Vertical velocities and vertical eddy fluxes derived from serial soundings at one station

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(Manuscript received 18 September 1972; in revised form 26 February 1973)

SUMMARY

Detailed 3-hourly soundings of the atmosphere from 0-30 km for one month (September/October 1967) over Laverton, Victoria (38°S) are analysed to obtain vertical velocities using the heat balance method. Results are assessed to be reliable from the point of view of error estimates, synoptic compatibility and statistical features for the whole month. The tendency for a reversal of vertical velocity between low and high troposphere occurs, with occasional very strong downward velocities (\( \sim 30 \text{ cm s}^{-1} \)) at the tropopause level. Convergence of vertical eddy flux of both momentum and sensible heat is shown to occur near the levels of maximum poleward eddy transport with magnitudes equivalent to 1.5°K/day and 3 m s\(^{-1}\)/day. Similar computations based on routinely available upper-air soundings at standard pressure levels at 12-hourly intervals are shown to give quite different (spurious) results. As a supplementary result, strong horizontal eddy transports are revealed in the middle stratosphere.

1. Introduction

The observational network deficiency is a fundamental problem in Southern Hemisphere meteorology. At present broadscale analyses, and hence prognoses, depend to a considerable extent on qualitative interpretation of satellite cloud photographs. In order to introduce a more quantitative element into this procedure attempts are being made to relate synoptic observations from land stations and ships to cloud patterns (Stretten and Troup 1973). The aim of these studies is the construction of composite three-dimensional structures of atmospheric systems which can be associated with various recognizable stages in cloud vortex development. In addition to directly measured synoptic quantities, a reliable method of estimating vertical velocities would considerably help the quantitative interpretation of cloud photographs and the construction of internally consistent horizontal velocity patterns.

Because, on the synoptic scale, vertical velocities are two or three orders of magnitude smaller than horizontal velocities and are highly variable, all estimates of synoptic scale vertical velocities are based on indirect methods. These can be classified into three groups, each being concerned with the balance of mass, vorticity or heat. The mass balance or kinematic method is based upon the Continuity Equation. Large-scale horizontal divergence is obtained from wind observations and integrated vertically to obtain vertical velocity. A spatial array of observing stations is required and, in addition, sophisticated filtering procedures are necessary (Eddy 1964) to obtain realistic vertical velocity patterns. The vorticity balance method in its most comprehensive form is incorporated in the so-called omega equation. This has been used in many studies, e.g. by Knighting (1960) for the North Atlantic-European area, Haltiner, Clarke and Lowniczak (1963) over North America and by Seaman (1969) over the Australian region – where particular attention was paid to the effects of varying grid-size and to the smoothing procedure.

Because both the above methods require a spatial array of stations, they have serious limitations in regions where the observational network is sparse. This is particularly true over most of the Southern Hemisphere.

The heat balance method is based on the evaluation of all terms in the thermo-dynamic energy equation except vertical advection. The method has several disadvantages. It requires an assessment of diabatic heating rates or involves the adiabatic assumption. Also

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it cannot be used when the lapse-rate is neutral, becoming progressively more inaccurate as this condition is approached. Additionally, in practice, the choice of using either a moist- or a dry-adiabatic lapse-rate is not clear cut. A compensating advantage is that by using the thermal wind relation the horizontal temperature gradient can be estimated from the observed wind-height profile; for this reason the method can be applied to single station soundings. Reasonably convincing vertical velocity patterns have been obtained in this way for a relatively small number of selected synoptic situations (Hubert 1953; Vuorella 1957). Although six-hourly soundings were used, the smoothing techniques applied reduced the effective separation of observations to 12-hours. Kreitzberg (1964) has used the method to study the vertical structure of a number of tropospheric synoptic systems, relying upon special ascents at about the time of a frontal passage to supplement the normal six-hourly wind and twelve-hourly temperature soundings. All these investigations have been confined to levels below about 300 mb and, with the exception of Kreitzberg’s, were based on observations at standard pressure levels.

The heat balance method has also been applied to higher levels, extending well into the stratosphere (Jensen 1961; Barnes 1962; Oort 1964; Dopplick 1971). Usually the object has been to calculate co-variances between vertical velocities and other measurements important in the overall energetics of the general circulation. Daily observations were used, but no assessment appears to have been made of the adequacy of the method when applied to observations 24 hours apart. Thus, although the results are generally consistent with each other and with an overall concept of general circulation mechanisms, they must be treated with a degree of reservation. There appears to be a need to examine the usefulness of applying the heat-balance method to more detailed and more frequent serial soundings, in particular, to investigate the effect of the time interval between soundings on the derived vertical velocities and subsequent vertical flux estimates.

A recent observational experiment in Australia has provided data to which the application of the heat balance method of estimating vertical velocities is particularly appropriate. These data are detailed and frequent (three-hourly) observations of wind and temperature from the surface through the troposphere into the mid-stratosphere at one station (Laverton, Victoria; 38°S, 145°E) over the period of about one month. Details are given in ‘The Laverton Serial Sounding Experiment’ (Commonwealth Bureau of Meteorology, Australia 1968) which will be referred to subsequently as the LSSS Summary. The results of this application are the subject of this paper.

The vertical velocity calculations described and presented here differ in several ways from previous applications of the same method to single station observations. They are based on a series of frequent, detailed and carefully processed soundings extending into the middle stratosphere. In this regard the data are unique. Because the results are sensitive to the vertical averaging procedure, an attempt has been made to select appropriate filtering and averaging processes. The reliability of results is assessed by means of an error analysis and their synoptic compatibility considered using derived vertical velocity height-time patterns for three periods within the sequence, each of which contains a discrete synoptic event. Finally, the entire results are used to calculate the average and standard deviation of vertical velocity and some vertical fluxes due to synoptic scale processes. In an attempt to determine the efficacy of using conventional single station soundings for vertical velocity on vertical eddy flux studies, these results are compared with similar computations based on the routinely available 12-hourly observations at standard levels.

2. Calculation of vertical velocities

(a) Method

Although the expression for the vertical velocity, \( w \), is well known, a delineation of the assumptions involved is not readily available. The derivation of the complete expression is therefore outlined in Appendix 1. The simplified form finally used in the following
computations was,

\[ w = \frac{\partial T}{\partial t} + \frac{fT}{g} \left( V \frac{\partial v}{\partial z} - u \frac{\partial v}{\partial z} \right) + \frac{Q}{C_p} \]

(1)

where \( u, v, w \) are the \( x \) (eastward), \( y \) (northward), \( z \) (upward) components of motion; \( t \) represents time; \( T \), temperature; \( f \), the Coriolis parameter; \( g \), gravity; \( C_p \), specific heat of dry air at constant pressure; \( J \), the mechanical equivalent of heat; and \( Q \), the rate of diabatic heating per unit mass.

The first term in the numerator represents the observed local rate of change of temperature. The second term is an approximation to the horizontal advection of temperature \( V \cdot \nabla T \). The third term represents the diabatic heating.

Diabatic heating occurs from three major processes: the convergence of the vertical turbulent flux of sensible heat brought about by motion on a smaller scale than can be resolved from observations, heat gained (or lost) by radiation, and the net liberation of latent heat from condensation and evaporation processes.

At present no realistic estimate of the convergence of turbulent heat flux can be made from synoptic observations. It has been ignored in the following computations.

For the calculation of net radiation in the atmosphere the detailed cloud structure must be known. This was observed during the experiment only from the ground. A radiation computation (Manabe and Strickler 1964) was applied to several notional cloud distributions and corresponding temperature and moisture profiles in order to obtain representative profiles of radiative cooling applicable during the observational experiment. Because of inadequate cloud observations, only a schematic profile was used in the vertical velocity computations. This was assumed to be constant throughout the period, and is shown in Fig. 1. It is consistent with the results of the ancillary radiation studies mentioned above.

![Figure 1. Assumed heating rate due to radiation.](image)

To be consistent with the thermal wind assumption and the filtering techniques described later, latent heating should be taken into account only if it is significant for synoptic scale processes. This, of course, is difficult to judge. An isolated thunderstorm precipitating at the observing site for some 20 min, if included in the computations, would obscure the effect of large-scale processes. In order to take some account of latent heat liberation, the device was adopted of examining rainfall at six rainfall observing stations within 10 km of Laverton and assuming that the minimum value for each three hour period represented the condensation due to synoptic processes. This was then distributed with height in accordance with an assumed weighting function (Fig. 2). This method of assessing the latent heat liberated was considered superior to the modified form of the expression
for \( w \) with the saturated adiabatic lapse rate in the denominator because of the practical difficulty of assessing where the atmosphere is saturated. In test computations saturation was assumed at various measured relative humidities between 70 per cent and 95 per cent but in no case was there a good correspondence between the assumed time of saturation and the observed surface precipitation. Vuorella (1957) in a case study over the British Isles has shown the large differences which occur in the derived vertical velocity fields according to whether the dry- or moist-adiabatic rates are used.

(b) Correction to temperatures

Associated studies have shown a pronounced diurnal variation of measured temperature which increases upwards to an amplitude of several degrees Celsius at 30 km (Talbot 1972). To some extent this is due to solar heating of the temperature measuring element. To avoid a consequential effect on computed vertical velocities, the temperatures were corrected by subtracting the average anomaly for time of day from observed individual values.

The local rate of change of temperature is taken to be the difference between successive corrected 3-hourly observations at the same level.

(c) Averaging procedure

To obtain the basic set of wind and temperature data at 100 m height intervals, preliminary smoothing and interpolation procedures were used. These are described in the LSSE summary. (The word 'preliminary' distinguishes these procedures from subsequent smoothing operations.) Sample sequential profiles of wind components are presented in Fig. 3 to illustrate the very minor nature of this preliminary smoothing.

Ancillary studies of these profiles show quasi-periodic wind variation with vertical dimensions of about 1 km, in agreement with the finding of Sawyer (1961). In addition, because in numerous cases a variation can be recognized in more than three or four consecutive 3-hour soundings, Sawyer's suggestion that they may have horizontal dimensions of the order of several hundred kilometers is supported.

Vertical velocity profiles computed from data which have been subjected to only preliminary smoothing show extreme fluctuations with a typical vertical scale of several hundred metres. These strongly reflect similar variations in the \( V \cdot \nabla T \) profile, although the lapse rate profile also shows this type of fluctuation. Some type of secondary smoothing was obviously required. The problem was: what form of smoothing is appropriate and should it be applied to the basic parameters (\( u \), \( v \) and \( T \)) or to the vertical velocities after they were computed from data which had only preliminary smoothing?

To be consistent with the thermal wind assumption involved in the expression for vertical velocity, ageostrophic motions ideally should be filtered out. This is not possible
because various types of ageostrophic mechanisms act over the complete spectrum of motion. However, a significant proportion of the contribution from these fluctuations probably will be removed if a vertical smoothing procedure minimizes variations with a vertical scale of less than two or three kilometers. Alternatively, it may be argued that the characteristic vertical dimension of geostrophic variations is 2 km and greater; therefore fluctuations of this wavelength must be retained by the smoothing process.

Equally weighted running means over vertical intervals of several kilometers would remove much of the smaller-scale features but would induce spurious fluctuations. To avoid this, a normally distributed smoothing function (Holloway 1958) was used with a filtering interval of $6\sigma$, where $\sigma$ is the standard deviation: for practical purposes a cut-off frequency of $\pm 3\sigma$ can be assumed. After tests with different filtering intervals it was decided to use an interval of 3 km because this retained fluctuations with vertical dimensions of about 2 km and did not embrace too large a vertical portion of the atmosphere. Further tests on smoothing before and after the computation of $w$ showed significant differences due mainly to the compounding of small-scale variations in temperature, velocity and vertical shear in the expression for $V_T$. It was decided that more realistic results would be obtained by minimizing the small-scale vertical fluctuations with height in the basic data before computing vertical velocity. Sample profiles of vertical velocity are presented in Fig. 4 to illustrate the differences involved.

3. Reliability of results

The reliability of vertical velocities computed via Eq. (1) can be assessed in four ways:

(i) by comparing results with those obtained via different methods

(ii) by an error analysis of the uncertainties involved

(iii) by comparing the results for well defined synoptic situations with indirect or qualitative indications of vertical motion, and assessing the overall consistency

(iv) by considering, for the period of analysis, statistical summaries of processes involving vertical velocities, and assessing the meaning and verisimilitude of the results.

At the time of the Laverton Serial Sounding Experiment numerical models were not used on a day to day basis for the Australian region. Therefore using the 'omega-equation' for the first of the above possibilities was not practical. Also, observing stations ad-
Following Kreitzberg (1964), errors in the expression for $w$ can be represented by

$$
\left( \frac{\delta w}{w} \right)^2 = \frac{\delta T}{\delta t}^2 + (\delta V \cdot \nabla T)^2 + \left( \frac{\delta Q}{C_P} \right)^2 + \left[ \delta \left( \frac{\partial T}{\partial z} + \Gamma \right) \right]^2
$$

where $\delta \psi$ represents the uncertainty in $\psi$, and $\Gamma = g/lC_p$. Further, taking into account the sources of uncertainty and the accuracy of various measurements described in the LSSE summary, the following values were used to assess the fractional uncertainty in calculations of $w$. (A detailed consideration of the factors involved in these estimates has been given by Kreitzberg (1964).)

$\delta \partial T/\partial t = 0.3 \times 10^{-4} \, ^\circ C \, s^{-1}$; based on an assessment of errors in measuring the temperature and on the smoothing procedures used.
\[ \delta V \cdot \nabla T = 0.2V \cdot \nabla T \]: because the alternative form for calculating \( V \cdot \nabla T \) is \(-fT/g (\frac{\partial u}{\partial z} - u \frac{\partial v}{\partial z})\), it is appropriate to use a constant fractional error which reflects the probability that the higher the windspeed the greater the error in calculating advection of temperature.

\[ \delta \frac{Q}{C_p} = 0.3 \times 10^{-4} \degree C \text{s}^{-1} (35 \degree C \text{day}^{-1}) \]: probably represents an overestimate on rain-free days and an underestimate at some levels on days when much condensation occurs.

\[ \delta (\frac{\partial T}{\partial z} + \Gamma) = 0.4 \degree C \text{km}^{-1} \]: only errors in assessing the lapse rate have been taken into account here. Occasions when the atmosphere is saturated have been considered when arriving at the latent heat source error involved in the previous quantity.

The above values are smaller than those of Kreitzberg (1964) and are due to the special observing procedures for the experiment and the smoothing techniques used.

The fractional uncertainty in the determination of \( w \) thus becomes

\[ \left( \frac{\delta w}{|w|} \right)^2 = \frac{18 \times 10^{-10} + 0.04 (V \cdot \nabla T)^2}{\left( \frac{\partial T}{\partial z} - V \cdot \nabla T \right)^2 + 0.16 \times 10^{-10}} + \frac{0.16 \times 10^{-10}}{ \left( \frac{\partial T}{\partial z} + \Gamma \right)^2} \]

The following criteria were used to assess the degree of uncertainty:

\[ 0 \leq \frac{\delta w}{|w|} \leq 0.5 \quad \text{good} \]
\[ 0.5 < \frac{\delta w}{|w|} < 1.0 \quad \text{fair} \]
\[ |w| < \delta w \leq 3 \text{ cm s}^{-1} \quad \text{poor} \]
\[ |w| < \delta w > 3 \text{ cm s}^{-1} \quad \text{bad} \]

Using these ratings, more than 50 per cent of the \( w \) derivations at nearly all levels are 'good', and about 80 per cent are 'good or fair'. Relatively few are 'bad'.

4. Sample synoptic patterns

The synoptic sequence during the Laverton Serial Sounding Experiment contained two major long-wave troughs moving steadily across the continent of Australia, separated in time by a period of blocking activity associated with a 'cut-off' vortex over the Tasman Sea to the north-east of Laverton and a high-pressure/upper ridge complex to the south-east. Superimposed upon this general pattern, several short-wave features affected the area. The three most discrete synoptic events were centred on 30 September, 9 October and 19 October. These are briefly described below. Vertical velocities below 2 km are not shown because events in the lowest kilometre could not be adequately treated by the method and a 3 km weighted vertical filter was applied to the remaining results.

(i) 30 September 1966 (Fig. 5). On 28 September a system associated with a surface low pressure area which had been developing for several days began to move rapidly south-eastwards. The major surface trough and associated precipitation affected Laverton on 30 September. It was a typical synoptic event for this time of year.

The derived vertical velocity patterns in the lowest 5 km show a good correspondence with the passage of surface fronts. However, the main precipitation appears to be associated with upward motion, strongest at the 5 to 8 km level, a few hours before the passage of the surface trough. Immediately after this, marked downward motion in the lower troposphere occurs until the final cold front reaches the area.

This system is characteristic of many depressions skirting the southern edge of Australia in that it does not have a classical warm frontal structure, antecedent conditions did not lead to an established warm moist air mass ahead of the strongly active cold front. The neglect of evaporative heat sinks beneath precipitating clouds could mean that regions of descent in these conditions are not adequately identified – but this would apply princip-
Figure 5. Time sequences and synoptic situation centred on 1800 local time, 30 September 1966.

ally to the lowest levels for which the vertical weighting makes the technique inadequate in any case.

A noticeable feature is that the vertical velocity in the upper troposphere and lower stratosphere tends to be opposite in sign to that in the lower troposphere.

(ii) 9 October 1966 (Fig. 6). A transient surface high pressure system with a warm ridge of medium wave-length aloft moved steadily across the area from 8 to 10 October. Synoptic charts and satellite pictures showed several minor rapidly moving frontal features in the south-westerly stream ahead of the ridge.

Strong descent is calculated as occurring throughout most of the troposphere – apart from a brief period of upward motion at the time of the passage of a minor trough. There is again a tendency for reverse vertical motion to occur at the 8 – 10 km level.

(iii) 19 October 1966 (Fig. 7). A major surface depression associated with a deep upper trough crossed the area from 18 – 20 October. A surface trough had existed over the entire eastern half of the continent for several days previously. This resulted in a considerable supply of warm moist air which gave classical warm sector characteristics with associated warm front and cold front structure to the depression over south-east Australia.

These frontal features appear very clearly in the vertical velocity pattern. The upward motion areas have the classical slope, and strong downward motion occurs in the cold air after the major surface trough has passed. A most notable feature is the strong descent which is calculated to occur at about tropopause level throughout the passage of the system.
For all three synoptic occasions the consistency between the vertical motion patterns, the synoptic chart, surface pressure, present weather and cloud history at the observing station gives credence to the method of derivation. The sign and magnitude at mid-troposphere levels are in agreement with characteristic values obtained for similar synoptic situations in detailed numerical models. However, the reversal of vertical motion between low and high levels in the troposphere is not a feature which appears in numerical simulation. This, together with the large magnitudes of vertical velocity in the vicinity of the tropopause (usually at 10 km), requires confirmation by further observational experiments. It may be an indication that a more detailed vertical resolution at these higher levels would improve the performance of numerical weather prediction models.

At the 10 km level there is a general consistency between the vertical motion and the ozone measurements (c.f. LSSE Summary). Ozone maxima occur at these levels on 30 September/1 October and on 19/20 October after periods of strong downward motion. On 10/11 October an ozone minimum is measured after strong upward motion.

5. Some statistical results for the period of the experiment

The position of the observing site in relation to the climatic situation for October 1966 is represented by the longitudinal cross-section based on 150°W (Fig. 8) and the mean sea level pressure pattern (Fig. 9). It can be seen that Laverton lies only about 10° south of the centre of the main surface sub-tropical high pressure belt and the associated westerly wind maximum at 100 mb. The main track of baroclinic disturbances is well to the south.

The influence of the sub-tropical part of the general circulation structure is reflected in the mean zonal wind profile (Fig. 10), which shows a maximum well above the mean
tropopause level at about 10 km. During the experiment the main tropopause was usually just above the 300 mb level and its variation was associated with transient disturbances and the main baroclinic zone further south. The standard vector deviation of wind with its characteristic maximum at about tropopause level is a useful measure of synoptic scale activity. Fig. 10 indicates that this activity was above normal for this time of year in the upper troposphere although the mean zonal flow was much weaker.

(a) Mean vertical velocity

The derived mean vertical velocity profile (Fig. 11) fits in fairly well with what might be expected near the strongest part of the downward arm of the Hadley cell. The magnitudes throughout most of the troposphere are two or three times larger than the most recent observational studies of the Northern Hemisphere depict for similar latitudes (Kidson, Vincent and Newell 1969). This is probably partly due to sampling, in particular the bias introduced by the strong downward motion just above the tropopause on 18/19 October (Fig. 7). But it is also consistent with a correspondingly stronger Southern Hemisphere
meridional circulation. The clear sky pattern in mean cloudiness maps obtained from satellite pictures suggests that Melbourne lies near the southern extremity of that part of the wave number three standing eddy pattern in which enhanced downward motion can be expected to occur. The strong average downward motion in the upper troposphere is confirmed by tentative results from a mass divergence calculation for the month of October 1966 using average data from Melbourne and three surrounding stations. Profiles relative to the tropopause (set at its mean level of 10 km) are included in Fig. 11. The difference between the curves reflects the occurrence of strongest downward motion in the upper
troposphere immediately below the tropopause on occasions when the tropopause level is high. At levels above 12 km no systematic pattern can be resolved in the profile of average vertical velocity. The standard deviation of vertical velocities reaches nearly 10 cm s\(^{-1}\) just below the troposphere, but is only about 1 to 2 cm s\(^{-1}\) throughout the stratosphere.

(b) Eddy fluxes

Profiles of vertical and horizontal eddy fluxes of westerly momentum and sensible heat are presented in Figs. 12 and 13. The relation between these transient eddies and various features of the mean structure of the atmosphere will be the subject of a separate paper. Flux profiles have been calculated relative to both the ground and the tropopause level, but for the sensible heat fluxes the latter have not been extended above the tropopause level because the large vertical gradients of potential temperature in the stratosphere result in spurious variations of temperature being included in covariances when the tropopause reference level varies. In general these profiles (represented by dashed lines) confirm the conclusion that the strongest eddy activity occurs just below the tropopause.

(i) Westerly momentum (Fig. 12): The horizontal eddy flux maximum at about 10 km is well known, but the magnitudes are larger than either Obasi's (1963) zonally averaged 6-monthly seasonal results for 40°S or Priestley's (1951) annual figures for Auckland (36°S) at about the same latitude. A notable new feature is the large flux in mid-stratosphere, increasing with height at the upper limit of observations. This suggests the existence of important organized synoptic scale disturbances at these levels.

The vertical eddy flux profile indicates a strong flux convergence in the upper troposphere just below the level of maximum poleward transport by transient eddies, with downward flux above and upward flux below. The magnitudes of the vertical fluxes are quite large: the downward maximum at 10 km corresponds to 0.8 dyne cm\(^{-2}\) and the upward maximum at 7 km to 0.7 dyne cm\(^{-2}\). These results differ from the simple scheme depicted by Palmén and Newton (1969) from which a downward net eddy flux of about 1.3 dyne cm\(^{-2}\) at 500 mb can be inferred. Toroidal circulations may exist which result in a requirement for an upward net eddy flux at mid-troposphere levels in these latitudes.
Figure 11. Profiles of the average derived vertical velocity, \( w \), and its standard derivation, \( \sigma_w \). Dashed line represents values calculated relative to the tropopause level. Crosses represent mean vertical velocities retained from a mass divergence technique applied to the resultant winds for October 1966 at Melbourne and three surrounding stations.

Figure 12. Vertical profiles of the covariances representing local (transient) eddy flux of zonal momentum. Dashed lines refer to values computed relative to the tropopause level. Crosses represent the 1948/49 annual value for Auckland, 37°S 173°E (Priestley 1951). Empty and full circles represent Obasi's (1963) six-monthly zonally averaged winter and summer values for 40°S during 1958.

(Tucker 1960). Because the prevailing thermal wind is westerly, the net effect of small-scale convective fluxes will probably be downward*; hence any upward flux must be due to synoptic-scale disturbances or, on the global scale, to standing eddies.

(ii) Sensible heat (Fig. 14): A stronger poleward flux occurs in the upper troposphere

* Note: Moncrieff and Green (1972) have described a mechanism whereby organized convection can transfer momentum in an up-gradient sense, and have cited evidence that at least some severe storms accomplish this.
than indicated by Priestley's (1951) annual values for Auckland, 38°C. Strong fluxes also appear in the lower stratosphere, largely due to an increase in the variance of temperature at these levels, and in the middle stratosphere. At these highest levels of observation the rapid increase in the $\bar{u}'\bar{\theta}'$ covariance is due to increases in the variances of $u$ and $\theta$ and in the correlation between them. This may be partly a result of decreasing observations (206 at 25 km and 72 at 28 km), but in view of the consistency between this and the $\bar{u}'\bar{\nu}'$ and $\sigma$ profiles, it is more likely to be the result of increased synoptic activity in the mid-stratosphere.

The pattern which emerges as a requirement for synoptic balance is of a strong downward eddy flux of sensible heat in the upper few kilometers of the troposphere. Upward fluxes occur at lower levels, in the 5 - 8 km layer. The net result is of a vertical eddy flux convergence centred on the level of maximum poleward horizontal transport. There are signs of a similar but weaker convergence centred in the vicinity of the level of secondary maximum horizontal transport in the lower stratosphere. If there is some radiative heating in the vicinity of the tropopause – slight cooling has been assumed (Fig. 1) – the downward eddy flux could be somewhat less strong; but because of the large vertical gradient of potential temperature any decrement would be less than about 5 per cent. Also, at lower levels, if the net effect of clouds is such that the assumed radiative cooling in mid-troposphere is too high, this together with the neglect of small scale convective heating may mean that the upward eddy flux is stronger than depicted, but again by only a small amount.

These results are consistent with the deductions of Palmén and Newton (1969) that the relative importance of synoptic scale disturbances for the vertical transport of energy increases upwards and dominates in the middle and upper troposphere. The maximum upward flux of 4 cm$^2$K s$^{-1}$ at 7 km corresponds to a value of approximately 0.04 ly min$^{-1}$, some 30 per cent larger than the 500 mb value which can be inferred at these latitudes from the derived winter heat budget of Palmén and Newton. Because the observing site is near the equatorward edge of the main belt of extra-tropical disturbances in the Southern Hemisphere, the eddy fluxes can be expected to be stronger further south.

Thus it appears that the derived vertical velocities yield vertical eddy fluxes which are plausible in both order of magnitude and pattern. They indicate that, at this latitude and time of year, the effect of synoptic scale processes is to cause a vertical eddy convergence of
both enthalpy and zonal momentum centred on the levels of strongest poleward horizontal transport. Observed winds and temperatures suggest also significant synoptic activity with a typical period of four or five days affecting horizontal transport of momentum and heat in the mid-stratosphere.

6. **Comparison with 12-Hourly Derivations**

Routinely available upper air winds (at standard heights) and temperatures (at standard pressure levels) at 12-hourly intervals at Laverton for the period of the lss were used to obtain vertical velocities. From these standard level (st) data the average vertical velocity at each level, its variance and covariance with zonal velocity and potential temperature were obtained. The difference between these results and those derived from the more detailed 3-hourly soundings is represented in Fig. 14. In addition a set of the more detailed soundings at 12-hourly intervals was used to shed some light on the relative extent to which
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climatological statistics are limited by the large time intervals or by the poor vertical resolution.

Although there is some similarity between the st. results and those from the detailed 3 hourly soundings for both \( \bar{w} \) and \( \sigma_w \) (in the latter case the correspondence is fair), the flux profiles are quite dissimilar. Time height sections of derived vertical velocity suggest the reason for this. The strong vertical velocity patterns associated with transient systems were almost completely smoothed out because they occurred on time scales which cannot be represented by 12-hourly soundings. Conversely the structure of downward motion patterns, particularly in the upper troposphere appears to occur on a longer time scale which can be better represented.

On the other hand, the correspondence between results from the 3-hourly and 12-hourly detailed soundings is much better. This indicates that poor vertical resolution is more detrimental than poor time sampling – at least within these limits.

The net result of this pilot study is the suggestion that vertical velocities and particularly statistics involving their covariance with other parameters cannot be adequately derived from 12-hourly routine upper air observations at standard levels. However, the detailed individual ascents may be capable of giving statistical data of improved quality.

7. Conclusion

The compatibility between synoptic sequences and vertical velocity time-height sections together with the verisimilitude of the vertical eddy flux results give credence to this method of determining vertical velocity from detailed 3-hourly serial soundings. The reliability assessment adds to this but, as with all such studies, considerable subjectivity is involved at some stage or other in allocating values to various error sources. This is particularly true in taking account of latent heat release; yet ancillary studies have shown that reasonable changes (involving 20 per cent differences at some levels) in the weighting profile (Fig. 2) do not cause major differences in the vertical velocity patterns or in the flux profiles.

A notable feature of the synoptic results is the very strong downward motion shown at about tropopause level, associated with equally strong upward motion at lower levels and the passage of a surface low pressure area. This is allied with a general tendency for reverse vertical motion to occur between low and high troposphere. This latter feature is summarized in Table 1 which contains the correlation coefficients of derived 3-hourly vertical velocities at each pair of (kilometer) levels between 3 km and 12 km. These features certainly require confirmation. The strong downward motion at about 10 km appears to occur when a strongly developed sub-tropical jet stream at higher levels (\(~\sim 13 \text{ km}\)) and lower latitudes is in juxtaposition with the polar jet (\(~\sim 10 \text{ km}\)) associated with a transient mid-latitude system. Reiter and Whitney (1969) and Beaulieu (1968) have studied aspects of similar synoptic situations from the point of view of air mass interchange.

An important feature of the vertical flux results is the apparent tendency of synoptic scale features at this geographical location to accomplish a vertical convergence at the level of maximum poleward eddy transport. This seems to occur for both zonal momentum and sensible heat which have maximum horizontal transports at different levels. The magnitude of the vertical convergence at these levels is \(\sim 2 \times 10^{-4} \text{ cal cm}^{-2} \text{ s}^{-1}\) for heat (corresponding to \(\sim 1.5^\circ \text{K day}^{-1}\)) and \(\sim 2 \times 10^{-4} \text{ g cm}^{-2} \text{ s}^{-2}\) for momentum (corresponding to \(\sim 3 \text{ m s}^{-1} \text{ day}^{-1}\)). If these are at all representative of the latitude zone then obviously these synoptic scale vertical fluxes are of substantial importance in the general circulation. Although not directly involving vertical velocity derivations, a further result revealed by the detailed vertical resolution of the serial soundings is the indication that synoptic systems near the upper limit of observations (27 – 30 km, the middle-stratosphere) accomplish strong meridional eddy transports.

Finally, on the basis of these results, it would appear that routinely available upper air soundings at standard pressure levels at 12-hourly or lower frequency cannot be used reliably to obtain observational details of the vertical eddy flux structure in the troposphere.
ACKNOWLEDGMENTS

I gratefully acknowledge the assistance of the following CMRC staff members: Mrs. Mary Adams who did most of the programming and helped in the preliminary work; Mr. P. England, for programming; and Mr. W. Kininmonth who carried out the ancillary radiation calculations.

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Determination of the Expression for Vertical Velocity

From an expanded and re-arranged form of the first law of thermodynamics, the following expression may be obtained:

\[
w = \frac{-\frac{\partial T}{\partial t} - \mathbf{V} \cdot \nabla T + \frac{Q}{C_p} + \frac{1}{\rho C_p} \left( \frac{\partial p}{\partial t} + \mathbf{V} \cdot \nabla p \right)}{\frac{\partial T}{\partial z} + \frac{g}{f C_p}}
\]

where \( \nabla T \) represents the horizontal gradient; \( f \), the mechanical equivalent of heat; \( Q \), the rate of diabatic heating per unit mass; and other symbols take their usual meaning.

An evaluation of the term in the numerator involving pressure shows it to be generally two orders of magnitude smaller than the sum of the remaining terms. Thus to a good approximation

\[
\frac{-\frac{\partial T}{\partial t} - \mathbf{V} \cdot \nabla T + \frac{Q}{C_p}}{\frac{\partial T}{\partial z} + \frac{g}{f C_p}}
\]

In order to obtain an expression for \( \mathbf{V} \cdot \nabla T \) which does not involve a horizontal gradient, the following algebraic manipulation is required.

The horizontal component of the equation of motion is differentiated with respect to \( z \) and re-arranged to become

\[
\frac{\partial}{\partial z} \mathbf{V}_{ph} = \frac{1}{g} \left( \frac{\partial}{\partial z} \mathbf{F} - f \kappa \mathbf{k} \frac{\partial}{\partial z} \mathbf{V} + \frac{\partial}{\partial t} \frac{d}{dt} \mathbf{V} \right)
\]

where \( \mathbf{F} \) is the horizontal frictional acceleration.

Also, from geometric considerations

\[
\frac{\partial}{\partial z} \mathbf{V}_{ph} = \frac{1}{T} \mathbf{V}_{zT} + \frac{1}{T} \frac{\partial T}{\partial z} \mathbf{V}_{ph}
\]

whence, multiplying by \( \mathbf{V} \), substituting for \( \partial \partial z \mathbf{V}_{ph} \), and using the geostrophic relation \( \nabla_{ph} = -f \kappa g \mathbf{k} \mathbf{V}_{ph} \), this becomes on rearrangement

\[
\mathbf{V} \cdot \nabla_{zT} = \frac{T}{g} \mathbf{V} \cdot \frac{\partial}{\partial z} \mathbf{F} - \frac{f T}{g} \mathbf{V} \cdot \kappa \mathbf{k} \frac{\partial}{\partial z} \mathbf{V} - \frac{T}{g} \mathbf{V} \cdot \frac{\partial}{\partial z} \left( \frac{d}{dt} \mathbf{V} \right) + f \frac{\partial T}{\partial z} \mathbf{V} \cdot \kappa \mathbf{k} \mathbf{V}_{ph}.
\]

The physical interpretation of this equation is that the horizontal advection of temperature is balanced by

(i) \( T \kappa g \mathbf{V} \partial / \partial z \mathbf{F} \) the effect of internal friction, or eddy stresses.
(ii) \( -f T \kappa g \mathbf{V} \cdot \kappa \mathbf{k} \partial / \partial z \mathbf{V} \) the effect of the vertical shear of horizontal wind
(iii) \( -T \kappa g \mathbf{V} \cdot \partial / \partial z (d / dt \mathbf{V}) \) the effect of accelerating horizontal flow
(iv) \( f \kappa g \partial T / \partial z \mathbf{V} \cdot \kappa \mathbf{k} \mathbf{V}_{ph} \) an a-geostrophic term involving the difference between \( \mathbf{V}_{ph} \) and \( \mathbf{V}_{zT} \).

An assessment of the magnitudes of the friction, acceleration and a-geostrophic terms
showed that above the boundary layer they are generally one or two orders of magnitude smaller than the wind-shear term. Accordingly they were neglected and the approximate equation for vertical velocity in component form becomes:

$$w = -\frac{\partial}{\partial t} T + \frac{fT}{g} \left( \frac{\partial u}{\partial z} + u \frac{\partial v}{\partial z} \right) + \frac{Q}{C_p}$$

$$\frac{\partial}{\partial z} + \frac{g}{fC_p}$$