Electric field recovery after lightning as the response of the conducting atmosphere to a field change

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(Manuscript received 30 November 1971; in revised form 6 March 1972)

SUMMARY

Computations are presented to show that the rearrangement of space charge existing in the fair weather conductivity profile of the atmosphere can explain the observed rapid recovery of electric field after a lightning flash without invoking a core of high conductivity within the cloud or charge at the cloud boundary. An exponential conductivity profile accurately accounts for the first second of observed recoveries at the ground, predicting increasingly rapid recoveries at greater distances. The whole of the rapid recoveries observed above clouds is also explained, but the suggestion that recoveries become even more rapid at higher altitudes remains to be verified by observation.

The conductivity near the ground is less than that given by the exponential profile, and if the model is restricted to include only the clear air above the cloud, the complete recovery curve at all distances on the ground is explained, while recoveries above the cloud are unaffected. The slight but significant deviations from the exponential form of distant recoveries observed on the ground are also accurately forecast.

The simple Coulomb field of a regenerated charge within the cloud cannot result in more rapid recoveries after distant discharges than after close ones.

1. INTRODUCTION

The variation of the electric field at the ground during a thunderstorm often follows a characteristic pattern. The sudden changes in the value of the field usually occurring in less than one second have been identified with the neutralization of electric charge by a lightning flash. Until recently the electric field variations between the flashes, the so-called 'recovery curve', had been interpreted as the simple Coulomb field of the charge within the cloud as it increased in magnitude until another flash followed.

The exponential form of the recovery curve led Wilson (1929) to postulate a constant generation current with a dissipation current proportional to the charge regenerated. Observing that a typical recovery curve has a time constant of only 15 s Freier (1962) concluded that the dissipation currents were large and the conductivity within the cloud was very high. Colgate (1969) and Imyanitov, Evteev and Kamaldina (1969) proposed that this effective high conductivity results from turbulent redistribution of charge. Sartor (1967) computed values of field growth within the cloud and compared them with recovery curves observed at a distance from the storm. In an argument summarized by Mason (1971), Illingworth (1971), finding that the recovery rate of the field for distant flashes tends to increase with increasing distance from the storm, and to be uncorrelated with the frequency of the flashes, deduced that rearrangement of the space charge existing around and above the storm as a consequence of the conductivity gradient in the fair weather atmosphere has a profound influence on the distant recovery curves. For this reason and because close recovery curves were often irregular, he deduced that the recovery curves give no useful information about the thunderstorm itself.

Taking the pre-flash field level as zero and defining $T'$, $T''$ and $T'''$ as the times in seconds taken for the field to recovery to $1/\sqrt{e}$, $1/2$ and $1/e$ respectively of its peak value $(\Delta E)$ Illingworth found the following statistical relationships:

$$T' = (0.32 \pm 0.01) \Delta E^{0.47 \pm 0.04}$$

in the range $\Delta E = 10$ to 1,000 V/m. For values of $T'$ less than 5 s the following fits were obtained:

604
\[ T''' = (0.53 \pm 0.36) + (2.76 \pm 0.11)T' \]
\[ T'' = (0.02 \pm 0.09) + (1.67 \pm 0.03)T' \]

whereas for a perfect exponential we would expect \( T''' = 2T' \) and \( T'' = 1.39T' \).

In this paper we present a theoretical calculation for the recovery curves at various distances, to support the statistical deductions from observation that the recovery curves beyond 10 km are merely the response of the conducting atmosphere to the field change. Because of the vertical gradient in conductivity existing in the clear air, a steady charge maintained in the conducting atmosphere results in a space charge in the conductivity gradient, and at larger distances this space charge is increasingly effective in screening the field of the steady charge. A lighting discharge will remove the steady charge and the recovery curve caused by the rearrangement of the screening space charge is computed. For distant discharges the term ‘field decay’ would be a more accurate description than ‘recovery curve’ which was coined by early workers.

2. Mathematical formulation

Holzer and Saxon (1952) found the analytic solution for the steady state problem of the potential \( \phi \) due to a point charge embedded in a medium whose conductivity \( \lambda \) is increasing exponentially with height \( z \) as \( \lambda = \lambda_0 \exp(2kz) \), where \( k = 0.11 \text{ km}^{-1} \). Under these circumstances the current continuity equation \( \text{div} j = 0 \), reduces to the differential equation

\[ \nabla^2 \phi + 2k \frac{\partial \phi}{\partial z} = 0. \]

An appropriate solution in cylindrical polar co-ordinates \((r, z)\) for a charge at a height \( z_0 \) is:

\[ \phi = \exp(-k(z - z_0))[\exp(-kR)/R - \exp(-kR')/R'] \]

where \( R^2 = r^2 + (z - z_0)^2 \) and \( R'^2 = r^2 + (z + z_0)^2 \). An image charge is introduced to make the surface of the Earth \((z = 0)\) have zero potential. To obey the boundary condition of an ionosphere of potential \( V_o \) at \( z = \infty \), a term \( V_o[1 - \exp(-2kz)] \) should be added. This term alone represents the fair weather field and its variation with height. We can see that \( \phi, \nabla^2 \phi, \text{and } \partial \phi/\partial z \) are reduced to an increasingly small fraction of the Coulomb value at large distances by the exponential factors. The space charge responsible for this screening may also be calculated from \( \nabla^2 \phi \). The current continuity equation for the rearrangement of the space charge \( \rho \) in the absence of sources is

\[ \text{div} j + \frac{\rho}{\epsilon_0} = 0 \]

which reduces to,

\[ \nabla^2 \phi + 2k \frac{\partial \phi}{\partial z} + \frac{\epsilon_0}{\lambda} \frac{\partial}{\partial t} (\nabla^2 \phi) = 0 \]

where \( \epsilon_0 \) is the permittivity of free space.

We visualize a lightning flash to ground removing charge at \( z_0 \); thus the boundary condition at \( t = 0 \), just after the flash, is that the potential everywhere is the steady state potential less the Coulomb potential of the charge removed, that is:

\[ \phi = \exp[-k(z - z_0)][\exp(-kR)/R - \exp(-kR')/R'] - \left( \frac{1}{R} - \frac{1}{R'} \right) \]

and that at infinity, \( \phi = 0 \) everywhere. Because of the linearity of the equations we may ignore the term involving the potential of the ionosphere which remains constant.

It has been pointed out by Mann (1970) and independently by Saxon (1954) that the above differential equation is separable in \( r, z, \) and \( t \). Writing \( \phi = T(t)R(r)Z(z) \), we find

\[ T(t) = \exp(-mt) \]

where \( m \) is a separation constant and \( R(r) \) are Bessel functions of order zero with an imaginary argument and involving a second separation constant \( p \), which we
write as \( J_0(p, r) \). Changing the variable from \( z \) to \( A \), where \( A = 1 - m \lambda \exp(2kz) \), the \( Z(A) \) equation is

\[
\frac{\partial^2 Z}{\partial A^2} + \frac{1}{A} \frac{\partial Z}{\partial A} - \frac{p^2}{4k^2 A^2 (1 - A)^2} Z = 0,
\]
solutions of which are the hypergeometric functions which we write as \( F(m, p, z) \) involving both \( m \) and \( p \). So the general solution for \( \phi \) is,

\[
\phi = \sum_{m, p} a_{mp} \exp(-mt) J_0(p, r) F(m, p, z).
\]

Because we seek solutions decaying in time, \( m \) must be positive, but \( m \) and \( p \) are not necessarily integers so the summation will tend to an integral. The coefficients \( a_{mp} \) are defined by the boundary condition at \( t = 0 \).

\[
\int_m \int_p a_{mp} J_0(p, r) F(m, p, z) dp \, dm = \exp(-k(z-z_0)) \left\{ \exp(-kr) - \frac{1}{R} \right\}
\]

The problem of finding the correct linear combination of Bessel and hypergeometric functions has not been solved, so we reluctantly resort to finite difference techniques.

3. Finite difference method

Using Holzer and Saxon’s solution less the Coulomb potential, the values of \( \phi \), \( \nabla^2 \phi \) and \( \frac{\partial}{\partial t} (\nabla^2 \phi) \) which are known at \( t = 0 \), the value of \( \nabla^2 \phi \) at a later time may be predicted. Having solved for \( \phi \) with the new \( \nabla^2 \phi \) distribution we can find a new value of \( \frac{\partial}{\partial t} (\nabla^2 \phi) \), and so advance the solution in time. The variation of gradient of \( \phi \) with time at \( z = 0 \) should give the observed recovery. The region in which we wish to compare with observation is up to 50 km from the storm. Using a linear mesh, problems arise since the solution of Poisson’s equation involves the simultaneous solution of as many equations as there are mesh points. The solution of Poisson’s equation is defined by the boundary conditions which are \( \phi = 0 \) at \( z = 0, z = \infty \), and at \( r = \infty \), so for a finite number of mesh points the value of \( \phi \) must be set to zero at some finite \( r \) and \( z \), but at a sufficiently large value that the solution at 50 km is not greatly perturbed. But the need to resolve the variations of \( \nabla^2 \phi \) for small \( r \) and \( z \) places a limit on the size of the mesh in these regions, such that extension to regions of high \( r \) and \( z \) leads to an unmanageable number of mesh points. Because of this the following transformation was used:

\[
P = \frac{1}{1 + r/a}, \quad Q = \frac{1}{1 + (z - \frac{1}{2}b)/b}
\]

where \( a \) and \( b \) are scaling constants. As \( P \) varies from 0 to 1, \( r \) varies from \( \infty \) to 0; and as \( Q \) changes from 0 to 2, \( z \) varies from \( \infty \) to 0.

Fig. 1 shows this transformation with mesh points every 0.1 units of \( P \) and \( Q \), with \( a = 8 \) and \( b = 12 \). In \( P, Q \) space \( \nabla^2 \phi \) takes the form:

\[
\nabla^2 \phi = \frac{\partial^2 \phi}{\partial P^2} \frac{P^4}{a^2} + \frac{\partial \phi}{\partial P} \frac{P^4}{a^2} \left( 2 - \frac{1}{1 - P} \right) + \frac{\partial^2 \phi}{\partial Q^2} \frac{Q^4}{b^2} + \frac{\partial \phi}{\partial Q} \frac{2Q^4}{b^2}
\]

with a slightly different form at \( P = 1 \) (\( r = 0 \)), and

\[
\frac{\partial}{\partial t} (\nabla^2 \phi) = -\frac{\lambda}{\epsilon_0} \left( \nabla^2 \phi - \frac{\partial \phi}{\partial Q} \frac{2kQ^2}{b} \right).
\]

Using the Taylor expansion we express each partial derivative in terms of the nearest neighbouring mesh points and solve the resulting simultaneous equations. The boundary conditions are rigorously applied by setting \( \phi = 0 \) for \( P = 1 \), and for \( Q = 0 \) and 2.
For comparison the analytic expressions at \( t = 0 \) are:

\[
\nabla^2 \phi = 2k \exp[-k(z-z_0)] \left\{ \exp(-kR) \left[ (z-z_0) \left( \frac{k}{R^2} + \frac{1}{R^3} \right) + \frac{k}{R} \right] \right\} + \text{image term}
\]

and

\[
\frac{\partial}{\partial t} (\nabla^2 \phi) = -\frac{\lambda_0}{\varepsilon_0} \frac{\exp(2kz)}{R^3} 2k(z-z_0) + \text{image term}.
\]

4. Acceleration of Convergence

The computations were carried out on a 41 by 21 mesh with mesh points every 0.025 units of \( P \) and 0.1 units of \( Q \). For a given \( \nabla^2 \phi \) configuration solving for \( \phi \) involves the inversion of an 800 rank matrix. This is impracticable, but because only small changes in \( \nabla^2 \phi \) occur after each time step, a relaxation method suggests itself. Scanning the mesh points and successively over-relaxing the newly calculated \( \phi \) still leads to a very slow convergence rate. The Peaceman-Rachford Alternate Direct Implicit method (PRADI) described in Wachspress (1966), was found to increase the convergence rate by about one thousand times for the solution of Poisson's equation in \( P, Q \) space. In this method a whole row or column of mesh points is relaxed at once and the resulting simultaneous equations form a tridiagonal matrix which can be solved simply by the Thomas algorithm. A complete scan involves solving for each row, then each column in turn. After each complete scan a successive over-relaxation factor of 1.5 increased convergence by another factor of two.

5. Accuracy of the Solution

Using the initial analytic values of \( \nabla^2 \phi \), values of \( \phi \) were generated using the PRADI technique and compared with the analytic expression for \( \phi \) at \( t = 0 \). On a 41 \times 21 mesh the accuracy was better than 1 per cent out to a horizontal distance of 50 km, except for a narrow band of mesh points where \( \phi \) was changing sign. The accuracy of the \( \phi \) solution generated was primarily a function of the position in \( r, z \) space. A scale factor of 8 km in the horizontal transformation, and one of 12 km for the \( z \) transformation minimized the errors. The above conditions apply to \( \phi \) and \( \nabla^2 \phi \) values after removal of charge at heights of four and six km.
The simplest formula for predicting values of $\nabla^2 \phi$ at $t_1$ from the values at $t_0$ is the Eulerian one:

$$\nabla^2 \phi_{t_1} = \nabla^2 \phi_{t_0} + \frac{\partial}{\partial t} (\nabla^2 \phi)_{t_0} \cdot (t_1 - t_0).$$

Higher order predictors are more reliable but would involve too many matrix inversions. As a first approximation the Eulerian predictor was used, and the time steps judged sufficiently small when a halving of the time step resulted in no significant change in the $\phi$ variations with time. We may add that as monotonically decaying solutions were always found, the errors of this predictor can be expected to be less than for growing or oscillatory functions. Using the values of $\phi$ generated at $t = 0$ the values of $\partial/\partial t (\nabla^2 \phi)$ at $t = 0$ were computed and compared with the analytic solution. The errors were the same as the $\phi$ errors, apart from errors greater than 1 per cent for some mesh points at a height greater than 20 km. The total amount of charge involved at these heights and at horizontal distances greater than 50 km is so small as to have a negligible effect on values of $\phi$ over the mesh.

Thirty complete PRADI scans after each step gave a solution differing from the 20 scan value by less than 0.2 per cent for all time steps, and so 30 scans were used throughout computation. The time steps were halved until the solution up to a horizontal distance of 40 km agreed to within 0.2 per cent.

When the equations were solved again with the term involving the potential of the ionosphere included, no significant differences in the computed recovery curves were found. This is to be expected from the linearity of the equations and acts as another check on the solution. Similarly doubling and halving the scaling constants $a$ and $b$ in the transformation did not affect the recovery curves.

6. Computed recovery curves at the ground

The recovery curves computed were found to have the same fundamental properties as the observations. The recoveries became progressively more rapid with increasing distance, and were significantly non-exponential. The field variations with time were expressed as a fraction of the total field change of the discharge, taking the pre-flash field as zero. Fig. 2 shows some computed recoveries after the removal of a charge at 6 km. The curves were similar but slightly less rapid for removal of charge at 4 km, and after 1 s field values were higher by 4.2 and 1 per cent at distances of 40, 20 and 10 km respectively. Thus a cloud flash modelled by neutralization of charges at 4 and 6 km would give very similarly shaped recovery curves with distance. In one second the field from the regenerated charge within the cloud should be negligible.

If the recoveries are assumed to be exponential then at any stage in the recovery an apparent value of the time constant, $\tau$, may be calculated. For example, if the field decays to $1/\sqrt{e}$ in a time $T''$, the apparent value of the time constant at this stage in the decay is $2T''$. It was found that this apparent value of the time constant increased with increasing time agreeing qualitatively with observations. However, after 1 s the computed recoveries lost their exponential form at all distances, and the recovery became almost linear and very slow at about 1/2 per cent of the field change per 0.1 s beyond 20 km, with a consequent rapid increase in the apparent time constant.

To compare the observations with computation we assume a median value of 120 Ckm for the moment destroyed in cloud and ground flashes as realistic for England (Pierce 1955) and determine $\Delta E$ as a function of distance. Fig. 3 shows the apparent value of the time constant ($\tau$) after 1 s and gives reasonable agreement with $T''''$. A different value of the moment destroyed only alters the intercept, the slope is unchanged.

After 4 s the value of $\tau$ is too great by a factor of two and the disagreement worsens with time. The possible effects of the conductivity profile on the computations after the first second will be considered later.

Tamura (1954) used $\lambda_0 = 4.5 \times 10^{14}$ mho m$^{-1}$ and $k = 0.11$ km$^{-1}$ from the work of Gish and Wait (1950) for the conductivity profile in his recovery curve analysis.
FIELD RECOVERY IN A CONDUCTING ATMOSPHERE

TIME IN SECONDS

0 0.2 0.4 0.6 0.8 1.0

3 km

8 km

15 km

24 km

45 km

FIELD

Figure 2. Recovery curves on the ground computed at various distances after the removal of charge at 6 km altitude. The recovery is expressed as a fraction of the field change.

review of more recent experimental work by Woessner, Cobb and Gunn (1958), Kraakevik (1958), Paltridge (1965), and Cole and Pierce (1965) would suggest \( \lambda_0 = 10^{-13} \) mho m\(^{-1}\) and \( k = 0.11 \) km\(^{-1}\) as more realistic values. These values were used throughout these computations. Changes in \( \lambda_0 \) lead to changes in recovery times which are, to a first approximation, inversely proportional to \( \lambda_0 \).

7. Recovery curves above the cloud

When the conductivity profile is included, the steady vertical field above a charge at a height of 6 km observed at altitudes of 18, 24, and 34 km is only 35, 15 and 1 per cent of the Coulomb field. The screening effect of the space charge is not very sensitive to horizontal distances at these heights. Using the same accuracy criteria as before, the recovery curve computed after the removal of charge at 6 km is shown in Fig. 4. The recoveries become more linear after 0-8 seconds as do those on the ground, but the apparent value of the time constant \( \tau \) at times \( T' \) and \( T'' \) at 18 km was 10-2 and 12-8 s; at 24 km was 4-2 and 5-0 s; and at 35 km, 2-3 and 3-2 s. The recovery curves after a charge is removed at 4 km are very similar. For example, at a height of 24 km \( \tau \) at \( T' \) was computed to be 4-8 s.

Vonnegut, Moore, Espinola and Blau (1966) measured a recovery curve at a height of 21 km and estimated the relaxation time to be 7 s. They recorded a low potential gradient of negative sign before the flash, and a higher positive one afterwards just as in Fig. 4. They explained the observations in terms of the formation and rearrangement of screening layers in the conductivity gradient at the surface of the cloud. Their model would predict only
Figure 3. The apparent values of the time constant after one second of recovery. ×, charge removed at 6 km altitude; ○, at 4 km, (electric moment destroyed 120 Ckm). The solid line is the observational fit of the time to decay to 1/e of the peak value; $T'' = 0.89 \Delta E^{0.47}$.

Figure 4. Computed recovery 24 km above the storm after charge neutralized at 6 km.

slight variation with height. Further, the relaxation time of the charge in the cloud boundary may be the relaxation of the local field, but the field variation at a distance will be much less.

8. THE CONDUCTIVITY PROFILE

The simple model so far described accurately predicts the first second of observed recoveries at the ground. During this second only charges high above the ground change in magnitude. Fig. 5 shows the sign of the initial value of $V^2\phi$ and $\partial/\partial t(V^2\phi)$ in a section through the storm immediately after the removal of a positive charge at 6 km. Only nega-
Figure 5. The signs of $\nabla^2 \phi$ and $\partial / \partial t (\nabla^2 \phi)$ after the removal of positive charge at 6 km altitude. --- the boundary between positive and negative space charge, --- the boundary between positive and negative rate of change of space charge.

tive values of $\partial / \partial t (\nabla^2 \phi)$ will contribute to a recovery. Fig. 6 shows the value of the ratio of $\nabla^2 \phi$ at two seconds to its value at zero time. We can see an anomalous region at an altitude of 10 km and at a distance of 30 km where $\nabla^2 \phi$ has risen appreciably, whereas elsewhere values are virtually unchanged or have fallen considerably. The charge density at zero time is proportional to $\phi$ and in the anomalous region is about three orders of magnitude lower than the charge density within 5 km of the position of the pole. But a numerical test shows that it has a strong effect on the distant recovery curves on the ground. The anomalous region makes a negligible contribution to the field closer to where the pole was and where the principal charge rearrangement is occurring.

Figure 6. The ratio of the charge density two seconds after charge removed at 6 km altitude to the density before removal.
This anomalous region is predicted using the analytic solution at zero time, and its magnitude and position after two seconds are unchanged in r, z space even if the scaling factors in the P, Q transformation are changed by a factor of three. As a result we believe it is not a numerical instability. It occurs for all reasonable values of the exponent k in the conductivity profile.

The solution of Holzer and Saxon applies to a perfectly exponential conductivity profile throughout the atmosphere: whereas the measurements of Morita, Ishikawa and Kanada (1971) show that below 8 km the conductivity is much reduced below the value predicted by a simple exponential profile. This conductivity reduction will increasingly modify Holzer and Saxon’s solution below 8 km, and space charge below these heights will be less mobile. However, departures from the non-exponential form do not lead to an analytic expression for the steady state potential. Let us therefore use Holzer and Saxon’s only for charge rearrangement where \( \frac{\partial}{\partial t}(\nabla^2 \phi) \) is negative, that is, in the clear air above the cloud. We note that the total space charge in the anomalous region is small and has little effect on the field where most of the space charge is situated. With this assumption the recovery curves at the ground agree much better with all the experimental evidence, instead of only the first second of recovery as previously.

Assuming a moment of 120 C km is neutralized the computed values of \( T' \) for charges removed at 4 and 6 km are shown in Fig. 7, together with the observational data. The agreement for distant recovery curves is good but for values of \( T' \) greater than 5 s the computed recoveries are rather slower than observation. If we assume a linear regeneration of the field of the pole of 2 per cent per second, equivalent to a flash every 50 s, then the agreement with observation for \( T' \) above 5 s is much better. The linear regeneration of field alone would give a constant \( T' \) of 15 s for all field changes.

The recovery curve measurements of Michnowski (1969) and Malan (1967) in the range 50 — 100 km are also included in Fig. 7 for comparison. Their measurements of \( T'' \) and \( T''' \) are expressed in terms of \( T' \) using the relationships given in Section 1. Also shown are the theoretical predictions of Freier (1962) and Tamura (1954).

The recovery curves above the storm are unaffected by the conductivity approximation, the values of \( T'' \) and \( T''' \) at 18, 24 and 35 km altitude lying within one half second of those previously computed. Computations of \( T'' \) and \( T''' \) for values of \( T' \) less than five seconds are shown in Fig. 8. For comparison we present the observational fit for \( T' \) less than five seconds, and the relationships for a perfect exponential. We observe that the complete recovery is accurately predicted on this model, both with regard to the shape and the speed of the recovery. The close recoveries are still very slow. After 10 s the field at 0 and 8 km from the pole has only decayed to 91 and 75 per cent respectively of its original value, whereas at 45 km it is 17 per cent.

9. Discussion

Using the linearity of the equations we can simulate recoveries after cloud discharges. Any inclined dipole may be modelled by the superposition of a vertical and a horizontal dipole. Because distant recoveries after a flash to ground do not change rapidly in a few kilometres the horizontal dipole will make a negligible contribution. Thus distant cloud discharges may be defined by the heights of the charges involved, and because the equations are linear the decay of the space charge which was in equilibrium with each charge may be added to give the recovery curve. At a given distance from the cloud, more rapid recoveries are predicted after neutralization of charge at 6 km height than at 4 km. This would suggest that if the moment destroyed in cloud and ground discharges is similar, and if cloud discharges on the average involve charge higher in the cloud, then cloud discharges should have rather more rapid recoveries than those to ground. Illingworth (1971) could find no significant difference in his \( \Delta E \) versus \( T' \) fits for positive and negative field changes. However, Michnowski (1969), estimating the distance of storms from spherics returns, suggested that within a given distance range recoveries after negative field changes were more rapid than field changes in the direction of the fair weather field, implying more rapid
recoveries after cloud discharges if the polarity of the storm is positive. But it is not obvious that the division of the distance into 50 km ranges does not introduce such large standard deviations that the difference in the values quoted for positive and negative field changes is not significant.

The first tenth of a second of the computed recoveries of distant discharges is extremely rapid (Fig. 2), which supports Malan's (1965) suggestion that the negative J changes for distant flashes to ground are not caused by cloud activity.

Choosing higher values of \( \lambda_0 \), in the conductivity expression, \( \lambda_0 \exp(2kz) \), leads to more rapid recovery curves. This supports the deduction from observation (Illingworth 1971) that recovery curves in South Africa on a plateau one kilometre above sea level are more rapid than those at sea level in the U.K.

Using the linearity of the equations the effect of screening layers in the conductivity gradient at the edge of clouds can be neglected, providing the boundary screening layers do not move in time. Phillips (1967) visualized the rearrangement of these screening layers as being governed by the clear air conductivity at that height, and being responsible for the
10 to 15 s time constants observed. We may observe that whereas the field variation at the surface of the cloud may have this short time constant, at increasing distances the field variation caused by this rearrangement will not generally be the same and will not give increasingly rapid recoveries. For example, a spherically symmetric rearrangement of charge would give no observed field change at large distances.

Experimental measurements and statistical analysis of close recovery curves have shown them to be rather variable. One can visualize many processes affecting their form besides the regeneration of the charge destroyed. Examples are the rearrangement of space-charge in the fair weather conductivity gradient, and in the conductivity gradient between the cloud and clear air, the effects of local point discharges, and reversal distance complications for cloud discharges. At this stage it would appear difficult to identify the various components of the observed recoveries.

10. Conclusion

The field from the rearrangement of space charge in the exponential conductivity
FIELD RECOVERY IN A CONDUCTING ATMOSPHERE

The conductivity near the ground is considerably reduced below the level given by the exponential profile, and if the model is restricted to include only the clear air above the cloud even better agreement is found. It is difficult to give a rigorous justification of the assumption that the real conductivity profile of the atmosphere would not lead to an anomalous area of $\partial/\partial t(\nabla^2 \phi)$ at great distances, with its consequent effect on the later portions of the distant recovery curves on the ground. But the evidence of the first second of recovery and the agreement of the subsequent seconds with the assumption, supports the suggestion that distant recovery curves are the response of the conducting atmosphere to the field change.

Neither a highly conducting cloud core nor screening layers at the boundary of the cloud need be invoked to explain the observations. It would be of interest to compare recovery curves observed at very high altitudes to see if the recovery times do become shorter with increasing height.

ACKNOWLEDGMENT

The author wishes to acknowledge receipt of an N.E.R.C. Research Fellowship during the course of this work.

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