A note on the spectra of wind velocity components in the surface layer

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SUMMARY

Results of our recent investigation of spectral measurements of wind velocity in the surface layer are analysed in light of the hypothesis proposed by Kaimal et al. (1972). It is confirmed that in the inertial subrange the spectra for all the three components and varying stability converge to a $-2/3$ line at the high frequency end. At the low frequency end the spectra cluster together in random fashion in a certain bandwidth and do not show any ordered arrangement with $z/L$.

1. LIST OF SYMBOLS

- $f$: a dimensionless frequency $nz/U$
- $g$: acceleration due to gravity
- $k$: von Kármán constant, 0.4
- $L$: Monin-Obukhov length
- $n$: frequency in Hz
- $S_x(n)$: longitudinal power spectrum
- $S_y(n)$: lateral power spectrum
- $S_z(n)$: vertical power spectrum
- $T_s$: $-w\bar{u}_*$
- $U$: flow velocity
- $u, v, w$: fluctuating velocity components in the $x, y, z$ directions
- $u_*$: friction velocity
- $x$: $x$-axis lies in the along-mean-wind direction
- $y$: $y$-axis lies across the mean-wind direction
- $z$: $z$-axis lies in the vertical direction
- $z_0$: roughness length
- $z/L$: $kz(g/\bar{\theta})(T_s/u_*)^2$, a dimensionless height
- $\alpha_1, \alpha_2, \alpha_3$: constants appearing in Eqs. for $u$, $v$- and $w$-spectra
- $\varepsilon$: dissipation rate of turbulent energy
- $\phi_*^{(e)}$: $kz\varepsilon/u_*^2$, a dimensionless dissipation rate for turbulent energy

2. INTRODUCTION

Since the early 1950s, when modern recording and computing techniques became available, considerable effort has gone into the study of atmospheric turbulence and its spectral characteristics. The large amount of data obtained by various investigators show clear indication that spectra of wind velocity and temperature obey similarity theory in the high frequency range in the surface layer. Using this similarity theory a number of attempts have been made to bring together velocity spectra from many sites and heights and thermal stabilities (e.g. Lumley and Panofsky 1964; Berman 1965; Busch and Panofsky 1968) and to define their general behaviour in terms of similarity parameters. A number of spectral
forms have been suggested, all of which provide a reasonably good fit under near-neutral conditions. Recently Kaimal et al. (1972) have suggested a somewhat different approach based on the Kansas 1968 AFCRL research project. The purpose of this note is to apply their hypothesis to a new series of measurements and test the validity of their hypothesis as applicable to a somewhat different terrain.

3. Description of terrain

The field site (Grindsjön) is situated 30 km south of Stockholm. The terrain is rather flat with short grass in all the directions except west to north, in which direction is situated a small hill 5 to 10 m high above the mean level (the hill is at a distance of 50 m to the nearest mast supporting the sensors). On the hill are situated two small houses, some trees and bushes. The hill influenced the wind very significantly; this effect of hill on wind flow is shown in Fig. 1, in terms of roughness length $z_0$ ($z_0$ was calculated from the well-known logarithmic wind profile formula). When the wind blew from west to north, the roughness length peaked up (see Fig. 1).

![Figure 1. Roughness length for varying wind direction in Grindsjön.](image)

4. Instrumentation and data acquisition

Spectral measurements were made with a sonic anemometer of type PAT 311, constructed by Kajjo Denki Co., Tokyo, Japan. The temperature fluctuations were measured by the same instrument during this set of measurements. The value of $u_*$ and $L$ were derived from eddy flux and heat flux which both were measured by the sonic anemometer. Details of the instrumentation are described by Mitsuta (1966). The inherent errors of sonic anemometers and the effects of other errors such as bad alignment, path displacement and line averaging are discussed by Kaimal, Wyngaard and Haugen (1968). The sonic anemometer was supported on a mast 6 m high.
The data were filtered, sampled and digitized and partially evaluated on line on a mini computer, model 704 Raytheon. The entire data were stored on magnetic tape for subsequent analysis on IBM 360/75 computer. The measurements used in this analysis also include winds blowing from north-west.

The data were sampled for 15 minutes at a sampling frequency of 10 Hz and filtered with a double stage Butterworth low pass filter (cut-off frequency 5 Hz). The spectral density was calculated using Fast Fourier Transform using a standard library program.

5. NON-DIMENSIONALIZATION OF SPECTRA OF VELOCITY COMPONENTS

Recently Kaimal et al. (1972) have suggested a new approach for non-dimensionalizing the spectra and co-spectra of velocity and temperature. They first collapsed all spectra into universal curves in the inertial subrange and then observed the spectral behaviour at lower frequencies as a function of z/L. The argument runs somewhat as follows.

In the inertial subrange, according to Kolmogorov's law and using Taylor's hypothesis, the non-dimensionalized logarithmic u-spectrum may be written as,

\[
\frac{nS_n(n)}{u^2} = \frac{\alpha_1}{(2\pi k)^{2/3}} \left( \frac{z}{u^2} \right)^{2/3} \left( \frac{nz}{u} \right)^{-2/3}
\]  

(1)

For definition of symbols see List of Symbols. Eq. (1) may also be written as

\[
\frac{nS_n(n)}{u^2} = \frac{\alpha_1}{(2\pi k)^{2/3}} \phi_e^{2/3} f^{-2/3}
\]  

(2)

At f = 4 and taking k = 0.35 and \( \alpha_1 = 0.5 \) (according to Kaimal et al. 1972) we may write the above equation as

\[
\left[ \frac{nS_n(n)}{u^2} \right]_{f=4} = 0.12 \phi_e^{2/3}
\]  

(3)

Similar formulae can be written for the v- and w-spectra. However, it should be noted that the v- and w-spectra in the inertial subrange are higher by a factor of 4/3, as predicted by isotropy.

Kaimal et al. (1972) obtained the following formula for \( \phi_e \) from the Kansas data:

\[
\phi_e^{2/3} = \begin{cases} 
1 + 0.5|z/L|^{2/3} & -2 \leq z/L \leq 0 \\
1 + 2.5|z/L|^{3/5} & 0 \leq z/L \leq +2
\end{cases}
\]  

(4)

Table 1 shows the predicted values of \( \phi_e \) (obtained from Eq. (4)) and the measured values of \( \phi_e \) (obtained from the measured data using Eq. (3)).

<table>
<thead>
<tr>
<th>S. No.</th>
<th>( z/L )</th>
<th>Predicted from Eq. (4)</th>
<th>Measured values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.112</td>
<td>1.1162</td>
<td>1.4277</td>
</tr>
<tr>
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<td>1.1155</td>
<td>1.3503</td>
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<td>3</td>
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<td>0.9030</td>
</tr>
<tr>
<td>5</td>
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<td>1.2644</td>
</tr>
<tr>
<td>6</td>
<td>+0.006</td>
<td>1.0192</td>
<td>1.4552</td>
</tr>
<tr>
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<td>1.2391</td>
<td>1.7195</td>
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<tr>
<td>9</td>
<td>+0.030</td>
<td>1.3050</td>
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</tr>
</tbody>
</table>
Figure 2. Generalized $\nu$-spectrum for $z/L$ values ranging from $-0.112$ to $0.030$. The solid curve is the spectral curve for the neutral limit of the stable spectra and the hatched area constitutes envelopes of the unstable spectra of Kaimal et al. (1972).

Figure 3. Generalized $\nu$-spectrum for $z/L$ values ranging from $-0.112$ to $0.030$. The solid curve is the spectral curve for the neutral limit of the stable spectra and the hatched area constitutes envelopes of the unstable spectra of Kaimal et al. (1972).
In Eq. (3) if we now include \( \phi^{2/3} \) in the normalization of \( u-, v- \) and \( w- \) spectra we remove the \( z/L \) dependence in their respective equations (see Kaimal et al. 1972). This brings all the spectra, regardless of \( z/L \), into coincidence in the inertial subrange.

Writing the \( u- \) spectrum in this form we get, from Eq. (2),

\[
\frac{nS_u(n)}{u_*^2 \phi^{2/3}_*} = 0.30f^{-2/3}.
\]  

(5)

The plots of logarithmic \( u-, v- \) and \( w- \) spectra normalized in this manner are shown in Figs. 2 to 4. Also shown in the figures are the envelopes of the unstable spectra (\( z/L = -2.0 \) to \( <0 \)) and the neutral limit of the stable spectra (\( z/L > 0 \)) of Kaimal et al. (1972). Our spectra converge to a \(-2/3\) line at the high frequency end, but at lower frequencies we need to study the spectra in more detail.

The range of \( z/L \) for our measurements is from \(-0.112\) to \(0.030\). This means that all our measurements were made either in conditions of almost neutral stability or slight instability.

Kaimal et al. (1972) observed a systematic progression with \( z/L \) on the stable side, which broke down as \( z/L \) changed sign and became negative. The unstable spectra were confined to a hatched area (see Figs. 2 to 4) with no particular regard to \( z/L \). This last feature is confirmed by our results. However, this frequency range would be influenced by terrain roughness and it is most likely that the systematic ordering of the stable spectra according to \( z/L \) would be apparent only over a uniform site with a long fetch. Our terrain was neither uniform nor had a long fetch.

In the inertial subrange the level of our \( u-, v- \) and \( w- \) spectra is in agreement with that of Kaimal et al. (1972). However, at lower frequencies our spectra show higher energy than that of Kaimal for \( z/L > 0 \). It should be noted that in the unstable range the level

![Figure 4. Generalized w-spectrum for z/L values ranging from -0.112 to 0.030. The solid curve is the spectral curve for the neutral limit of the stable spectra and the hatched area constitutes envelopes of the unstable spectra of Kaimal et al. (1972).](image)
of spectra of Kaimal for u- and v-components was higher by a decade at lower frequencies which agrees with our results.

Another interesting feature in the results of Kaimal was the separation between the areas occupied by the stable and the unstable spectra (true only for u- and v-components). This indicates a sudden shift in the predominant scales of motion as \( z/L \) changes sign. This last feature is not apparent in our results, but this is perhaps due to the limited \( z/L \) range of our investigation and the inhomogenetiy of the terrain at our site.

As we go from \( u- \) to \( w- \)spectra, the maxima in the curves shift to higher frequencies and the data are clustered in a random fashion within a limited bandwidth without any regard to \( z/L \).

6. CONCLUSIONS

Our results confirm the following two features observed by Kaimal et al. (1972).

(i) In the inertial subrange the spectra for all the three components and varying stability converge to a \(-2/3\) line at the high frequency end and are relatively insensitive to site characteristics.

(ii) At the low frequency end the spectra cluster together in a certain bandwidth and do not show any ordered arrangement with \( z/L \). Further, the separation observed by Kaimal between the areas occupied by the stable and the unstable spectra for the \( u- \) and \( v- \)components is not apparent from our results perhaps due to our limited bandwidth (\( f > 0.02 \) Hz) and inhomogeneous terrain.

REFERENCES


