0.08 s. Throughout the discussion which follows, units of length are used; the mean wind speeds during each sample were used to convert the time scales of the records into length scales, according to Taylor’s hypothesis. Table 1 lists the run of wind, $L$, for each of the 42 samples.

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## TABLE 1. VALUES FOR \( l \) AND \( \zeta \) (IN METRES)

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<thead>
<tr>
<th>Run</th>
<th>Time</th>
<th>Ri(1-4)</th>
<th>( L ) (m)</th>
<th>( u_0 ) (m s(^{-1}))</th>
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<th>Temperature</th>
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The humidity fine-structure (\( e' \)) was extracted from the digitized temperature (\( T' \)) and refractive index (\( N' \)) data through the differential form of Debye’s equation (Bean and Dutton 1966):

\[
N' = -(7.6 \times 10^{-3} T^{-2} + 7.47 \times 10^{3} T^{-3} e)T' + 3.73 \times 10^{2} T^{-2} e' = AT' + Be'.
\]

where \( T, a \) and \( e \) are mean values of temperature (°K), total pressure (mb), and partial vapour pressure (mb). The coefficients \( A \) and \( B \) were derived from values of \( T \) and \( e \) obtained from a sling psychrometer. A standard barograph gave \( p \). It is fair to assume that the coefficients were constant for each sample.
3. Autocovariance analysis

The autocovariance of the four fluctuating variables was calculated for each sample. A characteristic scale of turbulence \( l \) was determined using a relationship suggested by G. I. Taylor

\[
l = \int_0^\xi \frac{R(r)}{R(0)} \, dr \tag{2}
\]

where \( R(r) \) is the autocovariance at lag \( r \) and where \( \xi \) is the value of \( r \) where \( R(r) = 0 \) for the first time. This is not the only scale in use (see for example Webb 1955; Houbolt, Steiner and Pratt 1964; Burns 1963) but it is appropriate here being strongly dependent on the small eddies. When \( \xi \) is small in comparison with the wind run, the scale length is determined mainly by the small eddies. The values of \( l \) and \( \xi \) are given in Table 1 together with the mean wind speed at 2 m and the Richardson number over the interval 1-4 m. No cases are included where \( L < 7.5 \xi \).

In Fig. 1, the scales are plotted as a function of atmospheric stability. The individual values have been averaged to reduce the scatter. The dashed extension of each curve for small \( \text{Ri} \) was obtained by plotting all the data including the few values of \( l \) for positive and zero \( \text{Ri} \) on a linear graph and then converting to the semi-log scale. Each stable case is indicated with a point and an arrow at the left side of the Figure; the respective \( \text{Ri} \) values are in brackets.

![Figure 1. The characteristic scales of turbulence, \( l \), plotted as a function of \( \text{Ri} \). The dashed curve is due to Webb from measurements at 29 m, reduced to the 2 m level by assuming \( l \) is proportional to height. The number of values averaged for each point is given; each error bar is the standard deviation of the mean. Five stable estimates are indicated on the left side of the Figure. Their respective values of \( \text{Ri} \) are given in brackets.](image)

Three results follow from this analysis:

(i) Each variable indicates a peak value for \( l \) at near-neutral conditions;

(ii) The humidity scales are about twice as large as the temperature scales at both heights; and

(iii) The scales increase with height for both variables.

The relationship between scale and height was examined in the light of Webb’s (1955) results. Webb has applied Eq. (2) to temperature fluctuations measured at 29 m. Assuming here a direct proportionality between scale length and height, his values were extrapolated to 2 m to give the curve included in Fig. 1. The overall agreement is good.

The large standard deviation for humidity at 4 m is probably a consequence of the length of
the samples; the characteristic scales are largest for this variable and are therefore most sensitive to the finite sample length.

It is a common practice to assume that \( R(\tau) \) has the form \( R(0) \exp \left( -\tau/\ell_c \right) \) (Gerhardt, Crain and Smith 1952; Panofsky 1962; and Kaimal and Hagen 1967). Substituting this expression for \( R(\tau) \) in Eq. (2) and integrating between 0 and \( \infty \) yields the scale \( \ell_c \). The scale length \( \ell_c \) is particularly convenient since it is just the value of \( \tau \) when \( R(\tau) = R(0)/e \), which can be readily estimated from tabulated data.

Webb (1955) has compared \( \ell \) with \( \ell_c \) for temperature, humidity and vertical and horizontal wind components and suggests that \( \ell_c \) is a good estimate of \( \ell \) to within 30 per cent. The present study makes available a much larger body of data with which this conclusion may be tested.

In Table 2, the average ratios \( \ell/\ell_c \) and their standard errors are given for the cases plotted in Fig. 1. For the humidity, \( \ell_c \) over-estimates the scale \( \ell \) by about 20 per cent. For the case of temperature fluctuations, the \( \ell_c \) values appear to be roughly the same as those of \( \ell \).

<table>
<thead>
<tr>
<th>TABLE 2. AVERAGE VALUES FOR THE RATIO ( \ell/\ell_c )</th>
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<tr>
<td>2 m ( \ell/\ell_c ) Number of samples</td>
</tr>
<tr>
<td>Humidity</td>
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<tr>
<td>Temperature</td>
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4. Spectral Analysis

The spectral estimates \( F(k) \) were obtained in the usual way from the Fourier transform of the autocovariance. The resulting values were corrected for high frequency attenuation due to the finite size of the sensor and the finite response time of the recording system. The estimates were then smoothed through the relation

\[
F(k) = 0.23 F'(k - \Delta k) + 0.54 F'(k) + 0.23 F'(k + \Delta k)
\]

where \( \Delta k \) is the wave number interval (cycles m\(^{-1}\)) at which the estimates were calculated and the primes denote unsmoothed values.

We will consider the temperature spectra first. The values of \( kF(k)/H \) are plotted against \( kz \) on a logarithmic scale in Fig. 2. Each curve is the average of samples with heat fluxes in the range

![Figure 2. Temperature spectra, \( kF(k)/H \) plotted as a function of \( kz \). Curves are the averages of samples with \( H \) in the ranges indicated.](image-url)
indicated. The number of samples in each average is given in brackets. With the exception of A, the curves are closely grouped at both levels and indicate that \( kK(k) \) is a function of \( H \). The apparent irregularity of curve A is a consequence of the inability of the fluxatron to measure small fluxes accurately; the fluxes may be underestimated by as much as 2 mW cm\(^{-2}\).

Since the values for \( kK(k)/H \) at any value for \( kz \) are about equal in Fig. 2(a) and (b), it follows that \( kK(k)/H \) decreases with height, as pointed out by Priestley (1960) for the case of free convection and confirmed by Tsvang (1963) and Myrup (1967) for a wide range of heights.

The treatment of the humidity data was analogous to the temperature discussed above. An estimate of the day-time latent heat flux, \( LE \), was obtained from the relationship

\[
LE = R - H - 0.08R
\]

where 0.08R is the value for the ground heat flux considered appropriate for the present site. The spectral density function was scaled with \( LE \) and the results are given in Fig. 3. Since Eq. (4) applies only between about 1000 hr and 1600 hr, early morning and late afternoon samples have been omitted from Fig. 3. Each curve is the average of those samples with vapour fluxes in the range indicated. The anomalously large values centred around \( kz = 1 \) in curve B of Fig. 3(b) are instrumental in origin.

![Figure 3. Humidity spectra, \( kF(k)/LE \) plotted against wave number, \( kz \). Curves are the averages of samples with \( LE \) in the ranges indicated.](image)

The curves for each height are not as closely grouped as in the case of temperature, probably reflecting, in part, uncertainty in our evaluation of \( LE \). However, it is clear that the magnitude of \( kK(k) \) at any \( k \) is a function of latent heat flux. In addition, a comparison of estimates from the two heights suggests a dependence of \( kK(k) \) on \( k \) as in Fig. 2. Here, the two curves A, which represent the smallest fluxes and hence the largest per-cent errors, are not in accord.

Kolmogorov’s theory of the inertial subrange predicts a range of wave numbers where \( F(k) \propto k^{-5/3} \) and \( \delta = 5/3 \). Experimental studies of this region have been reviewed by MacCready (1962) who suggests that the upper limit in the range, that is the largest eddy size (\( \lambda \)), is given by \( \lambda/\lambda = 0.6 \). Taking this result as a guide, the spectral slopes were calculated for \( k > 0.15 \text{ m}^{-1} \). \( k > 0.15 \text{ m}^{-1} \) was used at both heights since there was no apparent difference between a limiting \( k \) of 0.15 or 0.3 at 2 m. The results in Table 3 are average slopes for all the samples; the errors are standard errors of the means. There is good agreement with the predicted \( -5/3 \) slope in each case.

5. DISCUSSION

The main result to be drawn from this study of the humidity and temperature fine structure is that the two variables are very similar. There is one exception. Large-scale fluctuations contribute more importantly to the humidity variance than to the temperature and result in a character-
istic scale of the humidity field which is larger than the temperature scale (by a factor of about two). It should not be inferred that this difference in the two fields is universal, since the large scales are coupled to the surfaces and therefore may reflect a property of the site. An adequate explanation is not available.

| TABLE 3. The average exponent in the relation $F(k) \propto k^{-\delta}$ |
|-----------------|-----------------|-----------------|
| Humidity        | 2 m             | $1.69 \pm 0.06$ |
| Humidity        | 4 m             | $1.66 \pm 0.06$ |
| Temperature     | 2 m             | $1.72 \pm 0.05$ |
| Temperature     | 4 m             | $1.69 \pm 0.03$ |

The present study suggests that an estimate of the scale of turbulence for temperature is given by the value of $r$ where $R(r) = R(0)/e$ and for humidity by $0.8r$.

The following properties were common to the two variables:

(i) The spectra have a slope of $-5/3$ for $k > 0.15 \text{ m}^{-1}$.

(ii) For a given $k$, the spectral estimates increase with increasing latent heat flux (sensible heat flux for temperature).

(iii) The spectral estimates decrease with height. Evidence supporting this conclusion in the humidity case is not conclusive.

(iv) The scale of turbulence increases with height. Webb’s results suggest that the increase is linear with height for temperature.

(v) With increasing instability beyond near-neutral conditions, the scale of turbulence decreases. This feature is probably a result of a decrease in the horizontal wind and a subsequent reduction in elongation of eddies in the direction of the wind.

It should be emphasized that these results were derived for a range of $k$ limited by the length of the samples. This limitation is particularly critical in the determination of scales where care was taken not to include cases where the sample length was not adequate to define a scale. It is felt that little bias has been introduced by this omission since the estimates of $L$ in Table 1 are fairly well distributed throughout the recording period and hence represent a wide range of conditions.

Acknowledgments

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References


