Observations on certain features to be seen in a photograph of haloes taken by Dr. Emil Schulthess in Antarctica

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SUMMARY

A more complete theory than has been available before of the Lowitz arc and its associated upward pointing companion, is given. The theory of the supralateral arc to the halo of 46° is also given and it is shown that, at least in the case of the halo photographed, the suggestion put forward by Hastings (1920a) about the nature of the halo of 46° is untenable. It follows that his explanation of the arcs of contact to this halo have also to be revised. Alternative explanations are put forward.

LIST OF SYMBOLS

We shall adopt the following nomenclature. Since we are interested only in directions and not in the positions of rays of light entering and passing through the ice crystals forming the haloes, it is convenient to place the crystal at the centre of the celestial sphere and note the direction of the rays upon it.

O  the centre of the celestial sphere
Z  the zenith
SO  the direction of the sun – i.e. of the incident light
S'O  the direction of a point on the arc or halo being considered
HH  the horizontal plane
Σ  the elevation of the sun
μ  the elevation of S', any point on the arc under discussion
λ  the azimuth of S', measured from the vertical plane through the sun
RO  the direction of any axis in the crystal maintained in a certain plane (usually the horizontal)
β  the azimuth of R, measured from the solar vertical

Normal plane – that principal plane in the crystal at right angles to the refracting edge under discussion.

P  the pole of the face by which the light enters
P'  the pole of the normal plane
θ  the angular distance of the sun from the pole, P', of the normal plane
α  the inclination of any plane (usually the normal plane or a crystal face) to the horizontal

BO  the projection of SO on to the normal plane
AO  the projection of S'O on to the normal plane
Δ  the length of the arc SS' (measured, of course, along a great circle)
D  the angle between the resolved part, BO, of the incident ray in the normal plane, and AO, that of the emergent ray

TO  the projection of SO on to the horizontal plane
NO  the projection of S'O on to the horizontal plane
μ  the refractive index of ice
μ'  the 'Bravais' refractive index
i₁  the angle of incidence of the ray of light on to the face of entry, resolved in the normal plane
r₁  the corresponding angle of refraction
r₂  the angle of incidence within the crystal of the light ray on to the face of emergence, again resolved in the normal plane
i₂  the angle of emergence of the ray resolved in the normal plane.
ON CERTAIN ICE CRYSTAL HALOES

1. Introduction

The photograph with which the theories developed in this paper were compared was supplied to the author by Professor R. S. Scorer, who obtained it from the agency of Dr. Schulthess in New York. It is a similar picture to that published in Dr. Schulthess's book on Antarctica, and was taken at the same time. It shows a fairly complete display of all the haloes which have been recorded. The haloes of 22°, 46° and 90°, the parhelic circle, the 22° mock suns and the upper tangential arcs to the two inner haloes are easily distinguished.

A darker print (Fig. 1), obtained by rephotographing the original one, shows a number of different features while losing some others at the same time. These include the Lowitz arc and its companion, already commented upon elsewhere (Tricker 1970a). Dr. Schulthess informed the author that the photograph was taken with a Beck 'fish eye' lens, and although the characteristics of this lens are not known, the existence of so good a photograph of such a complete halo display enables a number of points, which have rarely been sufficiently accurately measured in previous visual recordings, to be debated. In particular it casts doubt on several of the views put forward by Hastings (1920a) in a celebrated paper. In his paper, however, only the results of calculations were presented, and it is not possible to follow them in detail, or indeed to find out how detailed they were. One of the objects in pursuing them further in the present paper has been to set out the theory sufficiently fully for it to be checked by anyone interested. The demands upon the spatial imagination are such that it is by no means easy to avoid mistakes.

Fortunately the photograph now to be discussed was taken with the sun at the centre of the field of view, so that in spite of distortions of the fish eye lens, the angles between the various radii vecortes from the sun to the features to be seen will be correctly reproduced. Many of these features can be reproduced by means of a water prism and their general character checked.

2. The arc of Lowitz

We will begin by considering the arc of Lowitz. The formation of the arc of Lowitz, which passes through the 22° mock sun and is tangential to the halo at a point of lower elevation, was correctly attributed by Hastings to refraction in hexagonal ice crystal plates which spin, or are randomly distributed, about a diagonal which is maintained horizontal during fall through the atmosphere. His theory, however, is clearly defective in that he considers only those crystals which, as they spin, pass through the horizontal in such a position as to give rise to the mock sun. It is a minimum deviation theory with the added assumption that the whole Lowitz arc is formed by crystals which spin about a fixed position in the horizontal plane. It is indeed, in this respect, little in advance of the discredited theory of Pernter (Pernter and Exner 1922) in which the arc is attributed to crystals oscillating in one particular plane, to the neglect of all the others.

Hastings also failed to predict the formation of the upwards sloping companion to the Lowitz arc. Fujiwhara and Oti (1919) also put forward a minimum deviation theory of the Lowitz arc which is not tenable (see Tricker 1970b) as well as theories of what they called pseudo Lowitz arcs. The essence of the minimum deviation method is to divide the crystals into groups, for each of which the distance $PS = \theta$ is the same. It is then assumed that the arc being calculated will correspond to the locus of the points of minimum deviation of these groups. This argument is adequate in certain cases only, such as in Fig. 2(a), where the arcs generated by each group are indicated, but a glance at Fig 2(b) will show that the locus of points of minimum deviation, CD, can depart widely from the envelope AB of the traces formed by each group, and which will, in fact, determine the visible arc.

A minimum deviation theory is adequate for that portion of the Lowitz arc which lies between the mock sun and the point of contact with the halo (Tricker 1970c). The passage of the arc through the mock sun and the position of the point of contact are, in fact, given accurately by such a theory. Beyond the point of contact with the halo, however, the theory is inaccurate. The availability of photographs renders more accurate calculations desirable.
Figure 2. Illustrating the fact that it is the envelope to the constituent traces, and not the locus of points of minimum deviation, which gives rise to a visible arc.

Figure 3. Refraction through a rotating crystal plate.
Figure 1. Print copied from a photograph taken by Dr. Emil Schultheiss in Antarctica using a Beck ‘fish eye’ lens covering an angle of approximately 200°. The contrast has been increased by printing on to hard paper. Unless a series of prints is made some phenomena are apt to be lost in such a process, since a single exposure cannot be correct for all, and any attempt at shading during printing is apt to lead to unwanted artefacts.
3. Theory of the Lowitz arc

To trace the Lowitz arc we shall assume that it is formed in hexagonal ice crystal plates which rotate, or are randomly distributed around a horizontal diagonal RR (Fig. 3), by light which enters a face, AB, adjacent to RR, and leaves by one, FE, which is parallel to it. We shall calculate the direction of the emergent light for all angles of incidence, and not for those rays only which give rise to minimum deviation. For that purpose we follow Bravais (see for example Tricker 1927d) who showed that light leaves a prism inclined at the same angle to the normal plane as when it entered, and that the projections of the rays on to the normal plane follow the same laws of refraction as do rays lying in that plane, so long as the 'Bravais refractive index'

\[ \mu' = \frac{\sqrt{\mu^2 - \cos^2 \theta}}{\sin \theta} \]

is substituted for the usual refractive index \( \mu \).

![Figure 4. The directions of Fig. 3 transferred to the celestial sphere.](image)

Let Fig. 4 represent the position on the celestial sphere when the normal plane is inclined at an angle \( \alpha \) to the horizontal, as the crystal spins about the direction R in the horizontal plane, the azimuth of R measured from the solar vertical being \( \beta \). Since the ray emerges at the same inclination to the normal plane as when it entered, the distance of the new point of origin S' from the pole P' of the normal plane is unaltered, so that

\[ SP' = S'P' = \theta. \]

Let us take \( \theta \) and \( \beta \) as independent variables and work out the locus of S' for a given \( \theta \) by varying \( \beta \). This is a matter of simple spherical trigonometry.

The right angled spherical triangle RST is fixed and thus RS is known in terms of \( \Sigma \) and \( \beta \). In turn the spherical triangle P'SR, in which P'R = 90°, is determined in terms of \( \Sigma \), \( \beta \), and \( \theta \), and so also is the angle RP'S (marked \( \delta \) in the Figure). The triangle SRB is then also known and thus too is the angle SRB.

If now the light rays in Fig. 3 be taken to represent the resolved parts of the rays in the
normal plane, we see that δ (being the angle between RR and the incident ray) is connected with the angle of incidence, \( i_1 \), by
\[
i_1 = \delta - 30^\circ.
\]

From the angle \( \theta \) we have the Bravais refractive index, \( \mu' \), and we can trace the rays, resolved in the normal plane, through the prism and calculate the deviation, \( D \), in that plane.

S'R is now calculable from the triangle P'S'R, fixing the triangle S'RA. This gives the angle S'RA. Finally we have the right angled triangle S'RN, since the angle S'RN = S'RA - \( \alpha \) and we know \( \alpha \) from \( \alpha = SRT - SRB \).

Thus we finally possess \( \mu \), the altitude, and NR + \( \beta \), the azimuth of S', in terms of \( \Sigma \), \( \theta \) and \( \beta \).

By giving a series of values to \( \theta \) and \( \beta \) we can calculate in this way the azimuth, \( \lambda \), and the altitude, \( \mu \), for points on the origins of the emergent beams. If these are plotted for constant values of \( \theta \) we obtain curves corresponding to crystals so placed that the angular distance of the sun from the pole of the normal plane is fixed. These are the groups of crystals which are considered in the theories, such as those of Perner, in which only positions of minimum deviation are calculated. If, on the other hand, we fix \( \beta \) and plot curves by allowing \( \theta \) to vary, we obtain traces on the celestial sphere produced by light which has passed through crystals spinning about fixed directions in the horizontal plane.

The choice of a projection on which to display the results in a plane figure is always a matter of some difficulty, since they all distort the appearance to a greater or less extent. The one adopted for this purpose in the following Figures is that of projection on to the tangent plane of the celestial sphere at some convenient elevation, the same as that of the sun in the case of the 22° halo, and 50° in that of the 46° halo. This would correspond to a photograph taken of the phenomenon with the camera aimed at this elevation (Tricker 1970e). In most of the Figures the sun is placed off centre, so that the middle of the picture is occupied, so far as is possible, by the phenomenon in question, and distortion reduced to a minimum.

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**Figure 5(a).** Lowitz arc for a solar elevation of 50°. Curves for \( \theta = \) constant. Projection on to the tangent plane to the celestial sphere at an elevation of 50°.
Figure 5(b). The same data plotted as curves for $\beta$ = constant.

Figure 5(c). Curves for $\beta$ = constant for a solar elevation of 30°.
Fig. 5(a) shows the curves drawn for values of $\theta = \text{constant}$, when the sun is at an elevation of $50^\circ$. For values of $\theta$ less than about $90^\circ$ the envelope follows closely the points corresponding to minimum deviation. For values of $\theta$ greater than about $100^\circ$, however, the curves no longer intersect and no envelope exists. An arc drawn through positions of minimum deviation would be completely misleading. It would appear that the arc would turn abruptly away from the sun after the point of contact with the halo was passed.

Fig. 5(b) shows the same set of results plotted in curves given by $\beta = \text{constant}$. These curves are the traces which would be drawn on the celestial sphere by light passing through crystals spinning about fixed directions in the horizontal plane. Hastings considered one of these only—roughly that corresponding to $\beta = 80^\circ$. The same envelope is obtained to the curves of Fig. 5(b), of course, as that given by Fig. 5(a), since both represent the same set of data.

Fig. 5 (c) and (d) show the curves for $\beta = \text{constant}$ with the sun at altitudes of $30^\circ$ and $0^\circ$ respectively. In Fig. 5 (d) there is strictly speaking no envelope to the curves at all. As $\beta$ diminishes the curves advance towards the halo until $\beta$ is about $70^\circ$. With further decrease in $\beta$ they become stationary and then retreat. In this case the visible arc would be formed in the stationary position. In Fig. 5 (c) the changes are, in character, between those displayed in Fig. 5 (b) and (d). At first there is an advance towards the halo, as in Fig. 5(d) but it is accompanied by a downwards displacement leading to an envelope, before the final stage of retreat is reached.

To complete the theory it is desirable to consider not only the position of the curves but also the distribution of the luminous intensity along them. This will be determined partly by the writing speed with which the trace is drawn, partly by reflection and refraction coefficients depending on the angle of incidence, and partly by geometrical factors which control the width of the beam transmitted, and depending on the shape of the crystals. Little meaningful calculation can be done on the last factor because of the complete absence of information about the dimensions of the crystals. The writing speed is,
however, lowest and the trace, therefore, brightest near the centre of the curves. Much the same is true of the net effect of the reflection and refraction coefficients. Furthermore, if the crystal plates are thin, so that light is 'piped' across by means of a series of total reflections off the end faces, then there would be an increase in illumination corresponding to \( \theta = 90^\circ \), because of direct transmission at this inclination. This again emphasizes the luminous intensity in the middle portions of the \( \beta \) curves.

The Lowitz arc is clearly recorded in Fig. 1 passing through the left hand mock sun. We now turn our attention to the theory of the upper arc, which is also visible in the photograph, passing through the same mock sun. This arc has already been reported upon (Tricker 1970a) and we now wish to provide a more complete theory of the phenomenon on the lines of that just given for the Lowitz arc.

4. Theory of the upper companion to the Lowitz arc

That an upper arc must be a possibility, if the foregoing theory of the Lowitz arc is correct, may be seen by calculating the points of contact of the two arcs with the halo. This can be done very simply. When the arcs touch the halo the light must suffer minimum deviation, and thus the normal planes of the crystals refracting the light must pass through the sun, and the crystals themselves must be so oriented as to deviate the light through the minimum angle – in this case 22°. Referring to Fig. 6, the ray LMNO is deviated through 11° at M on entering the crystal, and through a further 11° at N on leaving, the path within the crystal being parallel to the face which truncates the refracting prism. Crystals which rotate about the axis \( R_1R_1 \) give rise to the Lowitz arc already considered, and we see that the angle between the incident ray, LM, and \( R_1R_1 \) is 71° (i.e. 60° + 11°) and that the emergent ray NO is deviated so as to come from within the acute angle 71°. When the crystal rotates about the axis \( R_3R_3 \) it gives rise to the upper arc. The angle between LM and \( R_3R_2 \) is 49° (i.e. 60° – 11°) and in this case the emergent ray NO comes from within the acute angle 180° – 49°. Fig. 7 represents this state of affairs transferred to the
celestial sphere. The two positions of the normal plane are $R_1SR_1$ and $R_2SR_2$ respectively, and $SR_1 = 71^\circ$, and $SR_2 = 49^\circ$. The distances $SS'_1$ to the point of contact of the Lowitz arc, and $SS'_2$ that to the point of contact of the upper arc, will, of course, both be $22^\circ$. The two cases are not quite symmetrical about the horizontal through the sun, the upper arc touching the halo higher above it than the Lowitz arc does below. The two angles, $\sigma_1$ and $\sigma_2$, can be calculated.

For an elevation of the sun of $30^\circ$, corresponding approximately to the photograph,

$$\cos TR_1 = \frac{\cos 71^\circ}{\cos 30^\circ} \quad \text{and}$$

$$\cos TR_2 = \frac{\cos 49^\circ}{\cos 30^\circ}$$

$$\sin \sigma_1 = \frac{\sin TR_1}{\sin 71^\circ} \quad \text{and}$$

$$\sin \sigma_2 = \frac{\sin TR_2}{\sin 49^\circ}$$

These equations give $\sigma_1 = 78^\circ 30'$ and $\sigma_2 = 60^\circ 0'$. These angles can be measured on the photograph approximately, and give satisfactory agreement with the theory.

The theory of the upper arc follows closely that already given for the Lowitz arc. The crystal now rotates about the axis through the angles $F$ and $C$ (Fig. 3) so that the axis of rotation is at right angles to the normal to the face of entry, $AB$, and the angle $\delta$ is now given by,

$$\delta = i_1 + 90^\circ.$$ 

Otherwise the theory is the same as before and we can trace the arcs formed by crystals whose axes of spin lie in given directions in the horizontal plane. The curves are drawn in Fig. 8(a), (b) and (c), on the same projections and for the same elevations of the sun as before, namely $50^\circ$, $30^\circ$ and $0^\circ$ respectively. Their general character is very similar to the
Figure 8(a). The upper companion to the Lowitz arc for a solar elevation of 50°. Curves for $\beta = \text{constant}$. 

Figure 8(b). Curves for $\beta = \text{constant}$ for a solar elevation of 30°.
Figure 8(c). Curves for $\beta = \text{constant}$ for a solar elevation of $0^\circ$.

Figure 8(d). The Lowitz and upper companion area for a solar elevation of $50^\circ$ (the envelope of the curves in Fig. 5(a) and (b), and Fig. 8(a) respectively.

Figure 8(e). The same for a solar elevation of $30^\circ$.

Figure 8(f). The same for a solar elevation of $0^\circ$. 
curves for the Lowitz arc and calls for no further comment. Fig. 8(d), (e) and (f) display the envelopes to these individual curves, which form the visible arcs generated (i.e. the Lowitz arc and its upper companion).

5. **Upper arc of contact to the 22° halo**

We shall not comment on the arcs associated with the upper part of the 22° halo, which are visible in the photograph, except to point out why the theory given by Perner (Perner and Exner 1922) for the upper arc of contact is accurate although based upon the method of minimum deviation which we have criticized earlier. The reason is as follows.

The upper arc of contact originates in long hexagonal prisms which sink through the air with their principal crystal axes horizontal. They either spin or are randomly oriented about this axis. The refraction occurs in the long sides of the prisms and the normal planes are at right angles to the horizontal axes. For a given position of the axis, and thus also of the pole of the normal plane, light will emerge coming from a small circle on the celestial sphere passing through the sun and having its pole on the horizontal axis. In Fig. 9 a number of such small circles, A, B, C, D, E, F, and G have been sketched on the sphere, corresponding to different positions of the axis of rotation. The emergent light has been indicated by cross-hatching on the circles. The caustic curve formed by the total of such small circles will obviously pass through the positions of minimum deviation, a, b, c, d, e, f and g, which will therefore give the position of the upper arc of contact. Thus the theory, which has been amply confirmed, at least qualitatively, by observation, will be accurate.

![Figure 9. Sketch showing on the celestial sphere the production of the upper arc of contact to the halo of 22°.](image)

6. **The supralateral arc of contact to the halo of 46°**

We may now turn our attention to phenomena associated with the halo of 46° which, following Cavendish, is usually attributed to refraction in ice crystals randomly arranged and furnishing a refracting angle of 90°. This was questioned by Hastings (1920b) who
suggested that what was normally observed was not this halo at all but the supralateral arc of contact to it, which lies close to the expected position of the halo. The suggestion requires careful consideration since it is possible that it has influenced some subsequent observations (Goldie and Heiges 1968; see Tricker 1970e). To consider it we will trace the supralateral arc throughout its length, and not merely close to its point of contact with the halo.

![Figure 10. Refraction in ice crystals giving rise to the supralateral arc of contact to the 46° halo.](image)

The supralateral arc to the 46° halo is formed by refraction in ice crystals in the form of long hexagonal flat ended prisms, as in Fig. 10, which descend with their long axis, RR, horizontal — the same group of crystals in fact which give rise to the upper arc of contact to the 22° halo, although in that case the ends need not be flat. When the face ABba by which light enters is also horizontal, refraction produces the circumzenithal halo. When the face of entry is inclined at a small angle to the horizontal, the effect is to tilt the circumzenithal halo to the right or left, according to the direction of tilt. (Since rotation of the axis through 180° in the horizontal plane reverses the angle of tilt, both tilted arcs arise from the one angle of inclination as RR moves over the whole 360°.) As the angle to which ABba is tilted increases, the arc produced is tilted further and its arms gradually close up. When the tilt is such that the normal plane passes through the sun when in the position of minimum deviation, the arc is tangential to the 46° halo. This occurs provided the altitude of the sun does not exceed 22°. The supralateral arc will be the envelope of the arcs described above, each of which originates in crystals whose face of entry is inclined at a given angle to the horizontal. Near to the point of contact with the halo an approximate calculation on the basis of minimum deviation would suffice, but as the arc departs from this prediction further away, it is necessary to proceed by means of the envelope.

We see from Fig. 10 that the face by which light enters (ABba), that by which it leaves (ABCDEF) and the normal plane (npqr) are mutually perpendicular. Transferring the directions to the celestial sphere we obtain Fig. 11, which is drawn for the particular case of the point of contact with the halo. For the arc to touch the halo not only must the normal plane pass through the sun, but the angle of incidence, SP, on the face of entry, must correspond to minimum deviation in a 90° prism. That is $8^\circ - 68^\circ$ and SR = $22^\circ$. The exit face is vertical and there is one place only where this can occur, on each side of the halo, for a given elevation of the sun less than $22^\circ$. When the sun is on the horizon the point of contact is also on the horizon. As the elevation of the sun increases the point of contact ascends the halo and reaches the vertex when the sun is at an altitude of $22^\circ$, when both the supralateral arc and the circumzenithal halo become tangential to the 46° halo at this point.
With the sun at a greater elevation the arc no longer makes contact with the 46° halo at all, but it continues to be tangential to the circumsenithal halo, so long as the latter is formed. When the altitude of the sun exceeds 32° the circumsenithal halo collapses into the zenith, light then being totally reflected from the vertical exit faces of the prisms.
To calculate the constituent curves the envelopes of which form the supralateral arc, we proceed from Fig. 12 as follows, treating $\beta$, the azimuth of the axis of rotation as an independent variable. SR and the angle SRT are calculated from the triangle SRT, in terms of $\Sigma$ and $\beta$.

The angle

$$\text{SRP}' = \text{SRT} + \alpha$$

and hence we know SP' from the triangle SRP', the angle RP' being 90°.

The angle of incidence, $\theta$, on to the normal plane, from which we calculate the Bravais refractive index, is

$$\theta = SP' = S'P'.$$

From the right-angled triangle SBR we obtain $i_1$, the angle of incidence on to the face of entry of the rays resolved in the normal plane, in terms of SR, and thus in terms of $\alpha$, $\Sigma$, and $\beta$.

Tracing the refraction through the prism in the normal plane as before (but using the fact that $r_1 + r_2 = 90°$ in this case, of course) we obtain $i_2 = AR$, the angle of emergence, again resolved in the normal plane.

The triangle S'RA now being known, we have S'R and the angle S'RA. The angle S'RN follows from the relation,

$$\text{S'RN} = 90° - \text{S'RA} - \alpha.$$

Finally we then obtain the altitude, $\mu$, of S', and the value of RN from the triangle S'RN. The azimuth of S' is, of course, RN $- \beta$.

By giving $\beta$ a series of values from 0° to 180°, we obtain azimuth and altitude for points on the locus of the beam emerging from crystals tilted at the angle $\alpha$. Repeating this for a series of values of $\alpha$ gives the family of curves and the envelope which constitutes the supralateral arc.

This has been done for elevations of the sun of 8°, 22° and 30° respectively in Fig 13(a), (b) and (c). The resulting envelope (marked supralateral arc) has been drawn in Fig. 13(d), (e) and (f). The projection used in each case is on to the tangential plane of the celestial sphere at an elevation of 50°, to correspond to what would be obtained with a camera elevated at this angle. The sun has been placed at an azimuth of 30° off centre in the case of solar elevations of 8° and 22°, and 40° off centre for the solar elevation of 30°, in order to bring the arcs reasonably into the centre of the field of view.

While it might be possible, in the case of visual observation, to mistake the supralateral arc of contact for the halo of 46°, as suggested by Hastings (Hastings 1920), such an error could hardly be made in a photograph if the arc is of reasonable length and the correct projection used for comparison (Tricker 1970e). The ratio of the maximum to the minimum radius vector from the sun to points on the upper two quadrants of the supralateral arc is 1:22, 1:13, and 1:27, for solar elevations of 8°, 22° and 30° respectively, amply sufficient to distinguish the figure in these cases from a circle even visually, if it is fairly complete. The halo in Fig. 1, where the sun has an altitude of approximately 30°, is exactly circular. (Fortunately the photograph was taken with the camera aimed directly at the sun so that the circle is not distorted by the action of the lens.) In this particular case, at least, the suggestion of C.S. Hastings is clearly not operative.

In the studies of Liljequist (1952) in Antarctica in which the ice crystals present in low ice mists giving rise to haloes were examined photographically, usually both columns and plates were present when the halo of 46° was visible. He does, however, record one case in which only plates were present. The 46° halo in this case could hardly be the supralateral arc in such circumstances, since plates seem unlikely to give rise to it.

7. THE UPPER ARC OF CONTACT TO THE HALO OF 46°

The relation of the circumzenithal arc to the halo of 46° has long been confusing. With the sun at an elevation of 30° the circumzenithal arc lies about 3½° – seven solar diameters –
Figure 13(a). Supralateral arc to the halo of 46° for a solar elevation of 8°. Curves for \( \alpha = \) constant – i.e. curves traced out by light passing through crystals the normal planes of which are inclined at an angle \( \alpha \) to the vertical. The supralateral arc will be the envelope of these curves. The curve for \( \alpha = 0 \) will be the circumzenithal arc.

Figure 13(b). The same for a solar elevation of 22°.
Figure 13(c). The same for a solar elevation of 30°.

Figure 13(d). Supralateral and upper arc of contact to the halo of 46°, for a solar elevation of 8°.
Figure 13(e). The same for a solar elevation of 22°.

Figure 13(f). The same for a solar elevation of 30°.
above the halo. Lack of exact alignment of the crystals could reduce this gap but at the same time it is likely to make the arc pale and inconspicuous. There are, in fact two contact phenomena which may be expected to occur in this region of the sky. One is that of the supralateral arc and the circumzenithal halo already discussed in Section 6. The other is that of the halo of 46° and its upper arc of contact.

According to Hastings it is the first of these contact phenomena which has been observed. Against this is the further argument that if a circumzenithal arc arises from the long hexagonal crystals which produce the supralateral arc (a proportion being preferentially aligned with two prismatic faces horizontal) we would expect to find an arc of similar intensity formed by refraction in faces inclined at angles of 60° to the horizontal, such as AF₁ and BC₁b (Fig. 10). This would be the arc labelled α = 60° in Fig. 13(c). No arc approaching this intensity is to be seen in this position in the photograph.

Refraction at right angle edges in the rotating flat crystal plates, as in Fig. 14, will produce arcs of contact to the halo of 46°. Refraction at the edge AB, which is parallel to the axis of rotation, gives rise to an upper arc of contact. Its position is obtainable accurately by a minimum deviation calculation which has been given elsewhere (Tricker 1970f). The arc has been plotted in Fig. 13(d), (e) and (f). It is, of course, tangential to the halo of 46° and lies below the circumzenithal arc unless the sun is at an elevation of 22°, when the two coincide at the point of contact.

It may be of interest to note, in passing, that the figure containing a pair of unidentified arcs, given by Goldie and Heighes for the Berkshire halo of 11 May 1965 (Goldie and Heighes 1968) could be interpreted as showing the circumzenithal arc (arc a in their figure), the supralateral arc (arc b), the upper arc of contact (arc c) and the 46° halo itself (arc fd). Mr. Goldie informs the author however, that the arcs a and b, and those labelled c and fd were not observed on the same occasion by any observer and that it is possible that the arcs a and c were the same, and so also the arcs b and fd.

Two recent observations, however, appear to confirm unequivocally the occurrence of a true upper arc of contact to the 46° halo, as distinct from the circumzenithal halo. The first is by Hättinga-Verschure, who records the arc with a solar elevation of 33° (Hättinga-Verschure 1971). A circumzenithal arc, properly so called, is impossible with the sun at this elevation, and if, because of small imperfections in the alignment of the crystals, such an arc might appear, it would be unlikely to be tangential to the halo of 46°.

The second observation is contained in two excellent photographs taken by Dr. W. Evans of the University of Saskatchewan, so far unpublished, in which both circumzenithal halo and arc of contact appear to be present together. At a solar elevation corresponding to the photographs the two arcs are theoretically separated by rather less than the width of either. The photographs show a red arc making accurate contact with the red of the 46° halo and beyond this a blue arc. Between the two is a very conspicuous white area such as is to be expected where the blue of the arc of contact is superimposed upon the red of the circumzenithal halo. The phenomenon has been sketched in Fig. 15.

8. 46° MOCK SUNS

For a mock sun to be formed at the intersection of the parhelic circle with the halo of 46°, analogously to the formation of the common 22° mock sun, it would be necessary for a 90° refracting edge to be maintained preferentially vertical during the fall of the ice crystals through the atmosphere. It is not easy to envisage a mechanism which would secure this. Mock suns associated with the halo of 46° have, in fact, rarely been reported.

Lying just below the parhelic circle on each side of the 46° halo in the photograph by Dr. Schultheiss (Fig. 1) however, are two bright patches of light which might, at a cursory glance, be thought to be 46° mock suns. Since they lie below the parhelic circle they do not belong to the ordinary run of mock suns. Actually they coincide in position with the point of contact with the halo of an arc which would be produced by light which, entering the upper face of a rotating crystal plate, such as ABCDEF (Fig. 14), leaves by a prismatic face
adjacent to the axis of rotation, such as FAaf or BCcb. This provides a possible explanation of their formation. (Their position does not agree with an interpretation as the point of contact of the ordinary infralateral arc.) There are, indeed, indications in the photograph of what may be the arcs of contact themselves but their position has not been computed theoretically for the whole of their length and so they cannot be identified positively.

![Diagram](image)

**Figure 15.** Sketch of the circumzenithal arc overlapping the upper arc of contact to the halo of 46°, from a colour photograph taken by Dr. W. Evans (published in the June 1972 issue of *Weather*)

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**References**

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