A simple theory of certain heliacal and anthelic halo arcs.

The long hexagonal ice prism as a kaleidoscope

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(Manuscript received 18 December 1972; in revised form 8 May 1973)

SUMMARY

G. H. Liljequist and J. R. Blake have both recorded anthelic halo arcs on several occasions in Antarctica, with the sun at comparatively low elevations, which exclude on some occasions, and almost exclude on others, the mechanism put forward by Alfred Wegener to account for such arcs. A particularly complete display reported by J. R. Blake enables these arcs to be identified with those predicted by a very simple theory given in the following paper, to which are added some remarks about the formation of an anthelion.

1. Discussion

Two theories have been put forward to account for the formation of anthelic arcs. Both employ long hexagonal ice crystals sinking through the air with their long axes horizontal, and consider rays of light which enter by a side face, are reflected off the vertical end face and emerge through another or the same side face. Probably the more satisfactory of these theories was the work of Wegener (1925) who showed that the rays which give rise to the upper arc of contact to the halo of 22°, would, if internally reflected off the end face, produce arcs through the anthelion. They would be tangential to the 22° halo on one side of the celestial sphere and intersect at the anthelion on the other. No such arcs could arise, however, if the solar elevation was less than 14° and they would be unlikely to be observable unless it was considerably higher. The other theory was put forward by Hastings (1920). He employed the same group of crystals as Wegener and pointed out that when they also had a pair of sides vertical, a ray reflected off such a side face and the end face would emerge from the face of entry, coming from the anthelion. If such a crystal rotated about its horizontal axis the emergent ray would trace out an arc through the anthelion. A crystal rotating about the horizontal axis would pass through this position and give rise to an arc through the anthelion, the combination of those arising from all positions of the horizontal axis giving rise to resultant arcs, though it would seem very diffuse ones. The question of the descent of crystals with side faces vertical will be discussed at the end of this article.

It is remarkable that a much simpler theory, based on nothing more difficult than the solution of the schoolboy problem of images in inclined mirrors, should not have been considered. The nature of the arcs which this theory would appear to explain can be understood from accounts of two comparatively recent observations in Antarctica, one by Liljequist (1956) and the other by Blake (1961), both at low solar elevations.

Fig. 1 shows Liljequist's anthelic cross observed on 30 September 1950, with the sun at an elevation of 16°. The arms of the cross were estimated to intersect at an angle of 60°. Fig. 2 is by Blake and shows the much more complicated display he recorded on 2–3 December 1958, with the sun at an elevation of 13°. The arcs in which we are at present interested are those labelled 1, 2, 3, and 4. They were white in colour and Blake records that 'although not bright, these curves were certainly not faint, and were quite distinct.'

The present theory assumes the presence of long hexagonal crystals floating with their long axes horizontal and possessing flat ends—as with the other two theories. Light entering by an end face and lying within a cone of semi-vertical angle 58° round the normal, will be totally reflected from the side faces inside the crystal (Fig. 3) and it will be
Figure 1. Liljequist's Anthetic Cross (Liljequist 1956).

Figure 2. Fig. from J. R. Blake's 'Solar halos in Antarctica' (Blake 1961). The present article deals with the arcs labelled 1, 2, 3, and 4, and with the anthetic pillar.

Figure 3. A ray of light entering a long hexagonal ice crystal from within the cone of semi-vertical angle 58° will be totally reflected from the side faces inside.
Figure 4. On emerging by the end face opposite to that of entry, the ray will proceed in one of the six
directions SO, S_1O, S_2O, S_3O, S_5O, or S_7O (O being the centre of the celestial sphere – not marked in the Fig.).

Figure 5. After reflection at the end face and emergence through the face of entry, the ray will lie in directions
such as S'O, etc.

Figure 6. The figure on the back of the celestial sphere when solar elevation and distances from the anthelion
are small.
'piped' from one end of the crystal to the other, by a series of total reflections, with no loss of intensity on that account.

If the light emerges from the flat end face at the opposite end of the crystal from that by which it entered, the ray will lie in one of the six directions $S_0O; S_0O; S_0O; S_0O; S_0O; S_0O$, $O$ being the centre of the celestial sphere — as in Fig. 4. The centre $O$ has not been marked to avoid crowding the Figure. In this Figure $R$ gives the direction of the axis of the crystal, $S_0$ the position of the sun on the celestial sphere, and RA, RB, RC ... are the reflecting planes formed by the sides of the hexagonal crystal prism. In this Figure, $S_1$ is, of course, the image of $S_0$ in the plane RB, while $S_2$ is the image of $S_0$ in the plane RC and gives the direction from which the beam of light proceeds after two reflections. Similarly $S_1$ is the image of $S_0$ in the plane RA, and $S_2$ is the image of $S_0$ in the plane RF, and, like $S_1$, gives the direction from which the light proceeds after two reflections. The image $S_3$ is the image of the points $S_1$ and $S_2$ in the planes RD and RE, respectively, and is the origin of the beam after three reflections.

It is evident that the angles $S_0RS_1$ and $S_0RS_2$ are both equal to twice $S_0RB + 60^\circ - S_0RB = 60^\circ$. That is

$$S_0RS_1 = S_0RS_2 = 120^\circ.$$  

An additional remark is required about the refraction which occurs when the light enters by one end face and leaves by the other. Reflection off the side faces of the prism does not alter the angle of incidence of the light on to the end face within the crystal, apart from rotating the plane of incidence. Thus the refraction which occurs as the light emerges from the crystal is equal in magnitude to that which occurred on entry, but, of course, away from the normal instead of towards it. Thus $S_0O$ (Fig. 4) is the same as the direction, of the incident light on to the crystal (i.e. the direction of the sun on the celestial sphere) while the other images lie on a small circle whose pole is at $R$, passing through the sun. The theory is identical to that of the simple kaleidoscope.

So long as the horizontal crystal axis is unaltered in position, the images $S_1$ and $S_2$ are fixed in position whatever the orientation of the crystal about its axis. As the axis moves over the horizontal plane, $S_1$ and $S_2$ describe arcs in the sky which we shall seek to identify with Arc 2 in Blake’s diagram. If the ray is reflected in the rear face and finally emerges from the front face by which it entered, an arc passing through the anthelion will be formed, which we shall seek to identify with Blake’s Arc 4.

Fig. 5 shows the effect of the reflection in the end face (at right angles to OR). We obtain the origins, $S'$ and $S_1'$, of the rays SO and $S_0O$ after reflection, by drawing $SS'$ and $S_1S_1'$ parallel to OR. When the elevation of the sun is small and the image $S_1'$ lies close to the anthelion, the figure on the back of the celestial sphere may be represented by straight lines, as in Fig. 6. If we drop a perpendicular from the anthelion, $A$, on to the horizontal plane and produce $SR'$ to meet it at $B$, we have

$$R'S' = R'A = R'S_1' = R'B.$$  

The points $S_1'$, $A$, $S'$, and $B$, thus lie on a circle with centre $R'$, and the angle $S_1'AB$ is half of the angle $S_1'R'B$, and therefore equal to $30^\circ$.

Thus $S_1'$ lies on a line through $A$ making an angle of $30^\circ$ with the vertical. When $S'$ falls on the other side of $A$ we have a similar arc generated by the image $S_2'$. This, therefore, provides an explanation of Liljequist’s anthelic cross, the two arcs intersecting at an angle of $60^\circ$.

Figure 7. Figure for calculating the arcs generated.
If $\beta$ is the azimuth of the crystal axis $R$ relative to the solar vertical, we can easily calculate the position of the image $S'_1$ for any given elevation of the sun, $\Sigma$. There is no point in doing this by plane trigonometry, assuming the lines on the celestial sphere to be straight. Using spherical trigonometry instead we have from Fig. 7

$$\cos R'S' = \cos \Sigma \cos \beta$$

$$\sin \sigma = \frac{\sin \Sigma}{\sin R'S'}$$

$$\sin S'_1 N = \sin \mu = \sin (60^\circ - \sigma) \sin R'S'$$

$$\cos N R' = \frac{\cos R'S'}{\cos \mu}$$

Measured from the vertical through the anthelion the azimuth of $S'_1$ is

$$\lambda = NR' - \beta.$$ 

By giving $\beta$ a series of values between $-90^\circ$ and $+90^\circ$ the entire arc generated by $S'_1$ can be plotted. That generated by $S'_2$ (the reflection of $S_2$ of Fig. 4) is continuous with it and the two together produce the curves plotted for a series of solar elevations in Fig. 8.

When the sun is on the horizon the anthelion is at $A_9$ and we have the pear shaped

Figure 8. Anthelic arcs formed by the images $S'_1$ and $S'_2$. 
curve labelled $\Sigma = 0$, which also has an identical loop lying inverted below the horizon. When the sun is at an altitude of $10^\circ$ the antheleon is at $A_{10}$ and the upper loop has shrunk in size while the lower one below the horizon has grown. With the sun at an elevation of $30^\circ$ the arc just reaches the antheleon $A_{30}$ at a cusp and for greater elevations no loop is formed at all and the arc does not reach up to the antheleon.

The existence of the Parry arc shows that long crystals sometimes fall not only with the long axis horizontal but also with a pair of faces horizontal as well. In such crystals the inclinations of the reflecting planes ARD, BRE, and CRF (Fig. 4) are fixed and the images $S_1$ and $S_2$ can also give rise to arcs. One of the planes (ARD) will be horizontal and $S_2$ and $S_3$ are the reflections of $S_2$ and $S_1$ in it. The resulting arcs will thus be simply the reflections of those plotted previously in Fig. 8, in the horizontal plane. The resulting series of antheletic arcs formed by the reflections of these images in the end faces of the crystals have been plotted in Fig. 9 for a similar range of solar elevations.

![Figure 9. Sub-antheletic arcs formed by the images $S_2'$ and $S_3'$.](image)

Those rays which emerge from the rear face instead of being reflected in it, give rise to heliacal arcs. The corresponding figure on the front of the celestial sphere is identical to Fig. 7 except that the antheleon does not appear and $S'$ becomes $S$, the position of the sun itself. The azimuth of $S_1$ (in place of $S_1'$) measured from the point on the solar vertical underneath the sun (and not under the antheleon as before) is $NR + \beta$, while its altitude remains the same. The effect of this is to produce a much larger loop than before. The
Figure 10. Stereographic plot of the four arcs generated by reflections in the faces of long hexagonal prisms floating with their long axes horizontal.

Figure 11. A possible crystal shape, with one pair of side faces smaller than the others, which would float through the air with this pair of side faces vertical.

Figure 12. Indicating possible reflections in the inclined faces of such a crystal as that in Fig. 11, which would give rise to an anthelion and an anthelic pillar.
arc crosses the solar vertical above the sun when $\beta = 90^\circ$, at an azimuth of $180^\circ$ from the sun, or $0^\circ$ measured from the anthelion. The anthelic arc also crosses the solar vertical when $\beta = 90^\circ$, and since the altitude is the same for both arcs, they touch at this point.

On the same side of the celestial sphere as the sun the heliacal arc again crosses the solar vertical but at the sub-sun and not at the sun itself. It is, in fact, a sub-heliacal arc. Again Parry type crystals give rise to an arc arising from the images $S_1$ and $S_2$, which is the reflection of the $S_1S_2$ arc in the horizontal plane. Since the $S_1S_2$ arc passes through the sub-sun its reflection (the $S_1S_2$ arc) will pass through the sun itself. A similar argument to that given for the tangency of the heliacal and anthelic $S_1S_2$ arcs applies to the two $S_1S_2$ arcs, which are also tangential where they cross the solar vertical. For any given elevation of the sun below $30^\circ$ there will thus be an heliacal and a sub-heliacal arc together with an anthelic and a sub-anthelic arc. The two heliacal arcs are too large to be plotted on the same projection as that used for the anthelic arcs in Figs. 8 and 9. All four arcs, however, have been plotted on a stereographic projection in Fig. 10 for a solar elevation of $15^\circ$. The resemblance to Blake's diagram (Fig. 2) is striking, especially when it is remembered that the observed elevation of the sun was somewhat lower ($13^\circ$) and this would cause the outer sub-heliacal and sub-anthelic loops to contract and the inner heliacal and anthelic loops to expand.

Blake's observations on this halo are very numerous and detailed and in spite of being made from a mobile station, appear highly accurate according to the theory. No photographic record was taken of this unique display and so no further check of such differences as exist between theory and observation is now possible.

It is satisfactory to note that such differences are comparatively few. They occur near to the anthelion through which arc 3 was recorded as passing. Perhaps this could be ascribed to the presence of the parhelic circle, into which the arc could have appeared to merge. Otherwise the differences are remarkably small.

There remains to be discussed the presence of an anthelic pillar, which both Blake and Liljequist have recorded on other occasions as well, and we will conclude this article with some remarks about the formation of an anthelion and an anthelic pillar.

Attempts to explain the anthelion have usually been based upon the postulation of crystals sinking with the end face and one pair of side faces vertical. Reflection of a ray in two such vertical faces at right angles would reverse its direction in the horizontal plane. However, a crystal is only likely to sink in such an orientation if it is broader than it is deep, as a result of less development of the vertical side faces (Fig. 11) and this would seriously diminish the reflecting mechanism. However, three total reflections in the inclined side faces (or an odd number of three such reflections) together with one off the back face, would give rise to an anthelion, the ray emerging from the front face as before. The position of the image $S_\nu$ in question, and three possible reflections in the inclined faces, are indicated in Fig. 12, (where only construction lines, and not the paths of actual rays, have been drawn). Small variations in the alignment around the horizontal axis would cause $S_\nu$ to move over a small arc of a circle centred on $R$. For low solar elevations the predominant displacement, as $R$ moves over the horizontal plane, would be vertical, thus producing a vertical pillar through the anthelion. Such an anthelic pillar is plotted in Fig. 10, corresponding to a maximum departure from the preferred orientation of the vertical side faces, of $5^\circ$.

**References**


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