the maximum downward heat flux and its integral are very small, yet we find that about 25 per cent of the warming of the mixed layer is associated with entrainment of the upper fluid.

The apparent paradox is explained if we drop the assumption that \( \hat{\theta} \) is fully constant with height in the mixed layer (above the surface layer). Instead, let us define the average mixed-layer potential temperature by

\[
\hat{\theta}_m = \frac{1}{h} \int_0^h \theta dz
\]

(3)

without assuming anything about the vertical shape of \( \theta \). It follows from vertical integration of the simplified thermodynamic equation

\[
\frac{\partial \hat{\theta}}{\partial t} = -\frac{\partial}{\partial z} w' \theta'
\]

(4)

and application of Leibniz' rule, that

\[
\frac{\partial \hat{\theta}_m}{\partial t} = (w' \theta' - w' \theta'_n)h + \frac{1}{h} \frac{\partial h}{\partial t} (\theta_b - \hat{\theta}_m)
\]

(5)

where \( \hat{\theta}_b = \hat{\theta} \) at \( z = h \). In the experiment, the last term on the right of Eq. (5) accounted for a majority of the mixed-layer warming due to entrainment, because \( \theta_b \) was somewhat greater than \( \hat{\theta}_m \) as can be seen by the dashed profile in Fig. 1. (If desired, uncertainties in definition (3) associated with the warmer air very near the surface can be removed by performing the integration from a surface-layer height, \( z_s \), to \( z = h \), and dividing by \( (h - z_s) \) rather than by \( h \) in Eqs. (3) and (5).) As suggested by Fig. 1, \( \theta_b - \hat{\theta}_m \) was about the same size as the model jump, \( \Delta \theta \), and the last term in Eq. (5) was greater than \(-w' \theta'_n/h\). In the mixed-layer model, the last term in Eq. (5) does not appear, and the negative heat flux assumed at \( z = h \), \(-w' \theta'_n\), is spuriously large for compensation.

Our conclusion, as before, is that the actual negative heat-flux area is quite small in comparison with the positive area, but that \( k \), as determined from the experiments and defined in Eq. (2), is significant, and of order 0·25. We did not appreciate this latter fact at the time the paper was written. The implications are

(a) only a very small fraction of the kinetic energy generated by the heat flux in the lowest 85 per cent of the mixed layer is available to support entrainment;

(b) the warming of the mixed layer associated with this entrainment is nevertheless substantial and constitutes roughly 25 per cent of the warming induced directly by the surface heat flux; and

(c) representative measurements of negative heat flux at \( z = h \) can be expected to be several times smaller than the corresponding model value.

REFERENCES


* The National Center for Atmospheric Research is sponsored by the National Science Foundation.

THE EFFECT OF CHANGES IN SOLAR RADIATION ON CLIMATE

By H. B. Gordon and D. R. Davies

In a well-known paper Budyko (1969) has calculated changes in the mean ice position, averaged over land and sea, consequent upon given changes in the amount of total solar radiation absorbed in the atmosphere; the calculation is essentially based on a relatively simple radiation excess or
deficit consideration, changes in radiation absorption being assumed to be independent of latitude. He estimates that a 14 per cent reduction of absorbed solar radiation would lead to a movement of the ice edge from its present average position at 72° latitude to the Equator.

He uses an empirical relationship between the outgoing annual average atmospheric radiation, averaged over a given latitude circle, and the mean zonal surface temperature at that latitude. An empirical expression is also used for the annual average vertically integrated meridional flux divergence of heat. Extensions of this method of calculation by Budyko and Yasishohava (1971) and by Sellers (1969) also suggest a similar rate of dependence of mean ice edge movement on reduction in the amount of total solar radiation absorbed. In all these calculations, the computed temperature distribution, zonally and annually averaged, is very close to present day conditions. However, it is unlikely that these empirical functions will still form a good description of mean annual conditions, when there is an appreciable change in general circulation characteristics, which must take place when there is a significant shift of the mean ice edge position. The importance of this type of comparatively simple method of calculation clearly lies in making an estimate of the change of mean ice position for very small changes in radiation functions. Extrapolation of these results to global conditions which are very different from the present, is of course very speculative and can only be improved by the use of time dependent general circulation models. Existing general circulation modelling studies show that global scale flow characteristics are highly dependent on choice of heating function, and so it is an important exercise to test by Budyko’s method the dependence of calculated mean ice position on the heating function used in such studies. We have therefore employed numerical values corresponding to radiation functions used extensively in two-level general circulation studies (Smagorinsky 1963); long time period integrations based on these functions reproduce flow systems whose basic global scale properties are similar to those of the real atmosphere, and averages taken over long periods are also similar to available present day observed averages. Then, since these heating functions lead to a reasonably good simulation of the present time-dependent global scale flow characteristics, as well as the long term average, they are likely to remain physically valid for at least small changes of solar absorption, and hence to give a better based prediction of mean ice edge positional changes. We note also that the Budyko (1969) expression for outgoing radiation, \( I \), in langlies per day, is \( I = 3T_\alpha - 400 \), where \( T_\alpha \) is surface (zonal mean) temperature in K, whereas the equivalent values given by Smagorinsky (1963) can, after linearization, be expressed in the form \( I = 6.86 T_\alpha - 1540 \) at the Equator, with both coefficients increasing almost linearly with increasing latitude to the form \( I = 9.12 T_\alpha - 2020 \) at the pole.

Following Budyko, we consider a simple one variable spatial distribution problem, in which only the spatially averaged Northern Hemisphere latitudinal position of the ice edge limit is studied, and this is assumed to be an asymptotic equilibrium position, corresponding to a given fixed meridional distribution of incoming solar radiation. The asymptotic equilibrium ice edge positions are then calculated for a number of different spatially fixed heating distributions. If \( S_\alpha \) denotes the solar radiation arriving at the outer boundary of the atmosphere, \( A \) the total albedo, of earth and atmosphere, and \( I \) the net long wave outgoing radiation from the outer boundary of the atmosphere, Budyko equates the net gain or loss of radiation at a given latitude to an assumed linear temperature function \( \beta(T - T_\alpha) \) which is taken to represent the vertically integrated annual average meridional flux divergence of heat, the vertical integration being taken through the ocean and atmosphere; \( T_\alpha \) denotes the planetary mean of the surface temperature, \( T \) and \( \beta \) is a constant derived empirically from the present average temperature and meridional heat flux distributions. So we write, in appropriate units,

\[
S_\alpha(1 - A) - I = \beta(T - T_\alpha) . \tag{1}
\]

The outgoing radiation, \( I \), is specified in Budyko’s calculation in terms of a linear temperature function, with coefficients depending on mean annual cloudiness distribution. Numerical integration of Budyko’s expression gives a good simulation of the present surface temperature distribution, annually and zonally averaged, but it is not known whether this heating function would on substitution into a general circulation model lead to realistic time dependent global flow systems. In our calculation, we have used the form for \( I \) assembled for use in two-level general circulation models by J. Smagorinsky (1963) and expressed as

\[
I = (1 - \bar{E}) \sigma T_\alpha^4 + \nu_1 \sigma T_\beta^4 , \tag{2}
\]

where \( T_\alpha \) and \( T_\beta \) denote respectively the surface and the 500 mb zonally and annually averaged
temperatures, $\sigma$ is the Stephan-Boltzmann constant, $\Gamma$ the long wave absorptivity of the atmosphere (given numerically by Smagorinsky as a slowly varying function of latitude), and $v_1$ is an upward atmospheric transmissive parameter (also given as a slowly varying function of latitude). Since $\Gamma$ and $v_1$ change only slowly with mean atmospheric temperature, and as we are concerned with the response of $T$ to relatively small changes in $T$, it is reasonable in a first order approximation to assume that $\Gamma$ and $v_1$ are held fixed at a given latitude. This leads to

$$S_0(1 - A) - [(1 - \Gamma) \sigma T_\odot^4 + v_1 \sigma T_\odot^4] = \delta(T_\odot - T_{*p}),$$

where $T_{*p}$ is the planetary mean temperature at the surface, and to complete the formulation we use a surface heat balance equation in the form used by Smagorinsky (1963)

$$S_\odot + v_2 \sigma T_s^4 - \sigma T_\odot^4 = M + E_s + E_l,$$

where $v_2$ is a downward atmospheric transmissive parameter (held constant for a given latitude) and

$$S_\odot = (1 - A_\odot)(1 - A) S_0.$$

$A_\odot$ being the ground albedo, $x$ the opacity, $A_\odot$ the atmospheric albedo: the present numerical distribution of $A_\odot$, $x$, and $A_\odot$ are given by Smagorinsky (1963) and are strongly dependent on the mean ice edge position; $E_s$ and $E_l$ denote the fluxes of heat due to the eddy transport of sensible and latent heat from the earth's surface to the atmosphere, and $M$ is the flux divergence of heat from the atmosphere to the ocean. A suitable numerical value for $T_{*p}$ can be deduced from Eqs. (3) and (4).

As a check on our methods of computation we first carried out a preliminary calculation closely based on the Budyko scheme, as described in his paper (1969), using the empirical form he adopted for the outgoing radiation, but replacing his values for the spatial distribution of $S_0$ by those tabulated by Smagorinsky (1963). The $A$ distribution was made continuous in the region of the ice edge and replaces Budyko's discontinuous $A$ value of 0.32 on the Equator side of the ice and of 0.62 on the polar side; it also leads of course to a continuous variation of $T_\odot$ across the ice edge position. The calculated $T_\odot$ distribution corresponding to representations of the present distribution of radiation heating functions is shown in Fig. 1 as a function of latitude; curve A is based on the Smagorinsky outgoing radiation function and curve B is based on Budyko's (1969) outgoing radiation function.

The solar radiation $S_0$ was then reduced in successive steps of 0.1 per cent, the magnitude of the reduction being kept the same in all latitudes; we also assumed that $(M + E_s + E_l)$ is independent of small changes of solar radiation. The spatial distribution of $T_\odot$ was computed for a given reduction in $S_0$; these new $T_\odot$ values led to a new position of the ice edge (identified by a specific $T_\odot$ value) and the corresponding albedo, $A$, distribution was obtained by displacing the original albedo distribution curve to a lower latitude, so that the value of $A$ associated with the ice edge
is taken up at its new position. These albedo corrections led to a new distribution of the radiation functions, and hence to a new $T_e$ distribution, and a new ice edge position, etc. This iterative procedure was found to be rapidly convergent and was carried out by a routine computer procedure: it was used in all the calculations carried out requiring an adjustment of albedo parameterization associated with the various positions of the ice edge.

If $[\Delta S_o]_n$ denotes the change in planetary mean solar radiation, the ice edge position is calculated as a function of $[\Delta S_o]_n/[S_o]_n$, firstly using Budyko's expression for $I$ and is shown in Fig. 2, curve B, the results of Budyko's original calculation being shown in Fig. 2, curve A. We see that the slopes of the curves at the origin almost coincide and that, by simple extrapolation, a percentage reduction is $S_o$ of about $2\frac{1}{2}$ results in the ice edge approaching the Equator: this is reasonably close to Budyko's result of $1\frac{1}{2}$ per cent, as we have used slightly different basic data.

![Figure 2. Computed mean ice edge position with solar radiation changes: Curve A given by Budyko (1969); Curve B based on Budyko's I function and $S_o$ tabulated by Smagorinsky (1963); Curve C based on the I function tabulated by Smagorinsky.](image)

We then modified the outgoing radiation function, using the expression and numerical values given by Smagorinsky (1963), and repeated the calculation. The iterative procedure was again rapidly convergent; the ice edge equilibrium positions associated with successive percentage changes in $S_o$ are shown in Fig. 2, curve C. The slope of this curve at the origin is significantly different from that of curves A and B and, again by extrapolation, indicates that a 5 per cent reduction in solar radiation may be required before the mean ice edge position approaches the equator; this replaces Budyko's value of $1\frac{1}{2}$ per cent. These inferences from extrapolation are of course highly speculative.

**REFERENCES**


Department of Mathematics,
The University,
Exeter, Devon