Wind speeds were substantially lower at level 9 (about 1 000 mb), and the increases appeared to be greater in 0°–6°N than further north. Monthly average 850 mb wind speeds of about 10 m s⁻¹ are not uncommon at Christmas Island (2°N, 137°W) (Joint Task Force Seven 1960–61).

Table 2 of the paper showed that the increased evaporation, due in part to increased winds, was spread over 0°–18°N, 90°–180°W. It does not seem surprising that air should pick up more moisture as it flows into a region of warmer ocean than it would if it flowed more nearly parallel to the sea surface isotherms. Also, the more quickly it moves the more rapid this adjustment to the sea surface will be—in the equations used in the model this is expressed by the usually assumed dependence of evaporation on wind speed. It is in such a region that the evaporation increase shown in Table 2 occurred. At Christmas Island, to which Ramage and Murakami refer, these conditions may not apply because of the equatorward decline of sea surface temperature south of 5°N (not represented in the model (Fig. 3)).

(iv) I verified and discussed the model's 200 mb southerlies in Section 5(c) and see little need to add to that. Ramage and Murakami apparently prefer the evidence of two November's to that of 14 seasons (41 months) used in producing the Table 4 entries for Majuro and Johnston Island.

In conclusion, Ramage and Murakami put great stress on the effects of the equatorial wall. It was largely because of the possible effects of this that a large section was devoted to verifying the tropical and subtropical results. I do not pretend that the model provides a perfect simulation and I do not doubt that the assumption of equatorial symmetry affected the details of the results. However, the agreement between observed data and the computations shown in Section 5 of the paper seems to me to be strong evidence that the results are generally applicable to the real atmosphere.

REFERENCE


Meteorological Office, Bracknell, Berkshire.
12 December 1972.
\[ \Delta t = \text{average time interval between collisions for a hydrometeor} \]
\[ \tau = \text{relaxation time for charging of the hydrometeor in a constant field as defined by Mason (1972)} \]
\[ \theta = \text{collision contact angle, measured with respect to the positive direction of the electric field} \]
\[ v = \text{fall velocity of small neutral cloud particles} \]
\[ v'(t, t^*) = \text{fall velocity at time } t^* \text{ of small charged cloud particles which have undergone a collision at time } t \]
\[ V(t) = \text{fall velocity of hydrometeor at time } t \]

The following remarks concern Sections 7 and 8 of ‘The physics of the thunderstorm’ (Mason 1972), which deal with electrification produced by the collisions of polarized hail pellets with ice crystals and cloud droplets which rebound.

I wish to point out that the equations used by Mason to assign particle charges in the computations of the electric field growth rates apply only to the limiting case when the field does not grow. In rapidly growing thunderstorms fields the equations can be expected to give only order of magnitude estimates of particle charges.

Mason’s Eq. (3) (reproduced below), which gives the rate of charging of a hydrometeor, implicitly assumes that the time rate of change of the electric field is negligible,

\[ \frac{dQ_n}{dt} = -\frac{1}{\tau} (3 F R^2 \cos \theta + Q_0) \quad \text{(Eq. (3), Mason 1972)} \]

To obtain the charge on the hydrometeor, Eq. (3) is then integrated over time assuming again implicit that the field strength remains constant. Consequently, terms involving changes in the electric field strength are lost. It will be shown that if charging occurs in fields that are growing at the rates observed in thunderstorms, Mason’s (1972) and Latham and Mason’s (1962) procedure for computing charges is generally not accurate, and results in a considerable overestimate of the large hydrometeor charges, and an underestimate of the smaller cloud particle charges; it does not, however, produce order of magnitude errors in electric field growth rates, and thus the following comments are not expected to alter the general conclusions in Sections 7 and 8.

The following equations, whenever possible, will be expressed in Mason’s (1972) notations, except that in order to avoid too many subscripts I shall omit subscripts R and r and refer to the large and small particle charges and fall velocities by capital and small letters only.

The amount of charge transferred between a smaller neutral sphere and a larger charged sphere in presence of a polarizing field can be expressed as

\[ \Delta q = \gamma_1 r^2 F \cos \theta + \frac{\gamma_1 A^2}{1 + \gamma_2 A^2} Q_1, \]

where \( A = r/R \), and \( Q_1 \) is the charge on the large sphere before contact. The above equation is the same as Eq. (17) given by Latham and Mason (1962) except that the last term has been expressed in terms of the initial rather than the final charge on the large sphere (thus when \( F = 0 \),

\[ q = \gamma_1 A^2 Q = \gamma_1 A^2 (Q_1 - q) = \frac{\gamma_1 A^2}{1 + \gamma_2 A^2} Q_1. \]

The charge on the larger sphere after contact at time \( t \) is

\[ Q(t) = Q_1 - \Delta q = -\gamma_1 r^2 F(t) \cos \theta + \frac{1}{1 + \gamma_2 A^2} Q_1. \]

Similarly, \( Q_1 \) can be expressed in terms of its previous charge \( Q_2 \) and the field strength at the time of contact \( F(t_1), Q_2 \) in terms of \( F(t_2) \) and \( Q_3 \), and so on. If the large sphere has undergone \( n \) collisions with smaller neutral spheres, then

\[ Q(t) = -\gamma_1 r^2 F(t) \cos \theta \left[ 1 - \frac{1}{1 + \gamma_2 A^2} \frac{F(t_1)}{F(t)} - \left( \frac{1}{1 + \gamma_2 A^2} \right)^2 \frac{F(t_2)}{F(t)} - \cdots \left( \frac{1}{1 + \gamma_2 A^2} \right)^{n-1} \frac{F(t_{n-1})}{F(t)} \right]. \]

To evaluate the above expression we must know how the electric field has varied with time.
If we assume that the field is constant, then the charge on the large sphere is

\[ Q = -\gamma_1 \tau^2 F \cos \theta \left[ 1 - \left( \frac{1}{1 + \gamma_2 A^2} \right)^n \right] \left[ 1 - \frac{1}{1 + \gamma_2 A^2} \right], \]

\[ = \frac{\gamma_1}{\gamma_2} (1 + \gamma_2 A^2) R^2 F \cos \theta \left\{ 1 - \exp \left[ -\gamma_2 A^2 n \left( 1 - \frac{\gamma_2 A^2}{2} + \frac{(\gamma_2 A^2)^2}{3} + \ldots \right) \right] \right\}, \]

where

\[ \gamma_2 A^2 n = \pi \gamma_1 \tau^2 E^* \alpha \pi_r (V - v) t = \frac{t}{\tau}. \]

When \( A^2 \ll 1 \) and \( \theta = 45^\circ \), the above equation reduces to

\[ Q = -\frac{3}{\sqrt{2}} R^2 F \left( 1 - e^{-t/\tau} \right), \]

which is the same as Mason’s (1972) Eq. (4).

To obtain some idea of how the charge on the large sphere undergoing collisions in a constant field differs from the charge on the sphere colliding in a growing field, I shall assume that over some time interval the field can be approximated by an exponential function. (Physically this is not an unrealistic assumption, since observations show that the electric field, at least in the beginning, appears to have an exponential growth.)

Let

\[ F(t) = F(0) \exp (t/\eta). \]

If \( \Delta t \) is the time interval between collisions of a large sphere with the smaller ones, then

\[ Q(t) = -\gamma_1 \tau^2 F(t) \cos \theta \left[ 1 - \left( \frac{\exp (-\Delta t/\eta)}{1 + \gamma_2 A^2} \right)^n \right] \left[ 1 - \frac{\exp (-\Delta t/\eta)}{1 + \gamma_2 A^2} \right]. \]

Generally, for thunderstorm conditions \( \Delta t/\eta \ll 1 \), so that \( \exp (-\Delta t/\eta) \approx 1 - \Delta t/\eta \). For \( A^2 \ll 1 \) and \( \theta = 45^\circ \) the above equation reduces to

\[ Q(t) = -\frac{3}{\sqrt{2}} R^2 F(t) \frac{1}{(1 + \tau/\eta)} \left( 1 - \exp \left( -\frac{1}{\tau} - \frac{t}{\eta} \right) \right). \]

It can be seen that when \( t \) is large, this equation differs from Mason’s (1972) Eq. (4) by a factor of \( 1/(1 + r/\eta) \). For rapidly growing fields this fraction can be considerably less than one. If, as in Mason (1972), we take \( r = 300 \) s and assume that the field increases by a factor of 2-72 every 100 s, then when \( t \) is large the charge on the large sphere is four times less than it would be if computed using Eq. (4) in Mason (1972).

The charge on the small sphere can be expressed as

\[ q(t) = \frac{dQ(t)}{dt} \Delta t. \]

For an exponentially growing field

\[ \frac{d}{dt} Q(t) = \frac{3}{\sqrt{2}} R^2 F(t) \frac{1}{(\eta + \tau)} \left[ 1 + \frac{\eta}{\tau} \exp \left( -\frac{t}{\tau} - \frac{t}{\eta} \right) \right]. \]

(The above equation is the same as Eq. (3) in Mason (1972) when the field is constant, i.e., \( \eta \to \infty \).) Thus the charge on the small sphere is:

\[ q'(t) = \frac{\eta^2}{2\sqrt{2}} R^2 F(t) \left[ \frac{\tau}{\eta + \tau} + \frac{\eta}{\eta + \tau} \exp \left( -\frac{t}{\tau} - \frac{t}{\eta} \right) \right]. \]

It can be seen that when the field is constant, the charge on the small sphere is the same as given for \( q' \) in Mason (1972) p. 445. The latter tends to underestimate the charge and goes to zero when \( t \) is large, whereas \( q'(t) \) here retains a constant value when the field is growing. (If, for example, \( r = 300 \) s, \( \eta = 100 \) s, and \( t = 500 \) s, then the charge on the small sphere is about 4 times less when computed from Mason’s equation than when computed using the above equation.)

It should be noted that \( q(t) \) is the charge the small sphere acquires during a collision at time \( t \) in field \( F(t) \). Since second collisions are not considered (their probability being generally small), at any time \( t^* \) after this collision the small sphere will carry that same charge. To find the total
charge flux due to the small spheres, it is necessary to integrate the sphere charge, their number density and fall velocity over the time during which collisions have taken place. Thus at any time given time \( t^* \) the charge flux due to the small cloud particles should be expressed as

\[
\int_0^{t^*} q'(t) v'(t, t^*) \frac{d}{dt} [n_r'(t)] \, dt.
\]

Mason (1972), however, performs the integration to find only the number density of the small charged particles and implicitly assumes that at any given time \( t^* \) the charge on these particles is the same as if they all had collided at time \( t^* \) in field strength \( F(t^*) \). This procedure tends to overestimate the small particle charge flux. It is fortuitous that this overestimation tends to cancel out the underestimation of the small particle charges. Furthermore, since generally the numerical value for the charge flux due to the large hydrometeors is considerably larger than the value for the charge flux due to the smaller cloud particles, errors in the latter term can be expected to affect the electric field growth rate to a considerably lesser extent. Thus Mason’s computations of the electric field growth rates are overestimated, mainly because of his overestimation of the charges carried by the larger hydrometeors.

**References**


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14 July 1972.

**Reply**

Dr. B. J. Mason, in communicating the above note, comments as follows:

I agree entirely with the points made by Dr. Paluch about the consequences of assuming \( F \) to remain constant in calculating \( Q_R \). In fact I assumed both \( F \) and \( \tau \) to remain constant in order to obtain a manageable and physically obvious equation for \( dF/dt \) in terms of the precipitation intensity \( p \)—one of the few quantities we can actually measure.

Although this approximation results in overestimations of \( Q_R \) and in the rate of build up of the field, it does not, as Dr. Paluch agrees, materially affect my conclusions that a maximum field of about 4,000 V/cm, limited mainly by the balance of gravitational and electric forces on the particles, may be achieved by reasonable rates of precipitation within about 10 minutes. This is demonstrated by Fig. 1 which shows the results of much more accurate computations made on a computer using the more exact equations given below.

The equation for the rate of growth of the electric field, in the notation used by Mason (1972), is

\[
\frac{dF}{dt} = -4\pi \left[ \sum R n_r V_r Q_R - \sum n_r' v_r' q_r' + 10^{-3} (e^{4t/3} - 1) \right].
\]

(1)

The exact equation for \( Q_R \), the charge acquired by a hydrometeor of radius \( R \) is

\[
Q_R = \frac{3}{4\sqrt{2}} \int_0^t R^2(t) F(t) e^{\tau r} \, dt.
\]

(2)

and its falling speed relative to the air is given by

\[
V_R \approx \frac{20}{\sqrt{3}} \rho \rho_R - \frac{5FQ_R}{\pi R^2}.
\]

(3)

The precipitation intensity in the absence of an electric field is assumed to grow exponentially.

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with time, which would be consistent with the hydrometeors growing by accretion in a cloud of constant liquid-water content, according to

\[ p = \frac{4\pi}{3} \sum N_n R_n^3 \rho' = p_0 e^{\frac{t}{\tau}}. \tag{4} \]

With weighted mean values of \( R \) and \( V_n' \) related to \( p \) by \( \dot{R} = 1/20 \rho^{1/3} R^{1/4} \) and \( V_n' = 1/3 \rho^{3/2} g R^{1/4} \), and \( N_R \) chosen to satisfy Eq. (4) with \( \rho = 0.5 \text{ g cm}^{-3} \), we have

\[-4\pi \sum N_n V_n Q_n = 5.6 \times 10^{-3} \frac{p_{1/4}}{r(e^{t/\tau} - 1)} \int_0^t p_{1/2} F(t) e^{t/\tau} \, dt\]

\[-9.2 \times 10^{-5} \frac{F}{e^{t/\tau} - 1} \int_0^t \left[ \int_0^t F(t) e^{t/\tau} \, dt \right]^{2/5}.\]

The concentration \( n_n' \) of smaller particles that rebound from the hydrometeors and carry away a positive charge \( q_n' \) is given by

\[ \frac{dn_n'}{dt} = \sum E N_R \pi R^3 V_n n_n r_n, \]

and

\[ \nu_n' \approx \frac{F q_n'}{6 \pi \eta} \quad \text{and} \quad q_n' = \left[ \frac{\pi^2 F}{2 \sqrt{2}} + \frac{\pi^2 Q_n}{6 R^2} \right] \tau^2 \]

so

\[-4\pi \sum n_n' \nu_n' q_n' = 32 n_n r_n^3 F \int_0^t \left[ F - \frac{R^2(t) F(t) e^{t/\tau}}{R^2(r(e^{t/\tau} - 1))} \right] \, dt.\]
Putting $T = 150s$, so $p = e^{125} \text{mm}^{-1} \text{hr}$ and $n_r = 50/1$, $r = 50 \mu \text{m}$, $a = 1$ for ice crystals rebounding from hail pellets,

\[
\frac{dF}{dt} = 5.6 \times 10^{-3} \frac{e^{12500}}{r(e^{1250} - 1)} \int_0^t e^{12500} e^{125} F(t) \, dt
\]

\[-9.2 \times 10^{-5} \frac{F}{r^2(e^{1250} - 1)^2} \left( \int_0^t e^{12500} e^{125} F(t) \, dt \right)^2 \]

\[-2 \times 10^{-7} F \int_0^t e^{12500} \left[ F - \frac{\int_0^t e^{12500} e^{125} F(t) \, dt}{r(e^{1250} - 1)} \right] \, dt \]

\[-4 \times 10^{-3} (g^{105} - 1).\quad (3)\]

Eq. (3) has been integrated on a computer starting with $F = 5V/cm$ at $t = 0$ to give the curves of Fig. 1 for $r = 150s$ and $300s$ respectively. In the former case the field reaches 4200 V/cm after 700s when the precipitation intensity reaches 106 mm hr$^{-1}$ and the total rainfall is only 4.4 mm.

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