Multi-level measurements of turbulence over the sea during the passage of a frontal zone

By G. W. BRYANT and K. A. BROWNING
Meteorological Office Research Unit, Royal Radar Establishment, Malvern

(Received 17 May 1974)

SUMMARY

A Doppler radar technique originally proposed by Lhermitte (1968) has been used to measure the horizontal components of turbulence simultaneously at closely spaced height intervals from an altitude of 80m to 2500m. The observable range of turbulence scales was between about 200m and 5000m. Observations were made over the sea, with the radar situated on the Isles of Scilly, to minimize the effects of inhomogeneities in terrain.

The measurements reported in this paper were made during the passage of a warm frontal zone with strong surface winds; they show the change with height in the structure of turbulence simultaneously in both the planetary boundary layer and a free shear zone situated some way above it. Below 600m, in the planetary boundary layer, the two horizontal components of turbulence were rather similar in intensity, the total variance of horizontal velocity in the observed spectral range decreasing from 1.7m$s^{-2}$ at 80m to less than 0.1m$s^{-2}$ at 700m. For much of the observed spectral range the turbulence spectra were of the form $kS(k) \propto k^{-2}$, with $n$ decreasing from 1.2 below 300m to 0.7 at 700m. The lapse rate was slightly less than dry adiabatic in the planetary boundary layer with broken cloud above 300m, consistent with slight stability below 300m and some instability above. The decrease in the value of $n$ is thought to have been due to the decrease of stability with height. The spectral scale $k_n$ increased with height reaching a value of 1.6km at 900m. Above the boundary layer for most of the time there was a placid layer several hundred metres deep and this was surmounted by a free shear layer in which the variance of velocity increased again to typically 0.5m$s^{-2}$.

In the free shear layer the horizontal components of turbulence resolved parallel and perpendicular to the local shear vector were rather similar in intensity at scales less than 500m but were markedly unequal at scales between 500m and 2km. This is consistent with the occurrence of Kelvin-Helmholtz billows with a wavelength of 1 to 2km.

The Doppler radar measurements also enabled the vertical flux of horizontal momentum to be calculated. The quantity $\overline{u'w'}$ was found to decrease from about 0.2m$s^{-2}$ in the planetary boundary layer to about 0.1m$s^{-2}$ in the placid layer, before increasing again to about 0.1m$s^{-2}$ in the free shear layer. The energy dissipation rate $\varepsilon$ was estimated from the turbulence spectra in those cases where the spectral slope $n$ was not greatly different from 2/3. Values of $\varepsilon$ decreased from 10cm$s^{-3}$ at 400m to 2cm$s^{-3}$ in the placid layer, before increasing to a second maximum of about 20cm$s^{-3}$ in the free shear zone. The rate of mechanical production of turbulence was calculated from the Doppler radar measurements of momentum flux and shear and it was found to vary with height in a rather similar manner to $\varepsilon$.

1. INTRODUCTION

A proper understanding of the role of turbulence in vertical transfer processes requires a knowledge of the variation of the turbulence structure in the vertical. Unfortunately, very few multi-level measurements of turbulence have been made at heights appreciably above the surface layer, especially over the sea. Turbulence measurements aloft have been made from balloons (e.g. Readings and Rayment 1969 over land; and Thompson 1972 over the sea) and from aircraft (e.g. Warner 1972), but until recently these were usually confined to a single level at any given time; to investigate the variation of turbulence with height usually it has been necessary to resort to the unsatisfactory procedure of combining measurements made at different levels at different times. However, a technique using ground-based pulsed Doppler radar permits turbulence to be measured virtually simultaneously over a considerable depth of the atmosphere. Although less accurate than many in-situ measurement techniques, the conical scanning (or VAD) radar technique is capable of giving detailed information on both the wind field and turbulence from near the surface up to a
height of several kilometres. This paper describes the technique briefly and presents results obtained on the Isles of Scilly. The results reveal the detailed vertical distribution of winds and turbulence simultaneously in the planetary boundary layer (PBL) and through an overlying free shear layer (FSL) during the passage of a frontal system.

2. The conical scanning technique

The conical scanning technique, first devised by Lhermitte and Atlas (1961), permits wind fields to be measured in the presence of widespread precipitation using the precipitation particles as tracers of the air motion. The basis of this technique is for the radar beam to be rotated in azimuth at a constant low elevation (α) so that the line-of-sight velocity \( v_r \) of precipitation particles can be measured at one or more ranges while the beam describes a conical scan (Fig. 1). As the beam is rotated, the radial Doppler velocity, \( v_r \), of the returned signal follows an approximate sine wave with a maximum in the upwind direction and a

![Diagram illustrating the conical scan, showing the horizontally oriented circle described by a single range gate while the beam of the Doppler radar is rotated at a constant elevation angle.](image)

minimum in the downwind direction. It has been shown by Browning and Wexler (1968) that the velocity–azimuth-record obtained at a given range during a single conical scan can be Fourier analysed to provide a number of kinematic properties of the wind field. The amplitude and phase of the fundamental give the wind speed and direction, respectively, these measurements being averages over the perimeter of the scanned circle. Horizontal deformation of the wind field appears as the second harmonic, and horizontal divergence and precipitation fall speed* together produce the zeroth harmonic. By making measurements simultaneously at a number of ranges (altitudes), it is possible to obtain virtually instantaneous vertical profiles of these properties and hence to evaluate profiles of vertical velocity and vertical wind shear.

Lhermitte (1968) and Wilson (1970) have shown that the conical scanning technique also provides quantitative turbulence data. If turbulence is present on scales larger than the

* The precipitation fall speed as measured is the sum of the mean reflectivity-weighted terminal fall speed \( \bar{w} \) and the vertical air velocity \( w \) both averaged over the appropriate radar pulse volume.
beam width (typically 100–300m) but much smaller than the scanned circle, it will be evident as small scale fluctuations in the velocity–azimuth records. After the lower harmonics have been removed, the remaining velocity variance (mean square deviation from the mean) gives a measure of the turbulent energy within the appropriate spectral interval. The effect of fluctuation in the vertical velocity of precipitation targets due to inhomogeneities in their fallspeed is reduced by scanning at low elevation angles. Since the radar measures line-of-sight velocities it is effectively the horizontal components of turbulence that are measured by this technique. Both Lhermitte and Wilson made measurements in snow; Lhermitte used an elevation $\alpha = 10^\circ$ while Wilson was able to use $\alpha$ as high as $20^\circ$. The technique can also be used in rain, but it is then necessary to restrict $\alpha$ to lower values to minimize the effect of the greater variability in the terminal fallspeed of the rain drops. Although Stackpole (1961) has shown that rain drops are poor tracers of very small-scale turbulent motions, they are nevertheless good sensors of the mean wind within the pulse volumes sampled by most radars. The total intensity of turbulence on scales smaller than the radar pulse volume can be estimated using snow as the tracer from the instantaneous variance of velocities within each pulse volume after allowing for wind shear and other effects (Atlas 1964); in the present paper, however, we restrict our attention to turbulence on scales larger than the pulse volume and we make measurements in both snow and rain.

The high frequency fluctuations in the velocity–azimuth record provide a considerable amount of information. Lhermitte (1968), in a study of turbulence in the lowest 2km, measured the Doppler velocity at $3^\circ$ azimuth intervals for a sector $30^\circ$ each side of the wind direction in both the upwind and downwind directions. After carrying out a Fourier transform to obtain velocity variance per unit wavenumber $S(k)$ as a function of wavenumber $k$, he was able to plot spectra which, on a log-log scale, had a slope of $-5/3$ in good agreement with that expected from similarity theory. In fact, an average of the upwind and downwind spectra followed the relationship:

$$S(k) = Ce^3 k^{-5/3}$$

where $e$ is the energy dissipation rate and $C$ is the Kolmogorov constant. Lhermitte also showed that the difference in the Doppler velocity variances measured in the upwind and downwind directions is a measure of the component of the vertical flux of horizontal momentum resolved along the wind direction. To understand this result, it is helpful to refer again to Fig. 1. It is evident from the diagram that the horizontal component of velocity $u$ defined with respect to any axis AB acts in the same sense as the vertical velocity $(w + w_f)$ when resolved along the axis of the radar beam at A but in the opposite sense to $(w + w_f)$ when viewed at B. Thus one can see that any positive correlation between small-scale fluctuations in $u$ and $(w + w_f)$ will enhance the variance of Doppler velocity in the vicinity of A (compared with the variance due to fluctuations in horizontal velocity alone) whereas it will decrease the variance of Doppler velocity in the vicinity of B. In general, fluctuations in the terminal fallspeed of the precipitation $w_f$ will tend to be uncorrelated with the wind and henceforth we shall assume that, provided one averages over a sufficiently large amount of data, the only significant correlation will be that between $u$ and $w$, which is of course representative of the vertical momentum flux.

Wilson (1970) extended Lhermitte’s analysis and proposed an improved computational procedure for extracting turbulence intensity and momentum flux using data from the entire velocity-azimuth record. He divided the circle scanned at a given range into quadrants I to IV as shown in Fig. 1. Assuming horizontally homogeneous turbulence, he showed that the difference between the sum of the variances of Doppler velocity in sectors I and IV ($\sigma_1^2 + \sigma_4^2$), and that in sectors II and III ($\sigma_2^2 + \sigma_3^2$) is a measure of the correlation between $u$ and $w$; viz:
\[ \text{cov}(uw) = \frac{1}{4\sin 2\alpha} [(\sigma_1^2 + \sigma_2^2) - (\sigma_3^2 + \sigma_3^2)] \]  

(2)

Similarly \[ \text{cov}(vw) = \frac{1}{4\sin 2\alpha} [(\sigma_1^2 + \sigma_2^2) - (\sigma_3^2 + \sigma_3^2)] \]  

(3)

After the low harmonics have been removed from the velocity-azimuth record, \( u, v \) and \( w \) may be replaced by the turbulence fluctuations \( u', v' \) and \( w' \) so that \( \text{cov}(uw) = u'w' \) and \( \text{cov}(vw) = v'w' \). Thus the data available from the conical scan permit the derivation of the momentum fluxes \( u'w' \) and \( v'w' \). From these values we can derive the rate of mechanical production of turbulence

\[ \left( \frac{dE}{dt} \right)_m = -\frac{u'w'}{\partial z} \frac{\partial \bar{u}}{\partial z} - \frac{v'w'}{\partial z} \frac{\partial \bar{v}}{\partial z} \]  

(4)

This expression is independent of the direction chosen to define \( u \) and \( v \). Shear terms \( \frac{\partial \bar{u}}{\partial z} \) and \( \frac{\partial \bar{v}}{\partial z} \) are measured by the Doppler radar in the same conical scans as the momentum flux.

The total variance \( \sigma^2 \), measured around the circle is, of course, equal to the average of the variances in the four sectors. Wilson showed that this, in turn, was related to the variances of \( u, v \) and \( w + w_f(\sigma_3^2) \). When the variances are computed by summation we have

\[ \sigma^2 = \frac{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_3^2)/4}{(\sigma_1^2 + \sigma_2^2 + 2\sigma_3^2 \tan^2 \alpha)(\cos^2 \alpha)/2} \]  

(5)

The term involving \( \sigma_3^2 \) is very much reduced by the use of a low elevation angle (\( \alpha \)). In the present study, the highest angle used was 12° (\( \tan^2 \alpha = 0.045 \)) in snow and 7° (\( \tan^2 \alpha = 0.015 \)) in rain. Fallspeed was measured throughout the period by pointing the aerial vertically at regular intervals and it was estimated that fallspeed variance in the range of wavelengths applicable to this study did not exceed 0.25 m s\(^{-2}\), even in rain and it was usually very much less. Thus this term cannot be expected to contribute significantly to the results. The term used in this study as a measure of the horizontal component of turbulent energy \( (\sigma^2/\cos^2 \alpha) \) is therefore equal to an average of the variance of the \( u \) and \( v \) components.

The optimum elevation for the measurement of momentum fluxes, using Lhermitte and Wilson's method, is 45° so that the need to use lower elevation angles for reliable turbulence measurements reduces the accuracy of these results. Also the values of momentum flux obtained by this method are contaminated by inhomogeneities in the horizontal distribution of turbulence intensity. A patch of locally intensified turbulence, for example, will increase or decrease the measured flux, depending on the sector in which it appears. If the patch in question is moving through the area being scanned, its effect can be removed by averaging the results from several consecutive conical scans. On the other hand, if the inhomogeneity in the distribution of turbulence is due to some effect of topography, the manner in which it would affect the results would not change much from scan to scan. Therefore, to provide meaningful estimates of momentum flux, these measurements should be carried out over flat land or the sea.

3. NATURE OF THE DATA

This study forms part of a larger project, aspects of which are described by Browning, Hardman, Harrold and Pardoe (1973) who discuss the precipitation patterns and associated mesoscale windfield as derived from radar, radiosondes and dropsondes. The principal
observations to be reported here were made on 18 January 1971 on the Isles of Scilly using a mobile Doppler radar. The radar has a pencil beam $2^\circ$ wide to the half power points, a peak transmitted power of 10kW and a pulse length of 1μs (Bahns and Whyman 1966). The Isles of Scilly are 40km west of the mainland of SW England and have a maximum height of only 50m. The four largest islands, with a combined area of about 15km$^2$, lay beneath the volume scanned by the radar (Fig. 2).

![Figure 2](image.png)

*Figure 2. Sketch of the Isles of Scilly showing the location of the Doppler radar and the circles which it scanned at radii between 5 and 11km. The line 200–020$^\circ$ is along the surface wind direction.*

The beam of the Doppler radar was rotated at 1rev/min with the elevation angle changing at the end of each revolution through a 14-minute cycle. The returned signal was gated into ten range gates each 150m long, spaced at intervals of about 1-2km, and was recorded on an FM tape recorder. The 14-min scanning cycle was a multi-purpose routine and the principal data used in this study were obtained at elevation 2°, 5°, 7° and 12°, which gave optimum data for turbulence measurements. Data at higher elevations were used as a check on the accuracy of the computed values of momentum flux but they were intended primarily for other purposes. Seven range gates were used for each scan at the four elevation angles to cover the height intervals 100-450m, 400-1000m, 900-1500m and 1500-2500m, respectively. The scans at the lowest two elevations (2° and 5°) were mainly in rain; scans at 7° and 12° were in snow. The outer gate at each elevation angle coincided approximately in height with the inner gate of the next elevation. The range of radii of the scanned circles varied from 5 to 11km (Fig. 2) and this is, of course, considerably less than the range of radii that would have resulted from a conical scan repeated at a single elevation angle. Subsequently, some of the data at an elevation of 2° were found to
give unsatisfactory results owing to ground echoes and so the 5° measurements in the close range gates were used to supplement the low altitude measurements.

The signal on each track of the tape recording was played through a mean frequency tracking device and the resulting mean Doppler velocity was recorded as a function of azimuth on an open-scale strip chart. An example of such a record is shown in Fig. 3. The velocity was extracted from the printed records at azimuth intervals of 0.5° and the 720 values from each scan were recorded on punched tape. This azimuth resolution, which was higher by a factor of four than that provided by the radar, was used to reduce reader error and to obtain a more accurate representation of the record. The results were then subjected to computer analysis. The first three (0 to 2) harmonics were subtracted to remove the effects of mean precipitation fallspeed and mesoscale divergence, deformation and wind speed. The filtered values were next processed by a Fast Fourier Transform (FFT) routine so that a velocity variance spectrum could be plotted for every range gate in each scan. The spectra were partly smoothed by averaging over one to five successive harmonics, the number increasing with wavenumber. Subsequently, successive spectra at each altitude were combined during periods in which the turbulence structure was not varying greatly in time so as to produce time-averaged spectra with more degrees of freedom, and hence a greater reliability for each spectral point.

![Figure 3](image)

Figure 3. Example of the record of line-of-sight Doppler velocity \( v_d \) against azimuth, showing turbulent fluctuation in velocity superimposed on the essentially sinusoidal variations due to the uniform wind. This trace was obtained at 1015 GMT at an elevation angle of 5.0° and slant range 6.5km corresponding to a height of 400m. Velocity is positive (toward the radar) between 120° and 280° and negative between 290° and 100°: reliable data were not obtained elsewhere for values of \( v_d \) near zero.

The 720 filtered velocity values from each scan were also divided into four quadrants, as shown in Fig. 1, using the shear direction as the main axis (AB). Throughout the frontal zone the shear direction was fairly constant (at 310°) despite the rapidly changing wind direction; through most of the PBL the direction of the wind shear coincided with that of the wind, at about 200°. The 180 values in each quadrant were then filtered to remove wavelengths longer than 2.5km, and the covariances and total variance were computed to permit calculation of turbulent energy and momentum flux. The upper limit of wavelength was used to reduce the effect of the variation of the radius of the scanned circle with height and to remove a small effect due to the topography of the Isles of Scilly.

It will be seen from Fig. 2 that a line drawn along the 200° surface wind direction divides the scanned circles into two 180° sectors, one of which (NW) includes almost all the Isles of Scilly while the other (SE) is almost entirely over the open sea. Thus topographical effects of the Isles of Scilly can be avoided altogether by using only the data from the SE sector. However, as is shown later from a comparison of results for the two sectors (Fig. 8), these effects are in fact confined to wavelengths longer than 2.5km.

Finally the degree of anisotropy in the horizontal components of the turbulence in the PBL and the FSL can be investigated by comparing the component of turbulence
resolved along the appropriate shear direction with that at right angles to it. This was done by first taking two 64° sectors from each scanned circle centred in the upshear and downshear directions. The data were processed by FFT and the spectral coefficients for the two sectors were combined and then averaged for all scans obtained at a given altitude. This was repeated using two 64° sectors centred on a line orthogonal to the shear vector. Two sets of spectra were thus produced for each altitude and compared in order to isolate any dependence of turbulence intensity on shear direction.

4. RESULTS

(a) Wind field and temperature structure

The measurements described here were made during a two-hour period with mainly moderate frontal rain at the surface. Figs. 4(a) and (b) depict two tephigrams derived from radiosonde ascents made at Camborne, 80km downwind of the Isles of Scilly. The times indicated for these tephigrams are the times when the corresponding parts of the frontal system passed over the Isles of Scilly (GMT is used throughout the paper). The tephigrams show that an intense warm frontal zone was passing through the area. At the beginning of the period, there was a dry layer at the base of the warm front (Fig. 4(b)); however, the moderate rain that began reaching the surface at 0850 brought this layer near to saturation shortly after.

![Figure 4](image)

**Figure 4.** Tephigrams derived from radiosonde ascents at Camborne. The times shown are those at which corresponding parts of the frontal system were over the nearby Isles of Scilly.

Hodographs showing the wind profiles below 3km, as derived from the Doppler radar on the Isles of Scilly, are shown in Fig. 5. A distinct change in shear direction from 200 to 310° can be seen at about 1km. Below 1km, the shear vector is roughly parallel to the surface geostrophic wind; above 1km, the shear direction is related to the orientation of the warm frontal zone, which according to Fig. 4(a), extended from about 1 to 3km at 1025.

Detailed time-height information on wind speed and direction, provided by the Doppler radar, are plotted in Figs. 6(a) and 6(b), respectively. Although some data were available up to several kilometres, the results in this paper are restricted to the lowest 3km where the radar return from the precipitation had a high signal-to-noise ratio. The magnitude of the vector wind shear, computed from the results in Figs. 6(a) and (b), is shown in Fig. 6(c). This shows a band of increased shear at about 2km, corresponding to a free shear layer (FSL) in the middle portion of the warm frontal zone. The FSL gradually lowered as the warm
front advanced overhead. Accurate Richardson numbers could not be evaluated for the FSL because the temperature soundings were not obtained in the same location as the detailed wind measurements; however, it is thought unlikely that $Ri$ over 200m would have been lower than 0.5 except in the very shallow layer in Fig. 6(c) where the shear exceeds 4m s$^{-1}$ over 200m. At lower levels there was a region of weak shear down to about 600m below which the shear increases steadily toward the surface in the planetary boundary layer (PBL)*; the shallow layer of very strong shear in the lowest 100m is not shown in Fig. 6(c).

(b) Vertical distribution of turbulent energy

Fig. 7 shows a time-height cross-section of the variance due to turbulence with wavelengths in the range 200m to 2.5km, derived using Eq. (4). There is an obvious relationship between the distribution of turbulence in Fig. 7 and the wind shear in Fig. 6(c). The relationship is clearest after 0930 (period II) but tends to be obscured before then (period I). Before examining the distribution of turbulence in detail, it is helpful to consider the essential differences in the thermodynamic structure during these two periods. Fig. 4(b) shows that the wet-bulb potential temperature ($\theta_w$) decreased with height up to 1-6km. At the time corresponding to Fig. 4(b), the air above 1km was dry and boundary layer convection is unlikely to have extended significantly into the dry region. Shortly after this time, however, precipitation generated above the warm frontal zone extended downward

* We have defined the PBL as the region throughout which turbulent energy decreased upwards monotonically from its value at the surface. In the present study, this placed the top of the PBL at about 600m.
Figure 6. (a) Time–height section of wind speed with isotachs at intervals of 1 m s⁻¹, as derived from Doppler radar data. The line of heavy dots shows the axis of maximum shear associated with the warm frontal zone. The broken double line shows the approximate extent of descending dry air. At the top of the diagram, T₁ and T₂ indicate the times of the radiosonde ascents in Fig. 4 and H₁, H₂ and H₃ indicate the times of the hodographs in Fig. 5. (b) Time–height section of wind direction with isogons at 5 degree intervals, as derived from Doppler radar data. The broken double line shows the approximate extent of dry air as in Fig. 6(a). (c) Time–height section of wind shear over 200 m intervals, derived from Figs. 6(a) and (b). The vertical line at T₂ represents the depth of the warm frontal zone (cf Fig. 4(a)).
to the surface bringing the layer of dry air just below the warm frontal zone close to saturation in the manner depicted schematically by the double broken contour in Figs. 6(a) and (b). This probably resulted in the convective instability being realised over a deeper layer so that for a while the boundary layer turbulence reached up to the layer of turbulence associated with the FSL (period I in Fig. 7). Subsequently the layer with \( \theta_v \) increasing sharply upwards, associated with the warm frontal zone, lowered toward the surface (Fig. 4(a)) and the rapid decrease in the turbulent intensity with height during period II (Fig. 7) suggests that any boundary layer convection was confined below 600m. According to Browning et al. (1973, Fig. 7(c)) the lowest 1km was characterized by weak subsidence until 1100; typically it was in the range 0 to 10cm s\(^{-1}\) between 0930 and 1100. This would have helped to suppress boundary layer convection in period II.

The layer of relatively strong turbulence corresponding to the region of maximum shear within the warm frontal zone was centred near 2km. Maximum variance observed within the FSL was 0.65m\(^2\) s\(^{-2}\). Within the PBL turbulence increased downward until, at the lowest levels studied (80m), a variance of 1.7m\(^2\) s\(^{-2}\) was recorded. Turbulence intensity, defined as equal to \( \sigma_u/\bar{u} \), was therefore equal to about 0.1 at 80m. During period II there was a placid layer between the FSL and the PBL in which the level of turbulent energy fell almost to the noise level (estimated at 0.07m\(^2\) s\(^{-2}\)). Similar values close to the noise level were found above 2.4km, at levels above the FSL.

(c). *Time averaged turbulence spectra*

Although individual conical scans are capable of providing reasonable estimates of total variance, in order to derive meaningful turbulence spectra it was necessary to reduce the effect of inhomogeneities and to increase the number of degrees of freedom by averaging
over several scans at each altitude. This was done for all scans in period II, during which the turbulence structure was not varying greatly in time, and graphs were plotted for each altitude of $kS(k)$ against $k$ on a log-log scale where $k$ is the wavenumber and $S(k)$ is the spectral density function. Typical spectra are shown in Fig. 9(a) for four levels in and just above the PBL and in Fig. 9(b) for four levels in and just below the FSL. Before examining these spectra in detail, it is necessary to digress briefly to consider the influence of the Isles of Scilly.

![Graph showing $kS(k)$ against $k$ on a log-log scale.](image)

Figure 8. Time-averaged spectra for period II for 180-deg sectors separated by the 200-deg surface wind direction for a scanned circle of radius 3-4 km at height 500 m. The dashed curve (solid circles) is for the north-western sector, over the Isles of Scilly; the solid curve (open circles) is for the south-eastern sector over the open sea. There is a spectral scale of 10 km for the south-eastern sector. The effect of the Isles of Scilly was to increase the turbulence in the north-western sector at scales greater than 2-5 km.

Fig. 8 shows an example of time-averaged spectra derived from two 180° sectors formed by dividing the scanned circle along the 200° direction of the surface wind. The radius of the scanned circle used in deriving Fig. 8 was 6-4 km, so that the north-western half of the circle was located directly over the Isles of Scilly. The two spectra are essentially identical for scales shorter than 2 km but at longer scales the turbulence in the north-west sector exceeds that to the south-east. The spectra in Fig. 8 apply to an altitude of 500 m but similar results were found for time-averaged spectra at all altitudes for which the scanned circles had radii greater than 5 km. Although the difference between the two spectra in Fig. 8 is in itself insufficient evidence of a topographical effect, the fact that similar results were obtained with pairs of time-averaged spectra for other altitudes indicates that there was indeed a small but significant topographical influence due to the Isles of Scilly which was confined to scales in excess of 2-5 km.

One of the reasons for plotting the spectra in Fig. 9 is to locate the local maximum at some spectral scale $\lambda_m$, which provides an estimate of the size of the energy producing eddies. Unfortunately the topographical effect of the Isles of Scilly occurs at scales rather
Figure 9. (a) Time-averaged turbulence spectra in and just above the planetary boundary layer over periods I and II. Curves (a), (b), (c) and (d) correspond to heights 100m, 300m, 500m and 900m (which is above the PBL) respectively. Arrows indicate the estimated spectral scales, $\lambda_m$. (b) Time-averaged turbulence spectra in and just below the free shear layer for period II after 0930 GMT. One curve is for a height of 1.3km in the placid layer; the other curves are for heights of 1.6, 1.8 and 2.0km, representing increasing penetration toward the axis of the free shear layer.
close to the value of $\lambda_m$ in parts of the PBL and, therefore, in order to produce reliable estimates of $\lambda_m$, the data used in deriving Fig. 9(a) were taken only from the south-east sector (020–200°) situated over the open sea. Three of the spectra in Fig. 9(a) corresponding to altitudes 100, 300 and 500m show a distinct local maximum corresponding to the spectral scale $\lambda_m$. The spectral scale increased from 500m at a height of 100m to 1km at a height of 500m. However, the intensity of the local maximum decreased rapidly with altitude above 500m and as shown, at 900m altitude, it could scarcely be distinguished from the background spectrum. The spectra in Fig. 9(b), which correspond to altitudes 1-3, 1-6, 1-8 and 2-0km, are significantly different from those in Fig. 9(a). At wavelengths longer than 700m, the spectra coincide quite closely and in none of them is there any sign of the local maximum that was present in the PBL. At 1-3km, there is no spectral scale within the observable range of scales; $kS(k)$ continued to decrease with increasing wavenumber until it levelled off at 0.03m²s⁻². This minimum value, which varied from 0.02 to 0.03m²s⁻², was due to instrumental noise which was visible on the chart record; (this may be compared with a value of only 0.003m²s⁻² introduced in converting the velocity-azimuth record to punched tape). At altitudes above 1-3km turbulence with a spectral scale of less than 300m appeared; the peak was too close to the limit of resolution to be located accurately. As shown in Fig. 9(b), the intensity of this short wavelength peak increased with penetration into the FSL. Other spectra, including those for altitudes above 2-0km (not shown), indicated that this peak reached its maximum intensity at 2-0km, at the centre of the FSL.

Spectra for all levels above 800m were very similar for wavelengths longer than 700m with a slope $n$ of 0.7, increasing to about 0.85 in the FSL. This suggested the existence of a 'background' spectrum which was almost independent of height, with a spectral scale greater than 5km, too large to be measured by this technique. The same background spectrum may have existed below 800m but it would have been obscured by the much more intense turbulence associated with the small input scales in the PBL.

The variation of the spectral scale with height is summarized in Fig. 10. In the lowest 900m of the atmosphere, $\lambda_m$ increased with height according to a relationship of the form: $\lambda_m \propto z^m$ where $m$ is between 0.5 and 0.7. Above 500m, the uncertainty in the values of $\lambda_m$ is large and it is conceivable that this parameter was independent of height at these altitudes. Between 1-6 and 2-5km, in the FSL, the principal spectral scale was less than 250m.

Figure 10. Variation of spectral scale $\lambda_m$ with height $z$, derived from time-averaged spectra similar to those in Figs. 9(a) and (b). The solid curve in the PBL corresponds to a relationship of the form $\lambda_m \propto z^{0.6}$. 

D
Figure 11(a) and (b). Time-averaged spectra for period II comparing the horizontal components of turbulent energy resolved parallel and perpendicular to the shear vector (a) at a height of 400m, in the planetary boundary layer; and (b) at 2-0km, in the free shear layer. Solid curves represent the component parallel to the local shear vector; dashed curves represent the component at right angles to the shear vector.
(d) *Horizontal components of turbulence intensity resolved parallel and perpendicular to the local wind shear*

Any anisotropy in the structure of turbulence would be expected to be oriented with respect to the shear vector (this direction corresponds to the wind direction in the PBL but not in the FSL). Figs. 11(a) and (b) show time-averaged spectra derived for 64° azimuth sectors about the shear direction (solid curves) and about the direction orthogonal to the shear vector (dashed curves). Fig. 11(a), corresponding to a height of 400m in the PBL, shows that, although the turbulence was slightly greater in the shear direction, the two spectra have much the same slope and spectral scale. Similar results were obtained at all levels in the PBL. However, the spectra in Fig. 11(b), which refer to a height of 2km, in the FSL, indicate pronounced anisotropy at scales larger than about 600m, although the relatively intense turbulence at shorter scales, which has already been noted as characteristic of the FSL, may well have been more nearly isotropic. This is consistent with the model shown in the inset in Fig. 11(b) in which Kelvin–Helmholtz (K–H) billows with a wavelength $\lambda_0$ between 1 and 2km are considered to break down into more nearly isotropic turbulence on scales shorter than 600m. The scanned circle intersected each K–H billow at an acute angle as one approached the upshear and downshear directions and so the peak in Fig. 11(b) occurs at wavelengths between 1·5 and 3 times the value of $\lambda_0$. According to the observations of Browning (1971), the ratio between crest-to-trough amplitude $(a)$ and the wavelength $(\lambda_0)$ of K–H billows in the atmosphere has a typical value of 0·25. Assuming a similar ratio in the present case this gives a crest-to-trough amplitude of about 400m, comparable with the dimensions of the largest ‘isotropic’ eddies. According to Miles and Howard (1964), the wavelength of the initial billows, $\lambda_0$, is 7·5 times the depth of the dynamically unstable layer $\Delta z$. Although the accuracy of this relationship is uncertain when applied to non-ideal vertical profiles of shear and stability, it gives a value of 200m for $\Delta z$ in the present case which is comparable with the depth of the layer of strong shear (Fig. 6(c)).

![Figure 12. Time-averaged profiles of energy dissipation rate $\epsilon$ for period II.](image_url)

Figure 12. Time-averaged profiles of energy dissipation rate $\epsilon$ for period II. estimated from the turbulence spectra. The dashed curve corresponds to the background spectrum (see text); the solid curve corresponds to energy input at scales related to those indicated in Fig. 10.
(e) **Time-averaged vertical profiles of energy dissipation and mechanical production rate**

For those spectra that were in fair agreement with the $n = \frac{2}{3}$ relationship, Eq. (1) has been used to find the energy dissipation rate $\varepsilon$ using a value of 0.15 for the Kolmogorov constant. Time-averaged values of $\varepsilon$ for period II are shown in Fig. 12; the dashed curve corresponds to the so-called background spectrum with $\lambda_n > 5$km whereas the solid curve corresponds to the turbulence associated with smaller spectral scales falling within the observable range of 200m to 5km. Between 400m and 500m, where the spectral slope begins to depart from $-\frac{2}{3}$, the high frequency end of the spectrum was used to estimate $\varepsilon$ and values for these levels have greater uncertainty. For the background spectrum the value of $\varepsilon$ was found to be about $2 \text{cm}^2 \text{s}^{-3}$, with no significant variation between a height of 500m and 2.5km. Superimposed on this, however, were regions of rather higher $\varepsilon$ due to small scale turbulence in the PBL and FSL. In the PBL $\varepsilon$ increased downward from $2 \text{cm}^2 \text{s}^{-3}$ at 700m to $10 \text{cm}^2 \text{s}^{-3}$ at 400m. Below 400m the slope of the spectrum was substantially different from $n = \frac{2}{3}$ and so $\varepsilon$ has not been estimated. In the FSL $\varepsilon$ could be estimated only crudely by assuming that the spectral scale was about 250m and that the $n = \frac{2}{3}$ law applied for shorter scales; the resulting values, which give an approximate lower limit to $\varepsilon$, reached a maximum of about $20 \text{cm}^2 \text{s}^{-3}$ at 20km, at the centre of the FSL.

Energy dissipation rates can be compared with the mechanical production terms derived from Eq. (5) using the method described by Wilson to find the momentum fluxes. As expected, values of momentum flux derived from individual conical scans showed large and erroneous variability at each level as a result of inhomogeneities in the distribution of turbulence, but averaging the values for all scanned circles at each altitude produced consistent trends of momentum flux with height. Time-averaged profiles for period II of the magnitude of the vector shear and of a component of the momentum flux are shown in Fig. 13(i) and (ii); each point is the average value derived from at least 10 successive conical scans. Above 1.2km, the lines along which the averages were taken were tilted slightly to relate the data to the axis of the FSL. In Fig. 13, $u$ is defined to be parallel to the shear

![Figure 13](image-url)

**Figure 13.** Time-averaged profiles of (i) wind shear; (ii) momentum flux; and (iii) energy production rate due to shear for period II. Profile (iii) has been derived from (i) and (ii).
vector, with momentum flux \((-\overline{u'w'})\) positive in the upshear direction; \(\overline{u'w'}\) is therefore the only component of momentum flux that contributes to the mechanical production of turbulence. (Although the component \(v'w'\) is non-zero, the product \(v'w'\, dz/dz\) is zero by definition.) The results are plotted in two portions, using a shear direction of 200° from sea-level up to 1.0 km and 310° from 1.1 km to 2.5 km. A correspondence between momentum flux and shear can be seen in both magnitude and sign. Throughout the PBL, momentum was transported downward to destroy the shear. In the placid layer, the momentum flux is not significantly different from zero. In the FSL, momentum flux is again in a down gradient sense so as to tend to destroy the shear throughout the depth of the layer. A few individual values of momentum flux differed in sign from the average, an effect which probably may be ascribed to errors due to inhomogeneity; several of these values occurred in the FSL just below 2 km and account for the low value of \(\overline{u'w'}\) at that level.

The time-averaged profile of the rate of mechanical production of turbulence calculated using Eq. (5) is shown in Fig. 13(iii). The profile is similar to that of \(\varepsilon\) in Fig. 12, computed from the spectra, but the mechanical production rate is smaller than \(\varepsilon\) at heights above 500 m. This effect is partly due to the fact that only a limited range of wavelengths is contributing to the results in Fig. 13 and so the two terms may be almost in balance. Also, since the spectra from which \(\varepsilon\) was derived combines the \(u\) and \(v\) components of turbulence, it is likely that the true value of the Kolmogorov constant is slightly different from that used here.

5. Comparison with other results

(a) Turbulence in the planetary boundary layer

Fig. 14 shows values of the main spectral scale \(\lambda_m\) for the horizontal component of turbulence plotted against height during period II of the present study (see the open circles

![Figure 14](image-url)  
Figure 14. Estimates of the spectral scale \(\lambda_m\) for the horizontal component of turbulence, indicated by open circles, superimposed on previous results for the vertical component in the planetary boundary layer.
in Fig. 14); these are superimposed on the results of other workers for the vertical component as summarized by Pasquill (1972). The recent results of Warner (1972) for a stable situation over the sea are also included. The present results agree well with most previous measurements except those labelled (a).

Fig. 15 shows a plot of energy dissipation rate $\varepsilon$ against height during period II superimposed on recent results obtained at Cardington as summarized by Pasquill. Between 400 and 600m the present results are consistent with data obtained for unstable conditions. A further comparison of the two sets of results suggests that from 600m to 800m on this occasion there was a transition to a stable situation, and above 800m the present values agree very well with the Cardington results for stable conditions. Detailed comparisons must, however, be made with some caution because previous measurements were made over land and in lighter wind conditions than in the present study. These aspects of the Cardington results have been discussed in more detail by Rayment (1973).

![Figure 15](image)

*Figure 15. Estimates of energy dissipation rate $\varepsilon$ from the present study, indicated by the dashed curve, superimposed on the results of Readings and Rayment (1969) as summarized by Pasquill (1972) for (a) near-neutral and (b) convectively unstable conditions.*

Fig. 16 shows a plot of spectral slope $n$ for spectra obtained during period II at all levels in the PBL. There is a large change in slope from $n = 0.7$ at 700m to 1.2 below 300m. The slope $n = 0.7$ corresponds to that expected for an inertial subrange. The value $n = 1.2$ is substantially in excess of that expected for an inertial subrange. A steep spectral slope has also been found by Thompson (1972) during tethered balloon measurements of vertical wind fluctuations ($w'$) over the sea at 140m altitude. Such a slope could be caused by "incipient turbulence" in which the inertial subrange is not fully developed but this seems unlikely over the sea where there are no irregularities in terrain which might give rise to preferred regions of generation of turbulence. The value $n = 1.2$ found at lower altitudes suggests that buoyancy forces may have been removing energy from the cascade and this value is in fact the slope that has been predicted for a buoyant subrange (Lumley and Panofsky 1964); this would be consistent with the stable lapse rate which was observed in the present case below 300m. Evidence of a transition from static stability below 300m to instability above 300m is provided by the tephigram in Fig. 4(a). This shows a lapse rate slightly less than dry adiabatic throughout the PBL, including close to the surface.
Figure 16. Variations of spectral slope $n$ with height in and just above the planetary boundary layer. The dashed curve corresponds to the background spectrum; the solid curve corresponds to energy input at the scales indicated in Fig. 10.

Despite continuous rainfall, mesoscale subsidence below the frontal zone was maintaining a relative humidity as low as 85% in the PBL so that the lapse rate was marginally stable with respect to small vertical displacements. However, surface observations at the Isles of Scilly indicated that there was a 3 or 4 oktas cloud cover at a height of about 300m (with a solid cloud cover at a level above the PBL) and, since the lapse rate was unstable for saturated ascent, the PBL can be regarded as partly unstable above this level.

Our inference of static stability below 300m is perhaps surprising in view of the strong low-level winds which would have the tendency to produce a well-mixed layer of near neutral stability. Although this inference of static stability is vulnerable to possible unrepresentativeness of the sounding in Fig. 4(a), it received support from measurements from sondes dropped from a Varsity aircraft in the vicinity of the Isles of Scilly at 0935 and 1121. Further support for the plausibility of stability at low levels was provided by the fact that there was a large horizontal gradient of sea surface temperature, amounting to about 2 deg C over 300km, upwind of the Isles of Scilly, and this would be expected to have helped maintain a stable lapse rate. Moreover, the surface wind recorded on the Isles of Scilly at an altitude of 50m remained as low as 10m s$^{-1}$ despite the wind at 200m being as strong as 20m s$^{-1}$. Such a strong wind shear could hardly have been maintained in the absence of static stability in the lowest part of the PBL. The static stability of the lowest 300m may have led to the turbulence close to the surface being contaminated by gravity waves which could also increase the value of $n$; however, it is difficult to distinguish between the two effects and we have not attempted to do so in the paper.

(b) Turbulence in the free shear layer

The spectral scale in the PBL increased in size to about 1-6km at the top of the PBL above which this particular energy input disappeared altogether. During most of the period of observation (i.e., period II) there was a placid layer above the PBL in which there was weak background turbulence with $\varepsilon \simeq 2cm^2s^{-3}$ and a spectral scale too large ($\lambda_m > 5km$) to be determined by the present method. The same weak background turbulence was also evident in the overlying FSL, where it was supplemented by markedly anisotropic 'turbulence' due to K-H billows. This agrees with the landbased radar measurements of Børresen (1970) who noted that, while turbulence at low levels was almost isotropic, at higher levels the intensity of the turbulence component resolved parallel to the shear vector was far
greater than the perpendicular component. The K–H billows in the present study had a wavelength of 1–2 km and produced a major source of turbulence with a spectral scale of 250 m or less. This spectral scale was too close to the resolution limit to permit accurate calculation of the energy dissipation rate but it appears that ε was probably close to 20 cm² s⁻³ in a layer 200 m deep over which the shear exceeded 4 m s⁻¹ (200 m)⁻¹ and in which the corresponding layer Richardson number may have dropped below 0.5.

ACKNOWLEDGMENTS

We are grateful to Drs. F. Pasquill, C. J. Readings and F. B. Smith for their helpful comments on the manuscript. Crown copyright; reproduced by permission of the Controller HMSO.

REFERENCES


Rayment, R. 1973 'An observational study of the vertical profile of the high frequency fluctuations of the wind in the atmospheric boundary layer,' Boundary Layer Met., 3, pp. 284–300.


