Coalescence in a weakly turbulent cloud

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SUMMARY

In a recent paper on the effect of turbulence on droplet collisions in clouds, Woods, Drake and Goldsmith (1972) proposed that the principal factor was the strong spectral peak of shear near the Kolmogorov microscale. This hypothesis led to an important simplification, namely that spatial and temporal fluctuations in the turbulent shear field could be ignored during an encounter between a pair of cloud droplets. However, the authors pointed out that their simple model of (isotropic, homogeneous) turbulence contained an assumption, namely that the shears are normally distributed, which would have to be re-examined if it turned out that droplet collision efficiency increased rapidly after a threshold value is exceeded. Subsequent measurements by Jonas and Goldsmith (1972) showed that this is indeed the case. In this paper we reconsider the efficiency of turbulent coalescence in the light of the new experimental data, and taking account of the intermittency of shear distribution. It is concluded that significant increases of collection efficiency will occur in clouds which are only weakly turbulent. For example, the collection efficiency for droplets of radii \( R = 20 \mu m \) and \( \tau = 9 \mu m \) in a cloud with energy dissipation rate \( \epsilon = 55.5 \text{ cm}^2 \text{s}^{-3} \) will be approximately 14 per cent compared with 2 per cent for the same droplets falling through still air. The shear zone in the wind tunnel experiment designed by Woods et al. (1972) is a realistic approximation to the shear zones in cloud turbulence.

1. INTRODUCTION

The wind-tunnel experiments of Woods, Drake and Goldsmith (1972) and of Jonas and Goldsmith (1972) have shown that the collection efficiency of droplets of various radii for droplets of 9 \( \mu m \) radius depends very strongly on the shear in the flow field through which the droplets are falling. The data of Jonas and Goldsmith show clearly that there is a threshold value of the shear, below which the collection efficiency is not affected. On the other hand, the collection efficiency increases rapidly as the shear exceeds the threshold value. In clouds with very intense turbulent motions, the threshold is exceeded frequently, but in weakly turbulent clouds the probability of exceeding the threshold shear is small. Calculations of the overall increase in collection efficiency thus require data on the probability distribution of shears in a turbulent cloud.

In turbulent flow at large Reynolds numbers, the power spectra of velocity derivatives have a peak at a wavenumber corresponding to about ten Kolmogorov microscales (Tennekes and Lumley 1972; Ropelewski, Tennekes and Panofsky 1973). The Kolmogorov microscale \( \eta \) is defined by

\[
\eta = (\nu \langle \epsilon \rangle)^{1/4},
\]

where \( \nu \) is the kinematic viscosity and \( \langle \epsilon \rangle \) is the mean dissipation rate of kinetic energy per unit mass. The value of \( \eta \) depends quite weakly on \( \epsilon \); for \( \epsilon = 10 \text{ cm}^2 \text{s}^{-3}, \eta = 1.35 \text{ m} \) for \( \epsilon = 100 \text{ cm}^2 \text{s}^{-3}, \eta = 0.76 \text{ mm}; \) for \( \epsilon = 1,000 \text{ cm}^2 \text{s}^{-3}, \eta = 0.43 \text{ mm} \) (here \( \nu = 0.15 \text{ cm}^2 \text{s}^{-1} \) has been used).

A representative value of \( \eta \) then is 1 mm, and a representative value for the linear dimensions of the filaments in which most of the fluctuating shear is concentrated may be estimated as 10 \( \eta = 1 \text{ cm} \) (for a detailed discussion, see Tennekes and Wyngaard 1972). A shear layer of approximately 1 cm thickness was used in the wind-tunnel experiments of Woods et al. (1972) and of Jonas and Goldsmith (1972); it is evident that those experiments were designed in an appropriate way.

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Since the power spectra of velocity derivatives are fairly narrow, it is not an unwarranted oversimplification to conceive the regions in which most of the fluctuating shear occurs as thin filaments that are rather similar to those observed in the smoke rising from a cigarette. Since the viscous dissipation rate is proportional to the square of the shear, these filaments are responsible for virtually all of the dissipation of kinetic energy that takes place in a turbulent flow. In isotropic turbulence, the following relations hold (Hinze, 1959):

$$\dot{\varepsilon} = 15 \nu \left( \frac{\partial u}{\partial x} \right)^2 = \frac{15}{2} \nu \left( \frac{\partial u}{\partial z} \right)^2 \quad (2)$$

Here, $u$ is the $x$-component of the velocity fluctuations; the co-ordinate system is chosen such that $x, y$ defines the horizontal plane, while $z$ points vertically upwards. The isotropic relations Eq. (2) indicates that the filaments with rapid shears indeed may be identified as dissipative eddies (Tennekes and Lumley 1972): the large numerical factors in Eq. (2) suggest that we have to keep in mind that many components are involved in the dissipative structure of turbulence.

This train of thought leads to the conclusion that the spectral structure of shear fluctuations plays a relatively minor rôle in the dynamics of the microstructure of turbulence. Since about 1960, investigators of the microstructure have focused their attention on the probability distributions of velocity derivatives (for a recent review, see Ropelewski et al. 1973). A still growing amount of evidence shows that the probability distributions of shear components such as $\partial u/\partial z$ is extremely non-Gaussian, with very small and very large excursions much more likely than in a normal distribution (e.g. Wyngaard and Tennekes 1970). This evidence has reinforced the belief that the dissipative eddies are thin filaments that occupy only a small fraction of the entire volume occupied by a turbulent flow, because the measured probability distributions are similar to those that would arise if one computed them on basis of a model in which the dissipative structure is assumed to be extremely intermittent. On these grounds, the turbulence community refers to this phenomenon as the intermittency of the dissipative structure.

It appears, therefore, that the isolated shear layer in the wind-tunnel experiments referred to above is quite representative of the kind of dissipative eddies that one encounters in a turbulent flow at large Reynolds numbers; in particular, it is fortunate that the shearing filament of the wind-tunnel set-up occurred in a flow field that had essentially no shear outside the frictional volume occupied by the filament, because that situation is quite similar to the one in which dissipative filaments occurring in turbulent flow find themselves. We conclude that efforts to compute the increase in overall collection efficiency in turbulent clouds, based on the laboratory data of Jonas and Goldsmith (1972) and on measured probability distributions of velocity derivatives in turbulent flows, do not involve undue amounts of speculation. Calculations of this kind have the additional attractions that they merely explore the consequences that can be drawn from available experimental results, and do not need to depend on questionable theoretical models of coalescence or of the turbulent microstructure.

2. Calculations

Not many measurements of the probability distributions of shears like $\partial u/\partial z$ have been made, but it is a widely held belief that they are quite similar to that of $\partial u/\partial x$ (Gurvich and Yaglom 1967). Measurements of the latter are quite abundant; in the calculations below we shall employ the data given by Tennekes and Wyngaard (1972), because those refer to a turbulence Reynolds number that is representative of those found in turbulent clouds (the Reynolds number based on the standard deviation of horizontal velocity and on the length scale of the largest eddies in clouds ranges roughly from $10^5$ to $10^6$).

The shear component $\partial u/\partial z$ will be called $s_1$; its standard deviation $\sigma_1$ is given by Eq. (2):

$$\sigma_1 = 0.364 \left( \langle \varepsilon/\nu \rangle \right)^{1/2} \quad (3)$$
The normalized probability density $\beta^* (x_1) = \sigma_1 \beta (s_1)$, where $x_1 = s_1/\sigma_1$ is the normalized deviation, satisfies the following integral conditions:

$$
\int_{-\infty}^{\infty} \beta^* (x_1) \, dx_1 = 1, \quad \int_{-\infty}^{\infty} x_1^2 \beta^* (x_1) \, dx_1 = 1. \quad (4)
$$

The collection efficiency, however, should not depend on the sign of $\partial \tau / \partial z$. For our calculations we thus need the probability distribution of the absolute value of $\partial \tau / \partial z$, which satisfies

$$
\int_{0}^{\infty} \{\beta^* (x_1) + \beta^* (-x_1)\} \, dx_1 = 1. \quad (5)
$$

If we define the symmetric part $\beta^s$ of $\beta^*$ as

$$
\beta^s = \frac{1}{2} \{\beta^* (x_1) + \beta^* (-x_1)\}, \quad (6)
$$

then

$$
\int_{0}^{\infty} 2 \beta^s \, dx_1 = 1. \quad (7)
$$

The distribution of $2\beta^s$ measured in a flow with a turbulence Reynolds number of about $4 \times 10^6$ is given by Tennekes and Wyngaard (1972); their curve is reproduced here in Fig. 1, along with the one corresponding to a Gaussian distribution. As is evident from those curves, the departure from the normal curve is quite pronounced.

![Figure 1](image-url)  

Figure 1. The probability density function of the absolute value of the shear component $s_1 = \partial \tau / \partial z$ in a turbulent flow at large Reynolds numbers (solid line). For comparison, a Gaussian curve is shown (dashed line).
In our calculations, we shall restrict ourselves to the shear-enhanced collection efficiency of 20 \( \mu \)m droplets for 9 \( \mu \)m droplets, because that is a case for which the threshold shear measured by Jonas and Goldsmith (1972) is fairly large, while gravitational coalescence does not yet play a major rôle. The collection efficiency of droplets larger than 20 \( \mu \)m rises rapidly with increasing radius because their terminal velocity is appreciable; the collection efficiency of drops smaller than 20 \( \mu \)m is relatively large — even in a weakly turbulent cloud — because their threshold shear is fairly small (Jonas and Goldsmith, 1972). The case in which the droplet radii are 20 \( \mu \)m and 9 \( \mu \)m thus is most sensitive to the probability distribution of the shear fluctuations in turbulent clouds. The laboratory data of Jonas and Goldsmith that pertain to this case are reproduced here as Fig. 2.

![Figure 2](image)

The increase in collection efficiency of 20 \( \mu \)m droplets for 9 \( \mu \)m droplets as a function of the shear in the laboratory experiment of Jonas and Goldsmith. The dashed part of the heavy line is an extrapolation of the data; it was needed for the calculations in Table 1. The exact shape of the extrapolated line has very little effect on the results because \( \beta^{**} \) decreases rapidly with \( S \).

The increase in overall collection efficiency is calculated as follows. The product of the collection efficiency at a particular value of the shear and the probability density at that value is a contribution to the probability that a 20 \( \mu \)m droplet collides with a 9 \( \mu \)m droplet.

The integral of that product, taken over all values of the shear, should be equal to the net increase in collection efficiency due to the shear fluctuations in a turbulent cloud:

\[
\langle \Delta E \rangle = \int_0^\infty 2 \beta^{**}(x) \cdot \Delta E(x) \, dx.
\]  

One problem immediately arises. In the laboratory experiments only one component of the shear, i.e. \( \partial u/\partial z \), was treated, while in reality the shear fluctuations of the turbulent microstructure have many components. The shear components are

\[
\frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial z}, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial z}, \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}
\]

in addition to those six, there are three components representing pure strain:

\[
\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial w}{\partial z}
\]

All of these may contribute to the collection efficiency. The data of Jonas and Goldsmith show that the collection efficiency does not decrease dramatically if the wind-tunnel is rotated by 45°; for lack of more comprehensive data we shall assume that the total increase in collection efficiency is equal to the product of the number of shear components and the
increase in collection efficiency computed for the \( \partial u / \partial z \) component:

\[
\langle \Delta E \rangle_t = 6 \langle \Delta E \rangle_1
\]

(9)

Here we ignore the possible contributions from the strain components \( \partial u / \partial x \), \( \partial u / \partial y \), and \( \partial u / \partial z \).

Since the assumption incorporated in Eq. (9) is a rather crude one, we shall compute \( \langle \Delta E \rangle_t \) also in a somewhat different way. If we assume that the shear filaments in a turbulent cloud are similar in shape to the one used in the wind-tunnel experiments, the local, instantaneous dissipation rate in such a filament is

\[
\varepsilon = \nu s_t^2
\]

(10)

where \( s_t \) is the magnitude of the shear, regardless of the direction in which it happens to occur. If Eq. (10) is an acceptable approximation, the standard deviation \( \sigma_t \) of the total shear is given by

\[
\sigma_t = (\varepsilon / \nu)^{1/3}
\]

(11)

Comparing Eq. (11) with Eq. (3), we see that the intensity of the shear fluctuations is appreciably greater than that of any one of its components. If we assume – again for lack of appropriate data – that the probability distribution of \( s_t \) is similar to that of \( s_0 \), we can compute \( \langle \Delta E \rangle_t \) directly, without having to be concerned about a multiplication factor.

For collisions between 20 \( \mu \text{m} \) droplets and 9 \( \mu \text{m} \) droplets, the threshold shear is 7 \( \text{s}^{-1} \) (see Fig. 2). We present here the results of our calculations for a case (a) in which \( \sigma_1 = 7 \text{ s}^{-1} \) and a case (b) in which \( \sigma_1 = 7 \text{ s}^{-1} \). Both of these are threshold situations in the sense that one would have to conclude that the collection efficiency was not enhanced if one did not take the probability distribution into account. The results are given in Table 1.

**TABLE 1**

<table>
<thead>
<tr>
<th>Case (a): ( \varepsilon = 55.5 \text{ cm}^2 \text{ s}^{-1} ), ( \nu = 0.15 \text{ cm}^2 \text{ s}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_1 = 7 \text{ s}^{-1} )</td>
</tr>
<tr>
<td>( \sigma_t = 19.2 \text{ s}^{-1} )</td>
</tr>
<tr>
<td>Case (b): ( \varepsilon = 735 \text{ cm}^2 \text{ s}^{-1} ), ( \nu = 0.15 \text{ cm}^2 \text{ s}^{-1} )</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>( \sigma_1 = 2.55 \text{ s}^{-1} )</td>
</tr>
<tr>
<td>( \sigma_t = 7 \text{ s}^{-1} )</td>
</tr>
</tbody>
</table>

In the Table, the first row for each case refers to the calculation based on one component, which is then multiplied by 6 according to Eq. (9), while the second row refers to the calculation based on Eq. (11). The results of the two methods of calculation are not identical, but they are close enough to warrant some confidence in the order of the magnitude of the effects of shear fluctuations. Clearly, in a cloud with \( \varepsilon = 55.5 \text{ cm}^2 \text{ s}^{-1} \) certainly not a case with extremely intense turbulence – the overall collection efficiency is increased by a surprising 12 per cent. The gravitational collection efficiency of 20 \( \mu \text{m} \) droplets for 9 \( \mu \text{m} \) droplets in still air is about 2 per cent (Jonas and Goldsmith 1972); we conclude that the shear fluctuations cause a seven-fold increase in the collision rate for this case.

Even in case (b), with a very small value of the dissipation rate, the collection efficiency is increased by some 2 per cent. Because the still-air collection efficiency also is 2 per cent, this implies that even a very weakly turbulent cloud can easily double the collision rate between 20 \( \mu \text{m} \) droplets and 9 \( \mu \text{m} \) droplets.
3. Conclusions

Our calculations show quite clearly that the shape and the velocity distribution inside dissipative filaments play a major rôle in the turbulence-enhanced collision rates between droplets. The problem that arises in this context is that the turbulence community has not reached a consensus on the structure of the typical dissipative filament. Having no other data, we had to assume that the shear layer in the laboratory experiments is representative of those found in nature. We did advance arguments showing that the laboratory set-up was well designed, but we cannot state for sure that the correspondence between the filaments in nature and that in the wind-tunnel is one-to-one as far as all details of shape and structure are concerned. Hence, our calculations, promising though they seem to be, must be considered to be exploratory, pending a much more specific definition of dissipative filaments in the microstructure of turbulence. Studies of this issue are continuing (e.g. Kuo and Corrsin 1971), but so far it remains speculation as to whether the filaments are mainly threads, ribbons, or sheets. In this context, Kovasznay (private communication) speaks of the Italian syndrome: are they vermicelli, macaroni, lasagna, or yet another kind of noodles?

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References


