On an observation of turbulent waves on the tropopause surface

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SUMMARY

An analysis of aircraft measurements at a well defined tropopause inversion is presented. It is found that there is turbulence on the inversion which is closely correlated to some form of wave activity.

The observations show that any aircraft flying at the level of a strong inversion may expect sudden changes in both temperature and horizontal and vertical wind due to wave motions on the interface, and that these changes can amount to the full temperature excursion through the surface (in this case 10°C); and the full shear (in this case 7 ms⁻¹) in a horizontal distance of about 1 km.

1. INTRODUCTION

This paper describes the results of an analysis of a Canberra aircraft flight made on 27 January 1970. It is of particular interest since the extremely well defined tropopause, which was measured in some detail by a 'porpoise' flight pattern, was found to have breaking waves embedded in it. Because of the large temperature increase with height across the tropopause inversion these waves are measured as similarly large temperature fluctuations (up to 8°C) by the aircraft in level flight. In this case the tropopause sloped along the flight path so that the level run passed from below the inversion into the warm air above it.

2. THE SYNOPSTIC SITUATION – 27 JANUARY 1970

At 0001 GMT 27 January 1970 the British Isles was covered by a weak ridge of high pressure at the surface, and an occluded frontal system was situated about 400 miles west of Ireland, running from 45°N 16°W to 55°N 22°W. During the day the ridge intensified, and moved eastwards into France, while the occluded front moved into western Ireland. Over the flight area surface winds were generally light and variable.

In the upper air above 500 mb there was a disrupted upper trough associated with a cold pool over eastern Spain, with the trough axis on the Greenwich meridian at midnight. By midday the trough had moved east, and a north-westerly jet stream, with maximum winds of 320° 50 ms⁻¹ moved over the area (See Fig. 1. The flight area is shown hatched). Along the trough axis the air was of polar origin, with a low, warm tropopause at approximately 450 mb (7 km), but as the north-westerly airstream advanced into the area, the tropopause rose to 200 mb (12 km). Fig. 2 shows the rawin sonde ascents at Aughton (05322) for 0001 GMT and 1200 GMT. It can be seen that by 1200 GMT the polar tropopause persisted in the flight area as a medium level inversion, and that there was a strong wind shear across it. Lenticular cloud was observed during the flight at an estimated height of 6 – 7 km close to the level of this inversion indicating that there was wave activity at these levels.

A cross-section along the line AA' in Fig. 1 is shown in Fig. 3(a) and (b). The jet core can be seen to be over the flight area, and as well as the marked shear at the mid-level inversion, the upper tropopause is well marked with a large reverse shear across it. The aircraft observations were in this upper tropopause.

3. AIRCRAFT INSTRUMENTATION

The instrumentation on the MRF Canberra has been described fully by Axford (1968). It is sufficient to remark that a differential temperature accuracy of 0.2 – 0.3°C (absolute
accuracy $\pm 1 - 2^\circ C$), and a differential gust accuracy of 0.2 ms$^{-1}$ (1 ms$^{-1}$ absolute) is obtained by the measurement system, which includes an inertial stable platform and a gust probe.

4. Flight plan

The total flight plan, shown schematically in Fig. 4, contains three straight and level downwind runs at 11.3 ± 0.05 km and two upwind legs during which the aircraft 'porpoised' between 10.6 km and 12.0 km. The runs were intended to be up and down wind, but, as can be seen, there was an off-set of 10 - 15 degrees. Each down wind leg took approximately 14 min, while the up wind runs took about 18 min. The starting time for each leg is shown in the diagram.

5. Meso-scale analysis of results

The flight was analysed to look at the meso-scale by taking measurements every 5 s (approximately 1 km in distance) along the five legs. The temperature measurements on
Figure 2. Temperature ascent for Aughton (03322) 27 January 1970. Thin full and dotted lines represent the temperature and dewpoint at 0001 GMT. Thick full and dotted lines represent the temperature and dewpoint at 1200 GMT. Pressure in mb, temperature in °C.

**TABLE 1. UPPER WINDS FOR AUGHTON (03322) 27 JANUARY 1970**

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277 315 72

Max
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238 300 85
Figure 3. (a) Temperature cross-section along the line AA' in Fig. 1. Temperature in °C. Tropopause and mid-troposphere inversion shown by dotted lines. Flight area shown hatched. (b) Wind speed and direction cross-section along the line AA' in Fig. 1. Full lines represent isopleths of wind speed in m s⁻¹. Dotted lines represent wind direction in degrees true. Flight area shown hatched.
porpoise legs (Runs 2 and 4) are shown in Fig. 5(a) and (b). During Run 2 the aircraft found what appeared to be a 'roll' disturbance in the tropopause near 11·3 km in which there was moderate clear air turbulence (CAT), and to the south-east of which (120 - 220 km on Fig. 5(a)) the well marked inversion broke down into a more isothermal layer. However, during Run 4 which occurred about 24 km south-west of Run 2, and 40 minutes later in time, the inversion showed no breakdown, but sloped down from north-west to south-east at an incline of order 1:600. Doppler winds measured at half minute intervals during these legs indicate that there is a wind maximum of 290° 46 ms⁻¹ just below the inversion becoming 300 - 310° 35 ms⁻¹ about 400 - 600 m above it.

It is instructive to try to build a three-dimensional model of the temperature structure within the volume defined by the flight plan (dimensions 200 km long x 36 km wide x 1·2 km high); on the assumption that the changes during the hour and a half of the flight are not significant. By using the temperature structures defined in Fig. 5(a) and (b) in combination with the temperatures measured on the three level runs, it is possible to obtain enough data to draw smoothed vertical cross-sections at right angles to the flight path at 25 km spacing. An example is shown in Fig. 6 for the 100 km section. Then by combining them together a three-dimensional model of the temperature structure is found (see Fig. 7). Of course it is realized that in fact the system will have moved eastward during the period, so that the model formed is distorted by this motion, but it does help to clarify the picture of a sloping surface separating the cold air at and below the tropopause at temperatures below -66°C from the warm air at above -60°C above it. The breakdown in the inversion found in Run 2 cannot be defined properly in the model because of the lack of additional measurements through it, and all that can be said is that it appears to separate a region where the tropopause inversion is intense from a region where it is less well defined.

Run 5 happened to be made over the same ground position as the porpoise Run 4, and, by good fortune, the level chosen passed from the cold tropospheric air through the sloping inversion into the warm stratospheric air above. During this transit there was considerable turbulence, and the fine scale structure of this turbulence is the subject of the next Section.

6. Micro-scale analysis of leg run 5

A five-minute portion of Run 5 was analysed at ½ s intervals (50 - 60 m distance) to obtain readings of temperature and vertical and horizontal gusts as the aircraft passed through the inversion. These results are shown in Fig. 8(a) and (b) while in (c) the wind vector deduced from the along- and across-track gust values is shown.

It is immediately clear that the inversion surface is the centre of considerable turbulent activity. As the aircraft approaches the inversion in the cold air it passes through two pulses of warm air (130 km and 132·5 km on Fig. 8(a)), each of which is preceded by a
Figure 5. (a) Temperature cross-section derived from Run 2. Isotherms shown in °C. Aircraft path shown by thick continuous line. (b) As (a) for Run 4.

Figure 6. Vertical cross-section at right angles to flight path at mid-point of flight plan. Temperatures shown in °C.
Figure 7. Three-dimensional schematic model of temperature structure found. Temperature isopleths shown in °C. Hatched areas signify as follows:—

- lower than −66°C
- between −62°C and −64°C
- higher than −60°C.

downward vertical gust. Then between 146 km and 164 km it passes through the inversion in slight to moderate clear air turbulence. In this region there appear to be waves or billows in the temperature profile with the turbulence mainly in the warm air. Note that the exact position of the inversion on this run can only be related approximately to Fig. 5(b) since the aircraft height is only known to within 50 m, and the inversion surface is not stationary on the micro-scale between Runs 4 and 5. Turbulence in this case is defined by the high frequency flutter (1–10 Hz) observable on the wind vane output traces, and is of too short a wavelength to show in Fig. 8. As the aircraft passes into the warmer air at 156 km and 158 km there are two further pulses of cold air associated with updraughts of 2–3 ms⁻¹ after which the aircraft remains in slight to moderate clear air turbulence to the end of the run. Looking at the horizontal gusts it is apparent that the cross-track variations, \( u \), in this case greatly exceed the along-track component. The deduced wind speed and direction shows that across the inversion the wind changes from 290° 47 ms⁻¹ to 297° 42:5 ms⁻¹ giving a shear vector of approximately 063° 7 ms⁻¹ which is at 72° to the aircraft track along 135°. It is apparent that as the aircraft passes from one side of the inversion surface to the other in the waves the observed horizontal wind also varies between the two values.

An enlarged version of the observation from 154 km to 158 km is shown in Fig. 9. In this diagram the potential temperature variation from 48°C to 58°C in the vertical is assumed to be about 90 m thick in line with the detailed measurements from Run 4 (Fig. 5(b)), and the potential isotherms are deduced from the aircraft traverse through the waves. It can be seen that if isentropic flow is assumed the observed vertical velocities correspond to air flow through the waves from left to right, and that the clear air turbulence is to be found in the wave troughs. There are a number of results which can be deduced from these measurements if certain assumptions are made about the orientation and cause of the waves.

(i) The layer thickness (\( \Delta z \)) is about 90 m, and there is a temperature variation across this layer of 10°C (\( \Delta \theta \)) and a shear of 7 ms⁻¹ (\( \Delta u \)). The average Richardson Number for the layer can thus be calculated as

\[
Ri = \frac{-g}{\theta} \frac{\Delta \theta}{\Delta z} \left( \frac{\Delta z}{\Delta u} \right)^2 \simeq 0.6.
\]
Figure 8. (a) Vertical velocities (w) and true air temperatures measured on Run 5. (b) Horizontal gusts measured along (u) and across (v) flight path on Run 5. Note that scale zero is arbitrary.
This can easily vary by a factor of 2 or 3 locally along the layer with small variations in \( \Delta \theta \) or \( \Delta \nu \).

(ii) If in Fig. 9 the flow is isentropic with regard to the potential isotherms the wave amplitude is approximately 100 m. Similar plots of potential isotherms at other points along the aircraft track confirm that this is the maximum amplitude measured.

(iii) The principal wavelengths observed vary from 1 km to 3 km. (Those shown in Fig. 9 are about 2.4 km.) However, the orientation of the waves to the aircraft track is not known. One possibility is that they lie perpendicular to the shear vector along 063°, and thus the aircraft is cutting through the wave surface along 135° at an angle of 18° to the wave trough axes. For this case the apparent wavelength would be 2.4 \( \sin 18° \) km or 745 m.

According to Miles and Howard (1964) a thin stratum of linear shear and density separating two thick layers will become unstable if Ri becomes smaller than the critical value of 0.25, and the instability causes growing waves of wavelength \( \lambda \approx 7.5 \ h \), where \( h \) is the thickness of the layer. These waves, generally known as Kelvin-Helmholtz (K-H) waves, will eventually roll up into vortices perpendicular to the shear vector which break giving turbulence.

It is realized that it is not possible to prove the existence of Kelvin-Helmholtz billows from a single traverse at an angle. However, it is interesting to consider the implications of the observations if they are used as a model. Fig. 10(a) shows a schematic model of a set of vortices with anti-clockwise rotation and an aircraft traversing from below to above them. Fig. 10(b) is an end on view of the vortices showing the velocity components of the wind normal to the billows. Note that in this case of reverse shear the billows are rotating backward.

K-H billows move downstream with the mean wind flow which in this case is 28.5 ms\(^{-1}\), and the aircraft took 10 s to pass from peak to peak in Fig. 9. Thus the wavelength of 745 m deduced above has also to be corrected for the wave motion of 285 m during the traverse giving a final wavelength of 450 m for the billow model. It can be seen that a value of \( \lambda_{\text{true}} = 460 \) m and a thickness \( h \) of 90 m fits the K-H wave model quite well.

Do the rest of the observations fit this model? First, it is noted that after a downdraught the aircraft penetrates the warm air from above the tropopause, and after an updraught it
passes into smooth cold air as demanded by the model. Second, the maximum amplitude observed of about 100 m is a little under \( \frac{1}{4} \) of the wavelength. According to Thorpe (1971) instability will always set in when billows grow to \( \frac{1}{4} \) of their wavelength. In the case of the rough estimates being made here the agreement appears quite good. Thorpe's observations also show that the largest vertical velocities found in laboratory models of billows are close to \( \frac{1}{4} \) of the velocity difference across the shear layer. Here again the vertical velocities of \( 2 - 3 \text{ ms}^{-1} \) and shear of \( 7 \text{ ms}^{-1} \) approximately fit the model.

Looking at the horizontal winds shown in Fig. 9 it is notable that in the warm air the wind direction veers temporarily to about 301°, 41 ms\(^{-1}\). This implies a temporary reduction in the wind component perpendicular to the vortex from 25.0 ms\(^{-1}\) to about 22.0 ms\(^{-1}\), and might be accounted for as an effect of reverse flow in the vortex.

The measurements show that the turbulence was in the warm troughs while the cold air was smooth. While penetrating the inversion this turbulence was intermittent and moderate, but even when the aircraft was continuously above the inversion slight turbulence continued to the end of the run.
(iv) Alternatively, a completely different model can be considered. Suppose there are gravity waves on the tropopause surface oriented roughly at right angles to the aircraft track moving upwind so that the air is flowing through them. Assume also that the warm air just above the inversion is turbulent because of a micro-scale breakdown of the flow in this region. Then the maximum vertical velocity in the waves would be $2\pi A c / \lambda$ where $A$ is the wave amplitude, $\lambda$ the wavelength and $c$ the phase velocity of the waves relative to the ground. In this case with $A = 100$ m and $\lambda \approx 2$ km (say), the maximum vertical velocity of 3 ms$^{-1}$ is achieved if $c \approx 10$ ms$^{-1}$.

Again the horizontal wind will back and veer as the aircraft passes through the shear layer, and the turbulent layer will appear whenever the aircraft passes into the warm air.

7. Conclusions

The observations show that the turbulence on a well defined tropopause inversion is closely correlated to some form of wave activity. A single aircraft pass through the waves is insufficient to define the wave structure fully, and plausible hypotheses such as Kelvin-Helmholtz waves or gravity waves both appear to fit the observations reasonably. More complicated flight patterns involving more than one aircraft are probably required to obtain a realistic three-dimensional picture of these phenomena. However, these observations show that any aircraft flying at the level of a strong inversion must expect sudden changes
in both temperature and horizontal wind due to wave motion on the interface, and that these changes can amount to the full temperature excursion through the surface (in this case 10°C), and the full shear (in this case 7 ms⁻¹). In the observations presented here the vertical gusts amount to approximately \( \frac{1}{3} \) of the total shear.

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References

