Precipitation growth at a cold front

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SUMMARY

A simple model of the growth of precipitation particles in cloud systems in which the air flow is known has been used to calculate the growth of hail pellets at a cold front. The calculated pellet sizes and densities, and surface rainfall rates, are consistent with observations. It is shown that in the 3 km deep frontal cloud the larger hail pellets were grown during at least two ascents in the strong updraught. The growth of large pellets depends on the airflow being such as to cause two ascents, and small changes in the airflow can prevent the growth of large hail pellets.

1. Introduction

Detailed observations of clouds using conventional radar techniques have made it possible to trace the regions in which precipitation develops. With Doppler radar it is possible to measure the air motion in precipitating clouds (Probert-Jones and Harper 1961; Caton 1963; Browning and Harrold 1969). Even before more direct evidence was available, Browning and Ludlam (1962) suggested that air motion in convective storms must permit the recirculation of growing precipitation particles in order that large hail can be produced. Analysis of large hail stones such as that by Macklin, Merlivat and Stevenson (1970) has confirmed that this recycling does occur.

Although there have been a number of attempts to infer the growth of precipitation elements in clouds from radar observations there has been little detailed attempt to obtain quantitative explanations of these results. Browning and Harrold (1970) used some results of Macklin (1962) on the density of ice accreted on hail to calculate the trajectories of typical hail pellets growing at a cold front, but there was no attempt to set up a detailed model of the physical processes or to calculate the effects of the structure of the air motion on the growth of the particles.

This paper describes an attempt to model ideas of the growth of precipitation particles and to use this model to explain observed precipitation growth. It is assumed that the air motion in the system in which the particles grow is known, that it is steady during the growth of the precipitation particles and that it is not affected by the growth of large particles. The effects of changes in the airflow are readily calculated and it is shown that in a particular shallow frontal cloud system the growth of hail pellets depended critically on an air flow which allowed recirculation to occur. Although the model of particle growth has only been used in the context of one particular cloud system in which the air motion is known it is clear that it could be included in a scheme for modelling cloud dynamics.

2. The precipitation particle model

This work is concerned to model the growth of individual large particles in a cloud composed of, in general, much smaller particles. Interactions between precipitation particles are therefore ignored. The method by which small numbers of cloud particles initially grow to become much larger than the mean size is not specified, although in a more general model it would be desirable to include these early stages. It is assumed that the precipitation elements are raindrops or hail pellets, that the cloud contains no ice crystals and that the precipitation elements are of radius greater than 50 μm.

The precipitation elements are assumed to be spheres of radius R and of mass M and
to be at a uniform temperature $T$. Growth is assumed to be by the deposition of concentric shells of low-density ice, high-density ice or liquid water according to whether the temperature of the particle is less than, equal to or greater than $0^\circ C$. The supercooling of raindrops to $-5^\circ C$ was considered.

The horizontal velocity of a precipitation element was assumed to be equal to the local horizontal component of the air velocity and the vertical velocity was obtained from the vertical component of the air velocity and the terminal velocity of the particle in still air. The terminal velocities of raindrops were obtained from the empirical formulae of Best (1950a) and Foote and Du Toit (1969). Hail pellets were treated as smooth rigid spheres using the method described by Mason (1957) but with a correction factor to take account of the surface roughness derived from the measurements of List (1959) and of Macklin and Ludlam (1961). The drag coefficient of a smooth sphere of radius and mass equal to that of the hail pellet was multiplied by a factor to obtain that from which the terminal velocity of the hail pellet was calculated. The factor was assumed to depend on the surface temperature of the hail pellet, increasing from 1.2 for a pellet temperature of $0^\circ C$ to 2.0 when the temperature fell below $-20^\circ C$.

The mass of a precipitation particle was assumed to increase by accretion and by sublimation if the air and surface temperatures were less than $0^\circ C$. Growth by condensation was not considered since, as stated earlier, the particles were of radius greater than 50 $\mu m$. The rate of change of mass of the particle is given by

$$\frac{dM}{dt} = \pi \int_0^R (R + R_d)^2 M_d V(R, R_d) E(R, R_d) N(R_d) dR_d + 4 \pi R D V_f (\rho_s - \rho_R)$$  

where the first term represents the gain in mass due to coalescence with cloud droplets of radius $R_d$ and mass $M_d$ and the second term represents the gain by sublimation. In this equation $V(R, R_d)$ is the difference between the terminal velocities of the precipitation particle and the cloud droplets, $E(R, R_d)$ is the collection efficiency obtained from the formulae of Scott and Chen (1970) and $N(R_d)dR_d$ is the number density of the cloud droplets. $D$ is the diffusion coefficient for water vapour in air and $V_f$ a ventilation factor depending on the Reynolds number of the falling particle. This factor was obtained from the results of Thorpe and Mason (1966). The vapour density at the surface of a hail pellet, $\rho_R$, and that at large distances from it, $\rho_s$, were assumed to be the saturated values with respect to ice at the surface temperature and with respect to water at the local air temperature respectively. The ratio of the sublimation term to the accretion term was typically in the range 0.05 to 0.5. Sublimation was sometimes important in the growth of particles in the radius range 50–120 $\mu m$ which grew to form hail pellets.

The changes in temperature of the particle are brought about by conduction to the air at temperature $T_e$, by the release or absorption of latent heat during any freezing or melting of the precipitation particle or of any accreted water and by any difference in temperature between the accreted water and the particle. The rate of loss of heat, $Q$, by conduction was assumed to be given by

$$\frac{dQ}{dt} = 4 \pi R C V_f (T - T_e)$$  

where $C$ is the thermal conductivity of the air. The accreted water was assumed to be at a temperature equal to that of the air. The change in temperature can then be calculated assuming that the specific heats of the solid and liquid parts of the precipitation particle are known. These specific heats were assumed to be independent of temperature and of the density of the ice. As stated earlier possible supercooling was considered in calculating the temperature change of a raindrop as was melting of hail pellets.

A very wide range of volumes for the density of hail pellets has been reported. It was necessary to make a number of simplifying assumptions in this work. It was assumed that ice formed when a supercooled particle froze had a density equal to that of solid ice. The density of ice deposited during wet growth was assumed to be equal to the density of solid
ice. The density of ice deposited during dry growth was calculated from the empirical relationship due to Macklin (1962):

$$\rho_{\text{ice}} = 4\cdot0 \times \left(-R_d V(R, R_d)/T\right)^{0.76}$$

(3)

where the temperature is expressed in degrees Celsius and the other parameters are in mks units.

The model described above is much simplified but it should be sufficient to examine the effects of large-scale motion in a cloud system on precipitation particle growth. The major deficiency in the model is its inapplicability to clouds containing ice crystals and this is the region where a major improvement could be effected. Other improvements could also be made and one of these, an allowance for shedding of water from the surface of a wet hail pellet has been considered. However, in the case in which the model has been utilized this was found to be of little importance and it has therefore been neglected in this report.

3. Application of the model to the growth of precipitation at a cold front

The precipitation growth model was used to calculate the development of precipitation at a cold front. A study of the cold front, using Doppler radar (Browning and Harrold 1970) provides an accurately determined frontal structure although with a lack of records of short period rainfall rates. A cross-section of the cold front, which passed over Pershore (Worcestershire) on 6 February 1969, is shown in Fig. 1. The passage of the front was marked by observations at several places in the vicinity of the radar site of hail pellets of high density and of radius 2–5 mm. A short burst of very heavy rain was also reported and there were occasional reports of thunder. Ahead of the surface front a wide belt of cloud gave rise to a 15 min period of slight to moderate rain while a longer period of moderate rain was associated with slope convection behind the front.

It was observed that there was little variability in the component of the air motion parallel to the surface front. The ascent at the front occurred over a long narrow area rather
than in a series of separate cells. For these reasons it was possible to obtain good estimates of the air motion at the front from the radar observations. The uniform structure parallel to the front made it possible for the motion to be treated as two-dimensional. Besides enabling accurate velocity determination the two-dimensional nature of the flow simplified the calculation of the trajectories of the growing precipitation particles. Another reason for the use of this front was the fact that little snow was observed to fall from the cloud although the temperatures were low. This suggests that the growth of ice crystals in the cloud could be ignored as is required by the growth model.

The cloud liquid water content was not measured but it was assumed that this could be calculated on the basis of saturated adiabatic ascent of the air, air below the frontal cloud being saturated by rain falling into it ahead of the surface front. The measurements indicated that 35 per cent of the inflowing water was precipitated out so that adiabatic ascent will over-estimate the true liquid water. In order to calculate particle growth by accretion it is necessary to know the radius of the cloud droplets in addition to the liquid water content. It was assumed in this work that the cloud was mono-dispersed. This assumption, which was justified by the close agreement between the calculated growth rates using it and the observations, is reasonable because the collection efficiencies between droplets of radius greater than 50 μm and smaller droplets, are insensitive to variations in the radius of the latter.

With these assumptions the growth of particles moving through the cloud system was calculated by integrating the equations of motion and particle growth described earlier. A wide range of embryo sizes was used and these were assumed to originate both within the rain near the surface front and in the rain ahead of the front. It was found that only those particles originating in the rain ahead of the front could grow into the small hail pellets. The paths and growth of typical particles originating in this region are shown in Fig. 2. The densities of unmelted hail pellets reaching the ground were all calculated to be greater than 0.85 g cm⁻³. As indicated in Fig. 2, the larger pellets were grown only during a recycling process involving at least two ascents in the strong updraught. It was demonstrated that the reason for the slow growth of particles other than those originating ahead

![Figure 2](image-url)
of the front was that they did not enter the region of strong updraught. All particles grow slowly except in this region. Another interesting result of the calculations was that the largest hail did not form on the largest embryos; often the converse was true.

The trajectories of the larger particles behind the surface front usually showed an ascent to a local maximum height about 1 km behind the front before their fall to the ground. This motion gave rise to a vault, a region deficient in large particles, about 1 km behind the front and extending to about 1.5 km above the ground. The presence of such a vault was noted by Browning and Harrold (1970) and is often observed in such situations. The numerical models of cloud systems of Srivastava and Atlas (1969) predicted that precipitation particle trajectories were concentrated in the region of strong updraught. The present work also demonstrates this, the concentration arising both from the convergence of the horizontal flow at the base of the updraught and from the recirculation of the larger particles in this region.

In order to verify that recirculation was necessary to grow large particles some of the calculations were repeated with the wind field adjusted to prevent recirculation. Fig. 3 shows the streamlines in the fields which were used. The first modification was to set equal to zero the air velocity at all points ahead of the front when the measured velocity was away from the front and the second modification was a smaller reduction of the circulation of the air

Figure 3. The streamlines in the three flow patterns which were tested for their effects on precipitation particle growth. The upper diagram is that assumed in Fig. 1 and modifications to this are incorporated in the lower patterns.
in this region. When either of these changes were made there was no recirculation of the precipitation particles and no hail of radius larger than 1 mm produced. It can therefore be inferred that in this frontal situation the air flow was important in the formation of large hail because it allowed recirculation of the precipitation particles.

The calculations have shown that the precipitation growth model can explain the growth of small hail in the shallow frontal cloud. However, much of the precipitation was rain so that attempts were made to calculate the rainfall expected from this cloud. It has been noted that the large precipitation particles reaching the ground in the vicinity of the front originated in the rain ahead of the front and it was observed that this rainfall intensity was of the order of 1–2 mm hr⁻¹. A spectrum of embryo sizes typical of the droplet spectrum in such rain was used to calculate the surface rainfall rate near the front due to the growth of these particles. The spectra which were used were line spectra approximating to the empirical spectra of Best (1950b) and typical spectra are shown in Table 1. Assuming rainfall rates of 1·0, 1·5 and 2·5 mm hr⁻¹ about 1 km ahead of the front, the total surface rainfall associated with the passage of the front was calculated to be 1·4, 1·5 and 2·3 mm. These figures agree well with the observed total of 1·8 mm at Pershore during this period.

TABLE 1. APPROXIMATE DROP SIZE SPECTRA FOR DIFFERENT PRECIPITATION RATES

<table>
<thead>
<tr>
<th>Drop radius/mm</th>
<th>Number of drops per g of air</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>rainfall rate 1·0 mm hr⁻¹</td>
</tr>
<tr>
<td>0·05</td>
<td>430</td>
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<tr>
<td>0·10</td>
<td>120</td>
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<tr>
<td>0·15</td>
<td>55</td>
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<tr>
<td>0·20</td>
<td>30</td>
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<tr>
<td>0·25</td>
<td>17</td>
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<tr>
<td>0·30</td>
<td>10</td>
</tr>
<tr>
<td>0·40</td>
<td>11</td>
</tr>
<tr>
<td>0·60</td>
<td>1·9</td>
</tr>
<tr>
<td>0·80</td>
<td>0·06</td>
</tr>
</tbody>
</table>

Fig. 4 shows the calculated rate of rainfall behind the front assuming rainfall ahead of the front of 1·5 mm hr⁻¹. A maximum intensity of 75 mm hr⁻¹ was obtained for a 40 s period after the passage of the surface front. There were unfortunately no sufficiently sensitive rainfall recorders in the vicinity of Pershore at that time to enable a comparison to be made. However, Browning and Harrold (1970) described a cold front similar to that under consideration at which a rainfall rate of 80 mm hr⁻¹ was measured over a 1 min period behind the front.

Another comparison between the results obtained using the precipitation growth model and the observations, can be made in respect of the terminal velocities of the large precipitation particles. Some unpublished measurements by Browning and Harrold of the terminal velocities of the precipitation above the cold front, obtained using a vertically pointing Doppler radar, indicate that particles with terminal velocities of up to 13 ± 2 m s⁻¹ were present about 1·25 km above the surface front. The highest calculated terminal velocities in this region were about 11 m s⁻¹.

4. SUMMARY AND CONCLUSIONS

The results described in this paper indicate that a simple model of precipitation particle growth can be used to give results which agree well with the observations. The
model is clearly capable of considerable refinement especially if it is to be used in more general applications.

Errors will be introduced by the simplification of the density of deposited ice, but these are small if the growth of hail occurs at temperatures close to 0°C. Shedding of water from the surface of wet hail is another factor which can be included, but in the application described there is little shedding of water as the hail is less than about 3 mm radius. The model ignored the effects of particle growth on the cloud so that it was impossible to determine whether the growth of particles was limited by competition for the available liquid water. The close agreement between the predicted and observed precipitation rates suggests, however, that even when as much as 35 per cent of the inflowing water is precipitated out this is not an important factor. The calculations described assumed that raindrops could be supercooled to −5°C. This is only an estimate of the possible supercooling but some other results showed that provided the permitted supercooling was not sufficiently large to prevent most of the precipitation particles freezing, in this case −8°C, there was little effect on the results.

The application of the model to the growth of precipitation at a cold front has given insight into the mechanisms involved in the production of hail and heavy precipitation from shallow cloud.

The importance of the cloud ahead of the front as a source of embryos for subsequent growth in this situation was suggested by Browning and Harrold (1970). However, even with such an adequate supply of particles, the structure of the air motion in the cloud system was a dominating factor. It was necessary for recirculation to occur in order to grow large particles. Browning and Harrold also suggested that the low freezing level was important in the growth of large hail. Calculations showed that if the temperature of the entire system was raised by 3°C then the maximum sizes of the hail pellets were decreased. This was a result of an increase in the density of the ice deposited on hail pellets at higher temperatures. Even small increases above the previously calculated values of about 0.85 g cm⁻³ upset the balance between the updraught and terminal velocity and decrease the growth rate.

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