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COMMENT ON THE PAPER BY A. K. BETTS 'NON-PRECIPITATING CUMULUS CONVECTION AND ITS PARAMETERIZATION'

By J. W. Deardorff, G. E. Willis and D. K. Lilly

The paper of Betts (1973) touches upon a misunderstanding concerning the negative heat flux to be expected in clear air at the top of a well-mixed layer heated from below. The laboratory measurements of Deardorff, Willis and Lilly (1969) disclosed that the area of negative heat flux near the top of the mixed layer, such as shown in Fig. 1 by the solid curve, is only two to three per cent of the area of the positive heat flux existing below. This result was interpreted by Betts to mean that the factor $k$ used in the model equation

$$\frac{\partial \theta_m}{\partial z} = (1 + k) \overline{w' \theta'/h}.$$  \hspace{1cm} (1)

was also very small in the experiments and therefore inapplicable to the atmosphere where $k$ is believed to be of order 0.25. In Eq. (1), $\theta_m$ is the potential temperature in the mixed layer, $\overline{w' \theta'}$ is the kinematic heat flux close to the surface, and $h$ is the depth of the mixed layer. The overbar refers to the average over a horizontal area which is much larger in scale than that of the turbulence. With this model equation, $\theta$ is treated as constant with height in the mixed layer but with a jump, $\Delta \theta$, at the top as shown in Fig. 1. In that case, the heat-flux profile must be linear up to $z = h$ as shown in the Figure. In both the model and the experiment, the actual height $h$ is conveniently defined as the level of maximum negative heat flux.

In Eq. (1) the parameter $k$ comes from the definition

$$k = -\overline{w' \theta'_m/w' \theta'},$$  \hspace{1cm} (2)

where $\overline{w' \theta'_m}$ is the model value of the kinematic heat flux at and just below $z = h$. From the dotted heat-flux profile of Fig. 1 it can be seen that $k \approx 0.23$; thus our experiments did not lead to a negligible value for $k$. The area of the small triangle of negative heat flux is, however, $(0.23)^2 \approx 0.05$ that of the larger positive triangle. Thus the ratio of potential energy production to kinetic energy dissipation is $k^2$, which may be negligibly small even though the contribution of the downward heat flux to the heat budget of the mixed layer cannot be neglected.

In our experimental results the situation is somewhat more complicated. It appears that both
the maximum downward heat flux and its integral are very small, yet we find that about 25 per cent of the warming of the mixed layer is associated with entrainment of the upper fluid.

The apparent paradox is explained if we drop the assumption that \( \dot{\theta} \) is fully constant with height in the mixed layer (above the surface layer). Instead, let us define the average mixed-layer potential temperature by

\[
\dot{\theta}_m = \frac{1}{h} \int_0^h \dot{\theta} dz
\]

(3)

without assuming anything about the vertical shape of \( \dot{\theta} \). It follows from vertical integration of the simplified thermodynamic equation

\[
\frac{\partial \dot{\theta}}{\partial t} = -\frac{\partial}{\partial z} w^' \dot{\theta}^'
\]

(4)

and application of Leibniz' rule, that

\[
\frac{\partial \dot{\theta}_m}{\partial t} = (w^' \dot{\theta}^' - \overline{w^' \dot{\theta}^' })h + \frac{1}{h} \frac{\partial h}{\partial t} (\dot{\theta}_s - \dot{\theta}_m)
\]

(5)

where \( \dot{\theta}_s \) is \( \dot{\theta} \) at \( z = h \). In the experiment, the last term on the right of Eq. (5) accounted for a majority of the mixed-layer warming due to entrainment, because \( \dot{\theta}_s \) was somewhat greater than \( \dot{\theta}_m \) as can be seen by the dashed profile in Fig. 1. (If desired, uncertainties in definition (3) associated with the warmer air very near the surface can be removed by performing the integration from a surface-layer height, \( z_s \), to \( z = h \), and dividing by \( (h - z_s) \) rather than by \( h \) in Eqs. (3) and (5).) As suggested by Fig. 1, \( \dot{\theta}_s - \dot{\theta}_m \) was about the same size as the model jump, \( \dot{\theta}_j \), and the last term in Eq. (5) was greater than \( -\overline{w^' \dot{\theta}^' }h \). In the mixed-layer model, this last term in Eq. (5) does not appear, and the negative heat flux assumed at \( z = h \), \( -\overline{w^' \dot{\theta}^' }h \), is spuriously large for compensation.

Our conclusion, as before, is that the actual negative heat-flux area is quite small in comparison with the positive area, but that \( k \), as determined from the experiments and defined in Eq. (2), is significant, and of order 0.25. We did not appreciate this latter fact at the time the paper was written. The implications are

(a) only a very small fraction of the kinetic energy generated by the heat flux in the lowest 85 per cent of the mixed layer is available to support entrainment;

(b) the warming of the mixed layer associated with this entrainment is nevertheless substantial and constitutes roughly 25 per cent of the warming induced directly by the surface heat flux; and

(c) representative measurements of negative heat flux at \( z = h \) can be expected to be several times smaller than the corresponding model value.

The Effect of Changes in Solar Radiation on Climate

By H. B. Gordon and D. R. Davies

In a well-known paper Budyko (1969) has calculated changes in the mean ice position, averaged over land and sea, consequent upon given changes in the amount of total solar radiation absorbed in the atmosphere; the calculation is essentially based on a relatively simple radiation excess or