Studies of turbulence in the surface layer over water (Lough Neagh).
Part I. Instrumentation, programme, profiles

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SUMMARY

Instrumentation is described which has been used over a body of deep water, with fetches of 8 km to 22 km, to provide (i) vertical profiles of mean wind $u$, potential temperature $\theta$ and specific humidity $q$ from the water surface to 16 m height and (ii) turbulent fluctuations of temperature and of the longitudinal and vertical components of wind and hence the vertical fluxes of momentum and sensible heat.

Logarithmic wind profiles in near-neutral conditions lead to a drag-coefficient $C_D$ of the water surface as a function of the wind speed $u_{10}$ at 10 m height given by $10^5 C_D = 0.36 + 0.1u_{10}$ ($3 \text{ m s}^{-1} < u_{10} < 16 \text{ m s}^{-1}$). Corresponding bulk transfer coefficients for heat and water vapour are not distinguishably different from $C_D$ in the observed range of $u_{10} z_0/v < 10^2$ (conventional nomenclature).

An analysis, following Webb (1970), of profiles in stable conditions ($0 < R_i < 0.16$) lends support to the utility of the log-linear form. Using the Monin-Oboukhov relation $\Delta X/\Delta z = X_{ref} (z/L)/(\nu z)$ where $X$ is either $u$, $\theta$, or $q$ and taking $\phi(z/L) = 1 + a z/L$, so that $z/L = R_i/(1 - a R_i)$, we find $a \approx 6$. In unstable conditions ($0 > R_i > 0.08$) the profile shape factor $S_Z = (X_{10} - X_Z)/(X_{10} - X_2)$ is greater for $\theta$ than for $u$, implying $\phi_\theta > \phi_u$ ($K_u > K_\theta$) but $S_Z$ has less well defined behaviour. The data on $S_z$ are not inconsistent with $\phi_u = (1 - 4 z/L)^{-4}$, $\phi_\theta = \phi_u^2$, as proposed by Dyer and Hicks (1970) and others but the shape factor is very insensitive to the numerical factor multiplying $z/L$ in these relations.

Analysis of the measurements of turbulent fluctuations and derived fluxes will be presented in later papers.

1. INTRODUCTION

This and some following papers will describe an investigation of mean and turbulent properties of the surface layer of the atmosphere over an extended body of 'deep' water, and of the form of the underlying water surface. The mean properties at issue are the wind speed $u$, potential temperature $\theta$, and specific humidity $q$, as a function of height $z$ to about 16 m above mean water level; the turbulent properties are the longitudinal and vertical components of wind $u'$ and $w'$ respectively, the air temperature $T'$, and the elevation of the water surface $h'$.

This paper describes the site, instrumentation, data gathering procedure and profile analysis. Later papers will present the results of the turbulence studies.

2. SITE, INSTALLATION AND GENERAL PROCEDURE

The observations have been made at a site 700 m offshore on the eastern side of Lough Neagh (54°40′N, 6°20′E), Northern Ireland, which is a roughly rectangular fresh water lake about 25 km south to north by about 15 km west to east and of rather uniform depth of about 15 m. The fetch in winds with a westerly component, to which observations have been confined in order to ensure a fully adjusted surface layer up to 16 m height, varies from about 8 km to 22 km. Wave energy is not reflected from the nearer shore but refraction of energy at adjacent promontories makes the shortest fetches somewhat indeterminate.

Most of the measuring equipment was installed on an angle-iron platform which was seated on a local mound at 8 m depth of water. It carried a scaffold tower of 12 m height with an extension pole rising to 16 m, the platform deck being about 2 m above mean water level (Fig. 1). Anemometers for the measurement of wind profiles up to the 8 m level were

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removed from the disturbing influence of the platform by installation on a separate 15 cm
diameter wooden mast some 25 m to the west of the platform.

Equipment was installed for the following measurements:

(i) Mean water level – by a float in a still well.

(ii) Wave amplitude – by a fine-wire wave recorder.

(iii) Mean wind speed at approximate heights of 1, 2, 4, and 8 m on the mast, and at

8 to 10 m and 16 m on the tower – by cup anemometers.

(iv) Mean temperature and wet bulb temperature at 0 m (sea), and approximately

0.5, 1, 2, 8, and 16 m, all with respect to dry or wet bulb temperature at 4 m – by thermo-
couples. The 16 m installation was duplicated to give added confidence to the topmost
point of the profile.

(v) Water temperature at about 2 cm below surface and air temperature at 4 m,

primarily for monitoring – by thermistors.

(vi) Fluctuations of longitudinal wind component, inclination of wind, and air

temperature at one or two of the levels 1-5, 4 and 12 m – by miniature cup anemometer,

miniature vane and fine-wire resistance thermometer respectively.

The 0.5 m level was commonly and the 1 m level occasionally below the level of wave
tops so that measurements on those occasions were either omitted or discarded.

All sensor signals were transmitted to shore where they were recorded on 16-channel
magnetic tape or suitably displayed. Features of individual sensor systems of special
interest are elaborated below.

The complete observational system as described above was the culmination of a
development extending over several years. In particular the fluctuation measurements
were introduced at a late phase and the technique of profile measurement of temperature
and humidity (see below) passed through phases of variable satisfaction and utility before
reaching its final form.

Observational runs were either of 15 min duration or, when fluctuation data were
recorded, of 10 min duration. It was very necessary to ensure that no rain or spray fell on
any dry bulb element immediately prior to or during a run. In early profile runs observations
were made of the 'surface' current by timing the displacement of 5 cm diameter × 3 mm
floats from the platform; currents of about 1/25 of the wind speed at a few metres height
were thus obtained.

3. Instrumental Technique

(a) Profile anemometry

Sheppard-type duralumin cups were used with photo-transistor counters in spherical
housings. The amplified signals operated high-speed electromechanical counters giving
unit count per cup revolution. Anemometers were mounted at the end of 1 m arms and
were interchanged in level as opportunity offered. They were calibrated in a wind-tunnel
before every field expedition and were periodically checked on site by side-by-side
exposure on a horizontal arm at 2.5 m over the water near the shore. Mean wind speeds
were thereby measured to a relative accuracy of about 5 cm s⁻¹ which is barely adequate
for the interpretation of profiles in the small gradients over water in other than strong
winds. There is some uncertainty about how much these anemometers overestimate mean
wind speed owing to gustiness of the wind.

(b) Profile thermometry and hygrometry

Because of the relatively small thermal gradients over the sea, it was our aim to keep
the errors in the measured temperature differences to within 0.01°C. Dry and wet bulb
thermocouples were made by welding fine manganin and constantan wires which were
matched for thermostatic power with the main 30 SWG lead wires and then aged by
passing current through them for a day or so. The junctions were enclosed in 20 SWG
stainless-steel tubing welded over at the ends and were insulated from it by an epoxy-resin
Figure 1. View of observation tower on Lough Neagh looking east to near shore. Profile anemometers are seen at about 16 m and 10 m above water level and thermometer housings at 16 m (duplicated) and a number of lower levels. Fluctuation measuring equipment is shown boxed and in canted positions at two lower levels on left and at 12 m at top of braced structure.
filling to a minimum of $10^8 \Omega$ leakage resistance. Each pair of steel probes, and the lead wires to which the fine wires were welded were mounted in an acrylic block for support in a radiation shield and on the mounting arm as shown in Fig. 2. Tests showed that a conduction error of $0.01^\circ C$ in a wind of $2 \text{ m s}^{-1}$ required the temperature at the base of the probe mount to be raised by $\sim 80^\circ C$.

![Figure 2. Construction of the radiation shield and mounting of the dry and wet thermojunctions in it. Detail of thermojunction construction shown inset.](image)

1. Brass support frame
2. 17.5 cm circular plate of steel-reinforced P.V.C. Top surface 1 coated polyester tape
3. Support pillars moulded into 4
4. Pressed polyester sheet black-painted on underside
5. Steel-reinforced P.V.C. 9 cm long x 6 cm wide. Top painted black; underside 1 coated tape
6. Dry bulb
7. Wet bulb with small wick
8. Large supply wick
9. Water bottle
10. Acrylic casting
11. Spring loaded flap to hold small wick to large
12. Stainless steel tube filled with epoxy resin
13. Thermocouple junction
14. Fine wire leads
15. Welded joints to heavier lead wires embedded in acrylic casting.

Thermocouple calibration was carried out by the immersion of junctions in oil-filled Dewar flasks which were placed in temperature-controlled environments. A temperature difference of up to $10^\circ C$, read to $0.1^\circ C$ from calibrated thermometers, was maintained between the two baths. The thermoelectric e.m.f. was found to be uniform to within $\pm 1$ per cent for all couples at $40-3 \mu \text{v per } ^\circ C$.

Each wet bulb was covered with a small replaceable wick of cellulose tissue (holding much more water than a similar muslin wick) which was fed from a large wick and reservoir. The full wet-bulb depression for the installation was obtained with a wind of $2 \text{ m s}^{-1}$, below which observations were not made. Even in the relatively clean air of our site the small wick needed replacement every one to two days to preserve full depression. The large wick remained good for some 14 days.

The radiation shield shown in Fig. 2 was designed to function with natural ventilation, laboratory and field tests showing that the radiation error in light winds and full sun did not exceed about $0.01^\circ C$ so that relative errors were insignificantly small.

The lead wires from each thermojunction passed to a heavily clad junction box at the
4 m level where they were individually joined to copper wires leading to an automatic switching system, again heavily encased, and thence to a fast-response DC amplifier (gain 10\(^4\)) feeding a digital voltmeter on shore. Thus, each thermocouple circuit when being read was completely isolated from the others. The following additional channels were switched into the amplifier: (i) a resistance equal to that of a typical thermocouple circuit to provide a zero and (ii) a reference 40·3 \(\mu\) V, equivalent to 1°C, derived from a mercury cell. There were sixteen channels in all.

A reading on any one channel was taken as follows. The output of the digital voltmeter, as a train of impulses proportional in number to the input offset by the equivalent of 1°C, was fed into a store. Thus zero in the store was at 1°C and a range of \(\pm 1°C\) could be accommodated (\(\pm 10°C\) for the 4 m to sea channel, whose initial output was resistively divided by a factor of ten for compatibility with the other channels). A train of 400 \(s^{-1}\) pulses was then fed to the store, counting up or down according to the sign of the temperature difference, until the zero was reached when the train stopped. This train was also fed, through a scale-of-eight divider, to one of two electromechanical counters according to sign where the reading was deposited. The automatic switch allowed all sixteen channels to be thus sampled in sequence in 1·5 s (the approximate time constant of the thermometer bulbs) and an adequate total of 400 samples additionally obtained in a 10 min run. The scale value, after allowing for zero drift and the reference reading, provided temperature differences to \(10^{-3}°C\) \((10^{-2}°C\) for 4 m to sea). Tests, including the comparison of the duplicated 16 m sensors, indicated reliability to somewhat better than \(\pm 0·01°C\) on a majority of occasions but, for unidentified reasons, appeared occasionally from internal evidence to be no better than about \(\pm 0·05°C\).

(c) Fluctuation measurements

The assembly of the fluctuation sensors is shown in Fig. 3. The miniature cup anemometer and vane were mounted horizontally on either side of the resistance thermometer between stainless steel supports and 29 cm upwind of a box (5 cm)\(^2\) \(\times\) 15 cm containing the electronic circuitry, level adjustors and a clamp allowing attachment to the tower.

The anemometer was formed of three plastic scoop-shaped cups, 2·5 cm long \(\times\) 1 cm wide, which were fastened to a spindle running horizontally in jewel bearings. In such a mounting, with the spindle normal to mean wind direction and the plane of the cups vertical, the response in terms here of instantaneous components \(u, w\) is essentially proportional to \((u^2 + w^2)^{\frac{1}{2}}\), which to good approximation is \((u + w)\) for \(w = 0\). The cup shape was designed to eliminate an aerodynamically induced periodicity in the rate of cup rotation which was found to be present with conventionally shaped cups. It provided moreover a bonus of an extra 14 per cent in the rate of cup rotation. One end of the cup spindle carried a 1·5 cm disc etched at 2° intervals which, with a miniature lamp and phototransistor in the support arm, gave 180 pulses per cup revolution and 360 pulses \(s^{-1}\) per m \(s^{-1}\) of wind. The distance constant of the system was 8 cm run of wind.

The wind vane, 0·6 cm \(\times\) 5·0 cm, was of folded 6 \(\mu\) m polyester sheet supported on 1·7 cm arms pressed from 50 \(\mu\) m acrylic film and attached to a horizontal spindle in jewel bearings. A loop of wire on the spindle varied an inductance buried in the side support and thereby the frequency of an oscillator centred at 10 kc \(s^{-1}\) and giving \(\pm 2·3\) kc \(s^{-1}\) for \(\pm 30°\) vane deflection. The vane was almost critically damped and had a distance constant of 10 cm. Its supporting frame was set up as closely as possible in the horizontal but the frequency for zero inclination of the vane in any one run was taken to be the average frequency during the run.

The resistance thermometer, of 12 \(\mu\) m Pt wire (255 \(\Omega\) at 10°C), was loosely wound on a 0·5 mm nylon filament and included in an oscillator circuit whose frequency varied from 1 kc \(s^{-1}\) to 30 kc \(s^{-1}\) for a temperature varying from 1°C to 30°C.

Prior to and during any run an observer was stationed on the platform to expose and check the performance of each sensor, to direct their supports into the mean wind at the
Figure 3. Assembly of fluctuation measuring equipment. Miniature cup anemometer and counting wheel at top left, fine wire resistance thermometer at centre and miniature vane at right with inductance loop over side support.
beginning of a run and check that they remained within 10° of it during the run, to warn of
rain, and not least to defend the instruments from cormorants and other predators.

The frequency signals from the platform, together with a 10 kc s⁻¹ time signal, were
recorded on the 16-channel magnetic tape running on shore at 75 cm s⁻¹. The tape was
later replayed at half speed into a counter timer, the time signal gating the system at 0·1 s
intervals of original time during which the pulses were counted and punched on paper tape
for later analysis. Resolutions obtained on the digital output for each 0·1 s reading were
3 cm s⁻¹ on the anemometer, 2 × 10⁻³ rad on the vane and 0·01°C on the thermometer.

4. Framework of profile analysis

The vertical variation of a mean property \( X(z) \) in the fully turbulent, constant flux
layer is given by the Monin-Oboukhov similarity analysis

\[
\frac{\partial X}{\partial z} = \frac{X_*}{\kappa} \phi_x \left( \frac{z}{L} \right)
\]

(1)

where \( X_* \) is a scaling parameter for \( X \) equal in individual cases to

\( u_* \equiv (\gamma / \rho) \) for \( X = \) mean wind speed \( u \), \( \phi_u = \phi_M \)

\( \theta_* \equiv H/(\rho c_p u_*) \) for \( X = \) mean potential temperature \( \theta \), \( \phi_\theta = \phi_H \)

\( q_* \equiv E/(\rho u_*) \) for \( X = \) mean specific humidity \( q \), \( \phi_q = \phi_v \)

\( \phi_x \) is an undetermined function of \( z/L \) for property \( X \) where

\( L \equiv -u_*^3/\left( \kappa \frac{g}{T} \frac{H}{\rho c_p} \right) \)

or, to be more precise particularly over water, with the heat flux \( H \) replaced by \( H +
0·07 LE \), where \( LE \) is the flux of latent heat associated with a rate of evaporation \( E \). Other
notation is conventional.

From Eq. (1) and the definition of eddy transfer coefficients \( K_M, K_H \) for momentum
and heat respectively in terms of flux and gradient

\[
\frac{K_H}{K_M} = \frac{\phi_M}{\phi_H},
\]

(2)

and from Eq. (1) and the definition of Richardson number \( Ri \),

\[
\text{Ri} \equiv \frac{g}{T} \frac{\partial \theta}{\partial z} \left/ \left( \frac{\partial u}{\partial z} \right)^2 \right. \]

where \( \theta \) is potential virtual temperature,

\[
\text{Ri} = \frac{z}{L} \frac{\phi_H}{\phi_M^2}
\]

(3)

The form of the functions \( \phi \) has long been at issue. It is, however, well established that
in the absence of buoyancy effects on transfer (\( z/L \to 0 \) \( \phi_M = \phi_v = 1 \), and somewhat less
certainly \( \phi_H = 1 \) also. Then Eq. (1) has the integral

\[
\frac{X - X_*}{X_*} = \frac{1}{\kappa} \ln \frac{z}{z_*}
\]

(4)

where \( X_* \) is the value of \( X \) at the underlying surface and \( z_* \) is the value of \( z \) to which the
log profile extrapolates at \( X = X_* \). For \( X = u \), \( z_* \) by convention is written \( z_\theta \) and is called
the roughness length of the surface. In general it is a function of the geometry of the
surface and of the length \( v/u_* \) where \( v \) is the kinematic viscosity of the air. For the tempera-
ture profile we shall write \( z_\theta = z_T \) and for the humidity profile \( z_* = z_q \).
It has been argued (Stewart 1961; Miles 1965; Phillips 1966) that the logarithmic wind profile might not strictly obtain over the sea because of the particular type of momentum transfer associated with the production of waves and because of the variable level of the water surface about the mean datum. Estimates of departures based on the above analyses indicate, however, that they are likely to be too small to be observed even at the lowest of our levels of measurement.

As $z/L = \zeta$, departs from zero, there is evidence, most cogently presented by Webb (1970) from profile data only, that

$$\phi_\zeta = 1 + \alpha_\zeta \zeta$$

(5)

provides a useful interpolation formula for $-0.03 \leq \zeta \leq 1$. Webb established Eq. (5) primarily from wind profiles but found similarity of profile form for $u(z)$, $\theta(z)$ and $q(z)$ within the given range of $\zeta$ and so inferred $\alpha$ to be independent of $X$. Webb, however, suggested different values for $\alpha$ for $\zeta < 0$ and $> 0$ of 4.5 and 5.2 respectively. To within its standard error in mean $\alpha$ of about 10 per cent it would appear reasonable not to make this distinction and to take $\alpha = 5.0$ which is not inconsistent with Businger, Wyngaard, Izumi and Bradley (1971) results.

Eq. (5) provides the well known log-linear profile

$$\frac{X - X_s}{X_s} = \frac{1}{\kappa} \left( \log \frac{z}{z_s} + \alpha_\zeta \zeta \right).$$

(6)

It also implies, given $\alpha$ independent of $X$,

$$\zeta = \frac{R_i}{1 - \alpha R_i}$$

(7)

Hence, if Eq. (5) retains validity for sufficiently large $\zeta$, a critical value of $R_i = 1/\alpha \approx 0.2$ is indicated, which is not out of accord with quite different lines of evidence. Also, from Eqs. (1) and (5), $K_\zeta = \kappa u_s \zeta / (1 + \alpha_\zeta \zeta)$. Then, with $\alpha$ independent of $X$ and allowing the use of Eq. (7), $K_s = \kappa u_s \zeta / (1 - \alpha R_i)$ which is consistent with the inferred critical value of $R_i$.

For unstable conditions, in the range $0 < -\zeta < 1$. Dyer and Hicks (1970) have shown that the simple relations

$$\phi_M = (1 - 16 \zeta)^{-1}, \phi_H = \phi_M^2$$

(8, 9)

represent their and some earlier data to within a few per cent in grouped mean values while Businger et al. (1971) have analysed other data with quite similar though not identical results. Some of Dyer and Hicks' data were also used by Webb in his analysis and there is evidently some inconsistency between Eqs. (8, 9) and (5) with $\phi_H = \phi_M$. But the very different upper limit of $-\zeta$ in the two cases is to be noted, while Dyer and Hicks state that in the Webb range $0 < \zeta < 0.03$ Eqs. (8, 9) represent the data used by Webb equally well (Eqs. (8, 9) imply $K_s/K_M = 1.12$ at $\zeta = -0.03$ instead of unity according to Webb). Eq. (9) implies $R_i = \zeta$ and our values of $R_i$ in unstable conditions all fall within the maximum value of $-R_i = 1$ provided for by Dyer and Hicks.

We shall attempt to exploit or test the relations (4), (6) and (8, 9) with our own profiles. Rather few of our profiles were accompanied by simultaneous measurements of fluxes and those few will be discussed in a later paper. We confine ourselves here therefore to profile data only. Even of these, many wind profiles were not accompanied by $\theta(z)$, $q(z)$—for instrumental reasons, or because of rain, or spray in high winds.

5. Aerodynamic roughness length and drag coefficient in near-neutral conditions

Because our sample of wind profiles is much larger than of $\theta$ and $q$ profiles, particularly in high winds, and since we wish to exploit the former to the maximum extent in regard to the drag-coefficients they imply, particularly for near-neutral conditions, we have assumed that a wind profile which plots logarithmically in the sense that it is thus well defined, with
no detectable curvature, is representative of near-neutral conditions. There were 233 such profiles from which values of $u_*$ and $z_0$ were deduced from the relation (4) using $\kappa = 0.41$ and the origin of height as given by the still-well. This procedure is admittedly not unexceptionable but it does allow us to cover a wide range of wind speed for which we have reason to believe – see below – that stability effects are marginal, particularly of course at the higher wind speeds. The current speed $u_c$ (≈ $u_*$) at the water surface was neglected. This does not affect the inferred value of $u_*$ but under-estimates $z_0$ by a small amount, ≈ $\kappa$ for $u_c \approx u_*$ – see also below.

Fig. 4 shows a logarithmic plot of $z_0$ as a function of $u_*$ from grouped data, a line of slope corresponding to $z_0 \propto u_*^2$ being also included. It will be seen that the slope decreases materially with increase of $u_*$ but barely falls to two, a suggested limiting value (Kitaigorodsky 1968), at the highest values of $u_*$. In the range of $u_*$, $z_0$ of Fig. 4 there is a variation of the roughness Reynolds number $u_* z_0 / \nu$ from below $10^{-2}$ to almost $10^2$. Further aspects of the form of $z_0(u_*)$ will be discussed more fully in a later paper in relation to the wave spectra observed simultaneously.

![Figure 4. Variation of roughness length $z_0$ with friction velocity $u_*$ from grouped data. Mean deviations and number of observations are shown for each point; also a line of slope corresponding to $z_0 \propto u_*^2$.](image)

The drag coefficient of a surface is conventionally and appropriately defined in terms of the wind velocity (at given level) relative to the surface, that is by $\left\{u_*/(u_{10} - u_*)\right\}^2$ for a reference height for wind of 10 m. In general, however, $u_*$ is not available either in the present series of observations or when using a drag coefficient to obtain oceanic surface stress in general application. We have therefore evaluated a drag coefficient $C_D$ defined by $(u_*/u_{10})^2$ and thus suited to our data and general application. It is less than the more properly defined coefficient by $C_D[(1 - u_*/u_{10})^{-2} - 1]$, ≈ $2C_D u_*/u_{10}$, ≈ $2C_D u_*/u_{10} = 2C_D^{12}$. Our grouped mean values of $C_D$ as a function of $u_{10}$ are shown in Fig. 5 and the relation is seen to be moderately well defined, the equation to the line entered by eye on the Figure being

\[10^3 C_D = 0.36 + 0.10 u_{10} \ldots \ldots 3 \text{ m s}^{-1} < u_{10} < 16 \text{ m s}^{-1} \]  \quad \text{(10)}

The values of $C_D$ for $u_{10}$ less than about 5 m s$^{-1}$ although most subject to relative error, are surprisingly low. Thus if the sea surface were aerodynamically smooth at these
wind speeds the conventionally defined drag coefficient would be expected to be about 0.8 to 0.9 \times 10^{-3} and the magnitude of the correction to $C_D$ given above, a few per cent only, would not lift our values into compatibility, within their scatter, with this value. We have failed to identify any source of systematic error in our measurements which would lead to too low values of $C_D$. For example a constant fractional error in wind speed, either from calibration procedures or from overestimation in gusty winds, would only produce a corresponding fractional change in the $u_{10}$ axis of Fig. 5. Too low a value of $C_D$ would result from using too high a datum of height but a shift of origin sufficient to increase $C_D$ by say 0.2 \times 10^{-3} at the lower wind speeds would have to be an order of magnitude larger than the error in surface level from the still-well ($< 1$ cm). There is no physical basis for setting the origin so low nor would the profiles then plot logarithmically.

Again, those apparently logarithmic profiles accompanied by temperature measurements failed to show any significant departure from the results of Fig. 5, either in the range of $u < 5$ m s$^{-1}$ or for $u > 10$ m s$^{-1}$, when classified according to Richardson number. This is not surprising since temperature differences, surface to air, were always less than 1°C in the first group ($|Ri| < 0.1$) and not often greater than 1°C in the second group ($|Ri| < 0.1$). For $|Ri| = 0.1$, the drag coefficients obtained by using a profile form (1) with $\phi_M$ given by Eqs. (5) or (8) would not differ by more than 5 per cent (negative Ri) or 10 per cent (positive Ri) from the values for neutral stability. For the Ri when $u > 10$ m s$^{-1}$ the differences would be much smaller.

The data of Fig. 5 and Eq. (10) are broadly consistent with less comprehensive data from our site quoted by Phillips (1966).

One naturally asks whether a relation such as in Eq. (10), derived from observations limited to the stated fetches (Section 2), is likely to have validity over the open ocean. The existence of a universal equilibrium range in the power spectrum of wind-generated waves at the higher frequencies (Phillips 1966), found also at our site on Lough Neagh, suggests that extrapolation may be possible. Certainly, we could detect no dependence of $C_D$ on fetch over our limited range of fetch (8 to 22 km) but that has no compulsion on its own. Kitaigorodsky (1968) has argued that $z^2 u_s / \nu$ should be a function of $g \sigma_h / u_s^2$ ($\sigma_h =$ root mean square vertical displacement of surface, $g =$ gravity), from which some dependence of $C_D$ on fetch might arise. Our values of $g \sigma_h / u_s^2$ lie generally in the range 15 to 30 but comprehensive observations in a significantly different range do not appear to have been thus analysed. See, however, the discussion of Wu (1969) at the end of this Section.
Evidence in support of or opposed to a wind speed dependence of $C_D$ such as Eq. (10) is inconclusive. Zubkovsky and Kravchenko (1967) found that their and other measurements of $u_*$ from $-u'w'$ up to the time of their writing supported a relation like Eq. (10) though the motion of a supporting buoy may have introduced systematic error in some of their results. On the other hand, Smith (1966, 1970), Hasse, Brocks, Dunkel and Görner (1966) and Weiler and Burling (1968), also measuring $-u'w'$, could detect no dependence within the wide scatter of their data. Neither could we (see later paper) by this technique used, however, over a materially smaller range of $u_{10}$ than in Fig. 5. Brocks, quoted in Hasse et al. (1966), found no significant variation in $C_D$ with $u_{10}$ from profile measurements on stabilized buoys in the North Sea and Baltic Sea but later measurements of Brocks quoted by Kraus (1972, p. 159) show a slight increase with $u_{10}$.

Wu (1969) has put together all open-sea data on $C_D$, independent of technique (wind profile, sea-surface slope, geostrophic departure of wind, but no $u'w'$ data), according equal weight to each determination. The results strongly suggest an increase in $10^3 C_D$ from about $0.7$ at $u_{10} = 1.5$ m s$^{-1}$ to $2.6$ at $u_{10} = 30$ m s$^{-1}$. Moreover for $u_{10} < 15$ m s$^{-1}$ Wu finds that suitably grouped data accord rather well, considering the notorious difficulty in application of the above techniques, with Charnock's relation $z_0 = a u_*^2 / g$ and gives $0.0156$ as the value of $a$ for best fit. The corresponding expression for $C_D$ in terms of $u_{10}$ for a logarithmic wind profile is

$$C_D = \left( \frac{\kappa}{\ln \left( \frac{g z}{a C_D u_{10}^2} \right)} \right)^2 \quad \ldots \quad z = 10 \text{ m}.$$  

Now, although we have not found agreement with Charnock's relation (Fig. 4) and have entered a linear relation for $C_D(u_{10})$ in Fig. 5, there is only moderate disparity between the line of Fig. 5 and the above formulation of $C_D$ with $a = 0.0156$. The latter gives greater values of $10^3 C_D$ for $u_{10} < 10$ m s$^{-1}$ by $0.2$ or less and for $16$ m s$^{-1} > u_{10} > 10$ m s$^{-1}$ greater by $0.1$ or less.

Wu considers that the open-sea data imply a discontinuity in $C_D$ at $u_{10} = 15$ m s$^{-1}$ from $1.9 \times 10^{-3}$ to a steady value of $2.6 \times 10^{-3}$ for $u_{10} > 15$ m s$^{-1}$ and argues a physical case for the discontinuity or at least for an asymptotic approach to a steady value depending on fetch. Our data throw little light on this question.

We conclude that the uncertainties about the behaviour of $C_D(u_{10})$ are being narrowed but there is clearly scope for more definitive observations. In the meantime a simple working relation like Eq. (10) appears to have some validity over the range of $u_{10}$ stated but is doubtfully extrapolable to materially higher wind speeds.

6. The relation of $z_T$, $z_\theta$ to $z_0$

The evaluation of fluxes of sensible heat and water vapour from bulk transfer relations, analogous to $\tau = C_{\tau} u_{10}^2$ for momentum, has considerable scope. Then, for the near-neutral condition as shown by Eq. (4) or more generally from the integral form of Eq. (1), the bulk transfer coefficient for entity $X$ depends on the surface parameter $z_\theta$ for $X$.

Simultaneous profiles of $u$, $\theta$, $q$ allow $z_T$, $z_\theta$ to be inferred for the associated $z_0$ provided the profile forms are known. Our simultaneous profiles are far fewer in number than our wind profiles and in near-neutral conditions the temperature differences are generally too small to allow of confident extrapolation to obtain $z_T$. We have therefore obtained the $z_\theta$ from such log profiles as were suitable and from profiles (mainly stable conditions) which satisfied the log-linear form (6) – see Section 7. There were 60 such pairs for $z_0$, $z_T$ and 28 for $z_\theta$, $z_\theta$.

Grouped results are presented in Fig. 6 in terms of $\ln z_0/z_T$ and $\ln z_\theta/z_\theta$, which are proportional to the respective sub-Stanton numbers (Owen and Thomson 1963), against $u_* z_0/\nu$. There is no clearly defined difference of $z_\theta$ from $z_0$ over the observed range of $u_* z_0/\nu$ nor of $z_T$ from $z_0$ except perhaps at the higher values of $u_* z_0/\nu$. These results are
not unexpected in that for $u_\infty z_0/\nu < 10$ the surface should be more or less aerodynamically smooth in which case the small differences between the molecular transfer coefficients would be unlikely to lead to material differences in $z_0$, $z_T$, $z_q$. When, however, $u_\infty z_0/\nu$ increases above 10, i.e. in the fully rough régime, $z_0$, $z_T$, $z_q$ become much greater than unity, as shown by Owen and Thomson (1963) and Chamberlain (1966, 1968). High winds would be needed at sea, productive both of relatively high $u_\infty$ and $z_0$ values, to enter the latter régime.

7. THE LOG-LINEAR PROFILE IN STABLE CONDITIONS

We have tested our $u$, $\theta$, $q$ profiles for their conformity with the log-linear form of Eq. (6) along lines used by Webb (1970). Representing the values of $X$ at any pair of heights $a$, $b$ by $X_a$, $X_b$ then in the log-linear range, from Eq. (6)

$$\frac{X_b - X_a}{\ln(b/a)} = \frac{X_\infty}{\kappa} \left( 1 + \frac{\alpha_x b - a}{L \ln(b/a)} \right)$$

(11)

If then $(X_b - X_a)/\ln (b/a) = y$, plots linearly against $(b - a)/\ln (b/a) = x$, using data from each pair of heights, $a$, $b$, the profile is log-linear (simply logarithmic if $y = \text{constant}$) and the intercept of the line on the $x$-axis is $-L/\alpha_x = x_0$ say, and on the $y$-axis is $X_\infty/\kappa$. When simultaneous $u$, $\theta$, $q$ profiles are available and each clearly conforms to the log-linear form, $L$ may in principle be evaluated from the three $X_a$ and so $\alpha_x$ from $x_0$. In general, however, this situation does not obtain (see below). We therefore also follow Webb by using Eq. (7), based on similarity of profiles, to replace $L$ by the appropriate function of $R_i$, namely $L = z/(R_i(1 - z/x_0))$ which leads to

$$\alpha_x = \frac{z}{R_i(z - x_0)}$$

(12)

There is then, of course, a slight inconsistency in providing for an $\alpha$ which may depend on the property $X$ from which it is evaluated.

The recognition of the log-linear profile from the plotting of data according to Eq. (11) has a subjective element which is exacerbated by the small gradients over water (relative to Webb’s overland). An example of a run in which all three of the $u$, $\theta$, $q$ profiles were confidently judged to be log-linear and extrapolation to $x_0$ was merited is shown in Fig. 7(a), another when curvature was regarded to be present in two of the three plots is shown in Fig. 7(b) and there were many others in which scatter prevented assessment. All our lapse cases, when gradients tend to be small, fell into the last category. The method was in any
Figure 7. Examples of simultaneous profiles, in stable conditions, of wind, potential temperature and specific humidity plotted to test for conformity with a log-linear relation: (a) Run 577, showing profiles judged to be log-linear; (b) Run 101, showing profiles judged not to be thus identifiable for $u$ (dots) and $\theta$ (circles) but log-linear for $q$ (crosses).

case found to be too inexact for application when $Ri_s < 0.01$ mainly because the relative error in $Ri$ is then too large.

We have 47 runs of $u$, $\theta$, $q$ in stable conditions for which our assessment of the profiles on the basis of the above $x, y$ plot is given in Table 1 according to the classification there shown.

### TABLE 1. Assessment of stable $u$, $\theta$, $q$ profiles in relation to $Ri_s$ (Richardson number at 4 m) and the following classification:

<table>
<thead>
<tr>
<th>$Ri_s$</th>
<th>$l$</th>
<th>$ll$</th>
<th>$o$</th>
<th>$l$</th>
<th>$ll$</th>
<th>$o$</th>
<th>$l$</th>
<th>$ll$</th>
<th>$o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 0.025</td>
<td>7</td>
<td>10</td>
<td>2</td>
<td>7</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>0.025 to 0.05</td>
<td>5</td>
<td>5</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>0.05 to 0.075</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>0.075 to 0.10</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>0.10 to 0.15</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>&gt;0.15</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>All</td>
<td>13</td>
<td>26</td>
<td>8</td>
<td>8</td>
<td>24</td>
<td>15</td>
<td>12</td>
<td>16</td>
<td>19</td>
</tr>
</tbody>
</table>

The evidence of the frequencies given in Table 1 is that, except at low $Ri_s$ where distinction of class $l$ from class $ll$ is a priori unlikely to be marked, the log-linear form is moderately well evident in our data.
The mean values of $a_x$ from Eq. (12) for the $ll$ cases in Table 1, excluding those for $R_i < 0.01$ or which give unacceptably large departures of $a$ form the median, are as follows:

$$a_u = 5.6 \pm 2.6 \ (20 \ cases), \quad a_g = 5.7 \pm 1.8 \ (17 \ cases)$$

$$a_q = 7.4 \pm 3.1 \ (9 \ cases)$$

The values of $a_u$, $a_g$ are moderately well in accord with Webb's mean value of $a_u$ of 5.2, bearing in mind the standard deviations which are comparable with Webb's. The value of $a_q$ is appreciably higher but in view of the small sample and large standard deviation it is much less to be trusted. We therefore take these results as supporting evidence for the log-linear profile in stable conditions and accord to $a$ the value of 6, though the difference from Webb and Businger et al. (1971) is hardly significant.

There were three runs in which simultaneous $u$, $\theta$, $q$ profiles were each confidently assessed to be log-linear so that Eq. (11) could be used to obtain $a_x$ and $L$ without invoking Eq. (12). The values of $a_x$ so obtained are given in Table 2 together with the values given by Eq. (12) for comparison. Measured $R_i$ and inferred $x/L$ ($z = 4$ m) from Eq. (11) are also included.

<table>
<thead>
<tr>
<th>Run</th>
<th>$a_u$</th>
<th>$a_g$</th>
<th>$a_q$</th>
<th>$a_u$</th>
<th>$a_g$</th>
<th>$a_q$</th>
<th>$R_i$</th>
<th>$(x/L)_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>3.8</td>
<td>7.2</td>
<td>5.7</td>
<td>3.4</td>
<td>4.7</td>
<td>4.1</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>b</td>
<td>2.4</td>
<td>6.2</td>
<td>7.2</td>
<td>2.2</td>
<td>3.7</td>
<td>3.7</td>
<td>0.16</td>
<td>0.22</td>
</tr>
<tr>
<td>c</td>
<td>6.2</td>
<td>4.1</td>
<td>9.6</td>
<td>8.1</td>
<td>5.9</td>
<td>10.7</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>Means</td>
<td>4.1</td>
<td>5.8</td>
<td>7.5</td>
<td>4.5</td>
<td>4.8</td>
<td>6.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The disparities in the values of $a$ for given $X$ as between the two methods are roughly within the uncertainties of the techniques and some support is provided to the conclusion of the previous paragraph. The sense of the difference between $R_i$ and $(x/L)_i$ is as expected.

8. Profile form in unstable and stable conditions

Following the examination in the previous Section of profile form in stable conditions we proceed now to consider unstable conditions and to present our conclusions on these and stable conditions in a single context.

Since we do not in general have measured values of the fluxes of momentum, heat and water vapour in company with our $u$, $\theta$, $q$ profiles we shall examine geometrical properties only of the latter in relation to stability but in a context of other work on profiles in relation to fluxes as discussed in Section 3.

From Eq. (1), the ratio of gradients of $X$ at two heights $a$, $b$ is independent of $X_0$ and given generally by

$$\frac{(\partial X/\partial z)_a}{(\partial X/\partial z)_b} = \frac{b}{a} \frac{\phi_a(a/L)}{\phi_b(b/L)}.$$  \hspace{1cm} (13)

Since gradients are difficult to estimate with any precision they have been replaced by finite difference approximations according to the following scheme.

We have formed a so-called shape factor $S_x = (X_4 - X_1)/(X_{15} - X_1)$ for each observed profile, the subscripts indicating levels of observations in metres. This shape factor is related to the ratio of gradients by assuming that $(\partial X/\partial z)_a$ for $a = 2$ m is given by $(X_4 - X_1)/(a \ln 4/1)$ and similarly $(\partial X/\partial z)_b$ for $b = 4$ m by $(X_{15} - X_1)/(b \ln 16/1)$ which are
exact for logarithmic profiles. Then

\[
\frac{X_4 - X_1}{X_{16} - X_1} = \frac{a \ln 4}{b \ln 16} \frac{(\partial X/\partial z)_a}{(\partial X/\partial z)_b} = \frac{1}{4} \frac{(\partial X/\partial z)_a}{(\partial X/\partial z)_b}
\]

so that, with Eq. (13),

\[
S_x \equiv \frac{X_4 - X_1}{X_{16} - X_1} = 0.5 \frac{\phi_x(2/L)}{\phi_x(4/L)}.
\]

Although we are essentially concerned with departure of profile form from the logarithmic the above procedure gives the ratio of the gradients and so of the \(\phi\)’s to within 6 per cent for gradients lying within a realistic range of height dependence of \(z^{-0.5}\) to \(z^{-1.4}\). This error is within the error of our determination of \(S\) which is estimated to be about 10 per cent.

Since the shape factor is a function of \(\phi\) it is a function also of \(Ri\) and Fig. 8 shows the observed dependence in 136 runs of \(S_u, S_\theta, S_q\) on \(Ri\), the latter being evaluated using finite differences for \((\partial X/\partial z)_4\) as given above though here the accuracy is poor. Before discussing these results we consider the right-hand side of Eq. (14) in the light of the relations (5), (7), (8) and (9).

![Figure 8](image.png)

Figure 8. Shape factors \(S_u, S_\theta, S_q\) (with mean deviations) as a function of Richardson number \(Ri\) for grouped data. Note displaced scale of \(S_u\) relative to that for \(S_\theta, S_q\). Dashed lines are entered for \(\gamma = 5, 16, 80\) in Eqs. (15) and (16) for \(Ri < 0\), and with \(\gamma = 6\) in Eq. (17) for \(Ri > 0\).

For the unstable conditions to which (8, 9) refer, with \(Ri = z/L\), (14) gives

\[
S_u = 0.5 \left( \frac{1 - \gamma Ri_4}{1 - \gamma Ri_4} \right)^{-\frac{1}{4}} = 0.5 \left( \frac{1 - \gamma Ri_4/2}{1 - \gamma Ri_4} \right)^{-\frac{1}{4}},
\]

\[
S_\theta = 0.5 \left( \frac{1 - \gamma Ri_4}{1 - \gamma Ri_4} \right)^{-\frac{1}{4}} = 0.5 \left( \frac{1 - \gamma Ri_4/2}{1 - \gamma Ri_4} \right)^{-\frac{1}{4}},
\]

where we have replaced the numerical factor 16 in Eq. (8) by \(\gamma\). On the other hand, for the stable conditions to which Eq. (7) refers Eq. (14) gives
\[ S_{u,e,q} = \frac{0.5(1 - \alpha R_i)}{1 - \alpha R_i} \]

which with Eq. (5) becomes

\[ S_{u,e,q} = 0.5(1 - \alpha R_i/2) \]  

(17)

Curves have been entered in Fig. 8(a), (b) from Eqs. (15), (16) and (17) as appropriate, with \( \alpha = 6 \) for \( R_i > 0 \) (Section 7) and with \( \gamma = 16 \) for \( R_i < 0 \) as given by Dyer and Hicks (1970) but also with materially different but otherwise arbitrary values of \( \gamma \) of 5 and 80 for comparison.

Fig. 8 shows that \( S_e \) is significantly greater than \( S_u \) for \( R_i < 0 \) but the scatter in observations is too great to say more than that \( S_u(R_i), S_e(R_i) \) are not obviously inconsistent with Eqs. (15) and (16) respectively. It is evident that \( S_e \) is very insensitive to \( \gamma \); an evaluation of \( \gamma \) to within 25 per cent requires \( S_x \) to be measured to about 1 per cent! Thus our plot can only be said to suggest a value of \( \gamma \) of the order of that found by Dyer and Hicks by more direct methods. We shall, however, present evidence in a later paper that our measurements of \( u^* w^*, w^* T^* \) within their narrower limits accord well with Eqs. (15) and (16) and \( \gamma = 16 \).

There is no significant difference in Fig. 8 between \( S_x \) and \( S_q \) for \( R_i > -0.25 \), which is in agreement with Dyer (1967) that \( \phi_q = \phi_q \) in unstable conditions. The two points for \( S_x \) at the largest negative Ri's show a marked drop below the values suggested by monotonic extrapolation from the other values of \( S_x \); this anomaly remains to be accounted for except in terms of experimental error.

We conclude that our profiles provide some support for the use of the expressions given in Section 4, with the numerical constants later evaluated or examined, to provide useful estimates of fluxes to or from the sea surface. The small gradients which usually obtain over the sea and the particular difficulties of their measurement make the test of transfer relationships more than usually difficult at sea. The invocation, however, of the surface condition for temperature and humidity, which is probably well defined in strong winds but not much considered in this paper, is a counterbalancing factor in obtaining useful estimates of flux.

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