Comparison of aerodynamic and energy budget estimates of fluxes over a pine forest

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SUMMARY

Values of the total vertical flux of sensible and latent heat over a level forested region, obtained from aerodynamic (profile-gradient) formulae appropriate to airflow over relatively smooth surfaces, are found to fall consistently short of independent energy-balance estimates by a factor of 2 to 3 in unstable and near-neutral conditions (Richardson number, Ri, in the range −0.4 to +0.01), whereas for Ri > +0.02 no similar discrepancy is detected. These results, based on tangents drawn to (semi-logarithmic) profiles at a height of about nine aerodynamic roughness parameters (z0) above the zero plane displacement level (d) of the forest, rely on the basic assumption that the value of d established in very nearly neutral conditions (|Ri| < 0.003), namely 0.76 mean tree heights, holds under all conditions of thermal stability.

Wake diffusion and thermal seeding effects are discussed as possible additional transfer mechanisms acting to reduce profile gradients immediately over aerodynamically rough surfaces. In terms of the former mechanism (assumed to operate below d + 2z0, or so), approximate empirical formulae are derived which attempt to quantify the observed discrepancy in terms of Ri and the proximity of the surface.

It is concluded that aerodynamic equations ought not to be used to give independent flux estimates close to aerodynamically rough surfaces.

1. INTRODUCTION

In the context of relating vertical fluxes to profile gradients in the surface boundary layer, the manner in which forced convective processes are modified by thermal convection and the degree to which this occurs have been under close theoretical and experimental scrutiny for some time (Pasquill 1949; Deacon 1949). Despite accelerated effort in recent years (e.g. Panofsky 1963; Swinbank 1964, 1966; Dyer and Hicks 1970; Webb 1970; Businger et al. 1971; Pruitt, Morgan and Laurence 1973), no consistent general solution to the problem has been found, perhaps because no such solution exists: each experimental site commends somewhat different values for the ‘constants’ in the various semi-empirical diabatic influence relationships. To a large extent, however, such of these differences as are not due to systematic error presumably are related to contrasts in the overall boundary conditions of research sites, not only in the sense of differing edge effects—brought about by peculiar topographical form or by discontinuity in aerodynamic roughness—but possibly also in terms of the characteristic stability structure of the entire planetary boundary layer over each site. Although it is not entirely certain that aerodynamic roughness itself can be ruled out as a possibly relevant variable, it is readily argued that such contrasts as do exist in surface roughness amongst the various research sites used, are unlikely to have had significant effects on the results. Measurements were made in almost every event at heights at least one order of magnitude larger than those of the surface roughness elements, i.e. at distances of the order of hundreds or even thousands of aerodynamic roughness parameters above each surface.

Stewart and Thom (1973) reported briefly on large discrepancies between energy balance determinations of vertical fluxes over pine forest and aerodynamic estimates employing currently acceptable flux-gradient relationships. We present here in detail these and

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further results, together with a critical scrutiny of the applicability of any flux-gradient relationship (diabatic or adiabatic) established over the relatively smooth research sites referred to above, to property profiles measured within a few aerodynamic roughness parameters of the tops of vegetation. Such drastic limitations in height of measurement are frequently imposed in field experiments by the rigorous fetch requirements of an imperfect or restricted experimental area, or by sheer scale when the vegetation is many metres tall; also, there is often the tacit assumption that sufficiently close to any surface, however rough, effects of free convection are minimized and amenable to fairly accurate description by one or other of the current semi-empirical formulae.

2. List of Symbols

A available energy; \( R_n - G - S - P \)
c level of action of aerodynamic drag on a vegetated surface
c\(_p\) specific heat of air at constant pressure
d zero plane displacement; 12.0m = 0.76h, derived in near-neutral stability conditions (\(|Ri| < 0.003\))
F stability factor; defined by \( A/(H_0 + (\lambda E)_0) \)
F\(_D\) wake diffusivity factor; defined by \( F/F_* \)
F\(_z\) stability factor for 'smooth' surfaces, or for \( \zeta > \zeta_* \)
g acceleration due to gravity
G energy flux into the ground; measured by soil flux plates
h mean height of trees
H sensible heat flux (+ve upwards), at \( z = z_R \)
H\(_0\) estimate of H from profile slopes, at \( z = z_R \), if stability corrections are ignored
k von Kármán's constant; 0.41
K eddy diffusivity
K\(_H\) eddy diffusivity for heat exchange
K\(_M\) eddy diffusivity of momentum
(K\(_M\)_0) value of \( K_M \) obtained if stability corrections are ignored
K\(_V\) eddy diffusivity for vapour exchange
K\(_{V, H}\) eddy diffusivity for vapour and heat exchange
L the Monin–Obukhov stability length; \(-\rho c_p T_u^2/[(kgH(1 + 0.070/\beta)]\); see Ri
P net rate of energy absorption by photosynthesis and respiration.
q specific humidity
q\(_*\) friction specific humidity; defined by \( \lambda E/(\rho \lambda u_*^2) \)
Ri Richardson number: \( \left\{ \frac{g}{T} \frac{\partial \theta}{\partial z} \left( \frac{\partial u}{\partial z} \right)^2 \right\} (1 + 0.070/\beta) \); the factor \((1 + 0.070/\beta)\), used by Dyer and Hicks (1970), accounts adequately for the small effect on stability of the humidity gradient associated with the vertical flux of latent heat (essentially it converts \( \theta \) to virtual potential temperature, and is exact at 12°C)
R\(_n\) net radiation; measured at \( z = z_R \)
S total flux of energy into storage between \( z_R \) and ground level
T absolute temperature
u wind speed
u\(_*\) friction velocity
z height above the ground
z\(_0\) roughness parameter; 0.93m = 0.06h
z\(_R\) reference level; height of net radiometers; 20.5m
\( \beta \) Bowen ratio; \( H/\lambda E \) or \( H_0/(\lambda E)_0 = (c_p/\lambda) \frac{\partial \theta}{\partial q} \) if \( \phi_H = \phi_V \)
ENERGY BUDGET ESTIMATES

\[ \zeta \] dimensionless height above \( z = d \); given by \( (z - d)/z_0 \)

\[ \zeta_R \] value of \( \zeta \) at \( z = z_R \); equal to 9

\[ \zeta_* \] value of \( \zeta \) below which wake diffusion may be important – a function of \( Ri \)

\[ \zeta_{*0} \] value of \( \zeta_* \) for neutral stability (\( Ri = 0 \)); estimated as 25

\[ \theta \] potential temperature

\[ \theta_* \] friction potential temperature; defined by \( H/(\rho c_p u'_* ) \)

\[ \lambda \] latent heat of vaporization of water

\[ \lambda E \] latent heat flux (+ve upwards) at \( z = z_R \)

\[ (\lambda E)_0 \] estimate of \( \lambda E \) from profile slopes, at \( z = z_R \), if stability corrections are ignored

\[ \rho \] density of air

\[ \phi \] stability function; subscripts \( H, M \) and \( V \) for sensible heat, momentum and water vapour exchange

\[ \phi_{V, H} \] stability function for vapour and heat exchange

3. PRELIMINARY THEORY

Over an aerodynamically rough surface the wind shear at height \( z \) from the substratum is

\[
\frac{\partial u}{\partial z} = \frac{u'_* \phi_M}{k(z - d)} \quad z > h
\]  

(1)

With neutral stability, \( \phi_M = 1 \) and the classical eddy diffusivity of momentum above the surface roughness elements is directly proportional to distance above \( d \), being given then by \( ku'_*(z - d) \).

With the usual assumption that the eddy diffusivities of momentum, water vapour and heat are identical when stability is close to neutral, expressions for the gradients of potential temperature and specific humidity above the surface roughness elements can be written, namely

\[
\frac{\partial \theta}{\partial z} = - \frac{\theta'_* \phi_H}{k(z - d)} \quad z > h
\]  

(2)

and

\[
\frac{\partial q}{\partial z} = - \frac{q'_* \phi_V}{k(z - d)} \quad z > h
\]  

(3)

which are similar to Eq. (1). (Differences in the vertical distribution of sources and sinks of momentum, mass and heat within an aerodynamically rough surface influence the magnitudes of the respective roughness parameters of the surface rather than its zero plane displacement (Thom 1972.) Although the stability functions \( \phi_V \) and \( \phi_H \) are almost certainly identical, they are not in general the same as \( \phi_M \); Dyer and Hicks (1970) conclude that in unstable conditions

\[
\phi_V = \phi_H = \phi_M^2 = (1 - 16z/L)^{-1}
\]  

(4)

an interrelationship strongly supported by Paulson (1970); whereas work by Webb (1970) and data discussed by Lumley and Panofsky (1964) indicate that an interrelationship of the form

\[
\phi_V = \phi_H = \phi_M = (1 + 5.2z/L)
\]  

(5)

is appropriate for stable stratification.

From the definitions of \( L \) and \( Ri \) it is easily demonstrated, using Eqs. (1) and (2),
that the ratio \( z/L \) is formally equivalent to the product \( (\phi_M^2/\phi_H) \cdot Ri \). In terms, therefore, of the Richardson number itself, which is much the more convenient parameter to employ in practice, Eqs. (4) and (5) take the respective forms

\[
\phi_{V,H} = \phi_M^2 = (1 - 16Ri)^{-\frac{1}{2}} \quad Ri - ve
\]

(6)

and

\[
\phi_{V,H} = \phi_M = (1 - 5.2Ri)^{-1} \quad Ri + ve
\]

(7)
in which \( \phi_{V,H} \) replaces \( \phi_V \) and \( \phi_H \). These are the stability functions adopted here as being representative of the more realistic semi-empirical forms currently in vogue; see e.g. the legend to Fig. 1. (The implied equivalence of \( Ri \) to \( z/L \) in the unstable regime is peripheral to this paper; it is not intended to express a general truth.)

From Eqs. (1) and (2) and the definition of \( \theta_\ast \), we can write

\[
\mathbf{H} = \mathbf{H}_0(\phi_H\phi_M)^{-1},
\]

(8)

where

\[
\mathbf{H}_0 = -\rho c_p k^2(z - d)^2 \frac{\partial u}{\partial z} \frac{\partial \theta}{\partial z}
\]

(9)
is the value found for the sensible heat flux when stability (or other) effects on the wind and temperature profile gradients are discounted. Similarly

\[
\lambda E = (\lambda E)_0(\phi_V\phi_M)^{-1},
\]

(10)

where

\[
(\lambda E)_0 = -\rho \lambda k^2(z - d)^2 \frac{\partial u}{\partial z} \frac{\partial q}{\partial z}
\]

(11)

If the sum of \( \mathbf{H} \) and \( \lambda E \) – the available energy, \( A \) – is known from energy balance measure-

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**Figure 1.** Current forms of the stability factor \( F = (\phi_{V,H}\phi_M)^{-1} \) as a function of Richardson number \( Ri \).

**UNSTABLE:** 1: Dyer and Hicks (1970), \( \phi_{V,H} = \phi_M^2 = (1 - 16Ri)^{-\frac{1}{2}} \), Eq. (6); 2: Pruitt et al. (1973), \( \phi_{V,H} = 0.885(1 - 22Ri)^{-0.4} \), \( \phi_M = (1 - 16Ri)^{-1} \); 3: Webb (1970), \( \phi_{V,H} = \phi_M = (1 - 4.5Ri)^{-1} \); 4: deduced from formulae of Businger et al. (1971), normalized to \( k = 0.41 \) from \( k = 0.35 \); 5: from KEYS function (see e.g. Panofsky 1963), \( \phi_{V,H} = \phi_M^2 = (1 - 18Ri)^{-\frac{1}{2}} \).

**STABLE:** (Authors as above): 1: \( \phi_{V,H} = \phi_M^2 = (1 + 16Ri)^{\frac{1}{2}} \), an inverted form of Eq. (6); 2: \( \phi_{V,H} = 0.885(1 + 34Ri)^{0.4} \), \( \phi_M = (1 + 16Ri)^{\frac{1}{2}} \); 3: \( \phi_{V,H} = \phi_M = (1 - 4.5Ri)^{-1} \), Eq. (7); 4: as above; 5: \( \phi_{V,H} = \phi_M = (1 + 18Ri)^{\frac{1}{2}} \).
ments, values of the product \((\phi_{V, H} \phi_M)^{-1}\) called the stability factor \(F\) by Stewart and Thom (1973), can be derived directly from appropriate profile-gradient data; i.e.

\[
F = (\phi_{V, H} \phi_M)^{-1} = \frac{A}{[H_0 + \lambda E_0]}
\]

(12)
as suggested independently by Holbo (1971). Curves of \(F\) as a function of Richardson number derived from various current forms of \(\phi_{V, H}\) and \(\phi_M\) are presented in Fig. 1: differences in \(F\) tend to be smaller than corresponding differences in \(\phi_{V, H}\) or in \(\phi_M\) alone.

Provided that horizontal advection of energy is negligible, or at least that any advection of sensible and latent heat be in the same ratio \(\beta\) as their vertical fluxes, a curve of \(F\) versus \(Ri\) for a particular site or a particular surface roughness can be used to estimate surface values of \(H\) and \(\lambda E\) from profile gradients alone when energy balance data are not available, or to find \(\lambda E\) from \(H_0\) and \(A\) when humidity gradients are suspect — the latter being a not unusual occurrence in practice.

Using Eq. (12), Holbo (1971) derived an expression for \(F\) in unstable conditions over a large, flat, unvegetated pumice deposit in Central Oregon, namely \((1 - 34Ri)^{0.55}\). In the range \(-0.6 < Ri < 0\), this differs by less than 10% from the corresponding Dyer and Hicks relationship \((1 - 16Ri)^{0.75}\) and lends support to our subsequent use of the latter to express \(F\) in unstable conditions over relatively smooth surfaces (Eq. (17)).

A simple interpretation of \(F\) in terms of the ratio of two eddy diffusivities is as follows: if the actual eddy diffusivity for vapour and heat exchange is defined by

\[
K_{V, H} = \frac{-A}{\rho \left( \frac{c_p}{\frac{\partial \theta}{\partial z}} + \lambda \frac{\partial q}{\partial z} \right)}
\]

(13)

and we let

\[
(K_M)_0 = k^2(z - d)^2 \frac{\partial u}{\partial z},
\]

(14)

which is the value found for the eddy diffusivity of momentum if the effect of stability on wind-speed gradient is ignored, it follows from Eqs. (8) to (12) that

\[
F = \frac{K_{V, H}}{(K_M)_0}
\]

(15)

4. Experimental results

(a) Recent measurements over pine forest

The data quoted here were obtained by the Institute of Hydrology's micrometeorological team in Thetford Forest, Norfolk. They refer to the same seven fine days in 1971 for which energy budgets for the forest have recently been published (Stewart and Thom 1973), plus two days in 1972 with particularly steady, near-neutral conditions. Full details of site and instrumentation are given by these authors and by Stewart (1971), Oliver (1971) and Oliver and Oliver (1973). We emphasize, however, that from the tops of the observing towers the trees present a uniformly rough, virtually level sward extending for at least 3km in almost every direction, whereas profiles of \(u\), \(\theta\) and \(q\) are measured within 15m or so of the top of the canopy. This satisfies even the most rigorous of suggested minimum fetch requirements (Pasquill 1972).

The experimental points on Fig. 2 give mean values of \(F\) and \(Ri\) for each hour on the seven sunny days, apart from those hours with either \(A\) or \(\{H_0 + (\lambda E)_0\} < 5\, \text{Wm}^{-2},\)
when exceptionally large errors in $F$ can easily arise – typically near sunrise and sunset and in cloudy conditions at night. Also included are values determined for two days in 1972 when conditions remained wet with low but steady values of available energy all day: hourly periods analysed for such days give much more accurate values for fluxes and therefore for $F$ than can otherwise be obtained under very nearly neutral stability conditions. All values of $F$ and $Ri$ correspond to $z = 20.5 m$, this being the level to which the available energy $A$ refers (Stewart and Thom 1973). In calculating $A$, larger and slightly more sophisticated corrections for the net consumption of energy by photosynthesis and respiration were applied than previously – namely, during the day, $P$ = the greater of $5 W m^{-2}$ or 2% of $(R_o - G)$, and at night $P = -1 W m^{-2}$. Values of $Ri$ were derived on the fine days from the expression in section 2, and on the wet days, when $\beta$ was often close to zero, from the equivalent finite-difference expression $(g/T)(\delta \theta + \frac{1}{6} \delta q) \delta z/(\delta u)^2$.

In finding $F$ from Eq. (12), values of $H_o$ and $\lambda E_o$ were deduced from tangents hand-drawn at $z = 20.5 m$ to curves of $u$, $\theta$, and $q$ versus $\ln(z - d)$, though on the wet days an equivalent finite-difference method was used. The zero plane displacement level $d$ was taken to be 12.0 m, close to 0.766, on all occasions, this being the value found in the usual way to permit of the basic semi-logarithmic description of the wind profile above the trees in conditions of very nearly neutral stability ($|Ri| < 0.003$).

Fig. 2 demonstrates that only in moderately stable conditions (0.02 < $Ri$ < 0.14 or so) is there any reasonable agreement between the values of $F$ derived for the forest and those calculated from Eqs. (6) and (7): in effectively neutral conditions there is a substantial discrepancy, amounting to a factor of about 2 in $F$, which cannot be ascribed to systematic error and which extends over the entire range of unstable data. Separate analysis of the unstable data demonstrates that $F$ is independent of wind direction and nullifies the disturbing hypothesis that perhaps the observed discrepancy is associated with local profile distortions generated by a particular upwind tree-crown geometry. This serves also to increase overall confidence in the horizontal homogeneity of the forest site.
(b) Discussion

As values derived for the factor $F$ are inversely proportional to the square of the distance $(z_R - d)$, the large observed discrepancy in $F$, on average a factor of 2.5 or so for $-0.4 < Ri < 0.01$, would not exist if the actual value of $d$ for the forest were about 7m or 0.45$h$, rather than 12m or 0.76$h$ — the value deduced initially from the 1971 wind data. It is essential therefore thoroughly to establish this latter value, as follows.

In almost neutral conditions of stability, the ratio $d/h$ is found to be close to 0.76 in each of the years of observation from 1970 to 1973 (e.g. Oliver 1971), regardless of wind strength (Oliver and Mayhead 1974) and for each of many combinations of five or six (out of the twenty available) sensitive photoelectric anemometers.

Thom (1971) has identified $d$ with the level of action $c$ of the aerodynamic drag on artificial vegetation in a wind-tunnel; and calculations for the forest in near-neutral stability conditions give $c/h = 0.76 \pm 0.04$, coincident with the conventionally derived value for $d/h$. Although $c$ varies with stability in response to associated changes in the shape of the wind profile within the canopy, it is in stable conditions that this effect is most pronounced: for $Ri > +0.2$, $c$ is about 0.9$h$. Even in the most unstable conditions, however, $c$ drops no lower than about 0.7$h$, a relatively insignificant change in terms of the value 0.45$h$ deduced above.

Also, Jarvis et al. (1975) summarize recent results for $d$ (and $z_0$) for eleven forest sites, encompassing eight species and a range in mean tree height from 10-4m to 27.5m. Average site values of $d/h$ range from 0.61 to 0.92 and the overall mean, excluding the Thetford data, is 0.79. (Corresponding values for $z_0/h$ range from 0.02 to 0.16 with a mean of 0.078, in comparison with 0.058 for Thetford.) We conclude, therefore, that 0.76$h$ is an entirely realistic value of $d$ for the Thetford forest site, so that the recorded discrepancy in $F$ is real.

In terms of the Prandtl–Kármán mixing-length model, to which in the meantime we adhere, there are two simple interpretations of conditions in the region of measurement ($4 < \zeta < 15$):

either

von Kármán’s constant $k$ has an effective value of about 0.6, rather than 0.41, so that equality of $K_{V,H}$ and $K_M$ in neutral conditions is retained;

or $k = 0.41$ is valid for momentum alone, with $K_{V,H}$ being approximately twice $K_M$ in neutral conditions (from Eq. (14)) and $k$ being effectively about 0.9 for water vapour and heat.

In section 5(b) we choose to pursue the latter interpretation. It is worthy of some comment here, however, that simply setting $d \approx \frac{1}{3}h$ in Eqs. (1), (2), (3), etc., rather than $d \approx \frac{1}{4}h$, leads to reasonable agreement between aerodynamic and energy budget flux estimates for the forest, and in particular to $F = 1$ at $Ri = 0$.

Certainly, many examples exist of successful aerodynamic flux calculations made over field crops, using values of $d/h$ of 0.5 or somewhat less: e.g. Penman and Long (1960), wheat; Udagawa (1966), barley; Lemon and Wright (1969), corn; and Grant (1970), barley, all in day-time conditions of almost certainly negative Richardson number and all for similar ranges of $\zeta$ to that over the forest. In particular, however, Lemon et al. (1971, Fig. 5) record a discrepancy of about a factor of two between aerodynamic and energy balance estimates of $K$ over corn, in unstable conditions. Despite replacing the small stability correction which they use by the more substantial value implicit in Eq. (6), a discrepancy of a factor of 1-6 remains which, if the value of $d$ employed, namely 0.65$h$ (D. W. Stewart, personal communication), is replaced by the possibly more realistic value 0.75$h$, reverts to a factor somewhat in excess of 2 (for $\zeta_R = 9$). Similar discrepancies are reported by Jarvis et al. (1975) for spruce, and by Mukammal et al. (1966) for the mature growth stages.
of a corn crop (see Fig. 4). In each event, however, the discrepancy is removed if \( d \) is simply endowed with a low enough effective value, in the region of 0.5\( h \).

Nevertheless, the empirical convenience of using a depressed value for \( d \) in aerodynamic flux calculations tends to obscure the physical significance of the necessity for so doing. This is that over tall vegetation, in the region of \( \zeta = 10 \), eddy diffusivities of water vapour and heat, and possibly also of momentum, must be much larger (except for \( Ri > 0.02 \)) than at similar distances above the relatively smooth (mostly short-grass covered) surfaces to which the relations illustrated in Fig. 1 specifically refer, and for which \( \zeta \sim 10^2 \) to \( 10^4 \).

5. Possible Transfer Mechanisms Close to Rough Surfaces

There are two mechanisms which could possibly operate to increase the eddy diffusivity close to aerodynamically rough surfaces to values substantially in excess of those based on the Prandtl–Kármán mixing length theory; namely thermal seeding and wake diffusion.

(a) Thermal Seeding

Analyses of the distribution of Richardson number in the upper parts of the forest canopy reveal that values just above \( z = d \) are typically several times larger than those at the canopy top \( (z = h) \), as demonstrated in Fig. 3. This is probably not an isolated result: in general, profiles of wind speed and temperature over aerodynamically rough surfaces all pass through a point of inflection at, or near, \( z = h \); if, also, they are roughly similar in shape below \( z = h \), as they are above, \( Ri \) must everywhere be approximately proportional to \( (\partial u/\partial z)^{-1} \), passing through a minimum close to \( z = h \) and increasing downwards as well as upwards from \( z = h \). It seems possible, therefore, that free convective thermals of characteristic dimension \( z_0 \) could originate within a vegetative canopy wherever \( Ri \) is relatively large (and negative) and emerge into the region of turbulent, boundary-layer flow above, where the additional mixing generated would serve to enhance \( K \) and reduce profile gradients, thereby increasing \( F \).

The existence of such plumes, with cross-sections of up to a few \( z_0 \), has been qualitatively demonstrated by the use of smoke trails (Oliver 1973). However, conditions at the time were at least moderately unstable (\( Ri \) probably \( < -0.1 \)), in contrast to the slightly unstable conditions \( (0 > Ri > -0.03) \) for which substantial effects are still implied by the large discrepancies then observed (Fig. 2). Moreover, Fig. 2 indicates not only that \( F \) exceeds unity at zero \( Ri \), having then a value of about 2.2 but also, that substantial enhancement of eddy diffusivity persists into the stable regime, to positive \( Ri \) values of 0.01 at least. The certain lack of free convection in this regime, coupled with the unlikelihood that free convection alone could produce the large observed discrepancies in the slightly unstable regime, tends to rule out thermal seeding as the overriding primary mechanism. The possibility that it occurs with substantially negative \( Ri \), however, cannot be ruled out: it may contribute to the large scatter of experimental points then observed.

(b) Wake Diffusion

Schlichting (1955, p. 504) reports that in the wake behind a row of heated bars, the temperature profile is 'wider' than the velocity profile to an extent implying that \( K_H \) there is roughly twice \( K_M \) (and comments that this supports G. I. Taylor's vorticity transfer (rather than momentum transfer) hypothesis). It is an attractive possibility, therefore, that in a region of limited depth (\( \zeta < \zeta_M \), say) over an aerodynamically rough surface, mixing generated by turbulent wakes behind individual roughness elements (here the
trees themselves) acts as an 'additional' diffusing mechanism, contributing more effectively to $K_H$ (and $K_V$) than to $K_M$, so that the profiles of temperature and humidity there are relatively less steep than the wind speed profile. It is necessary further to propose only that both the depth of the region affected by wake mixing and the absolute magnitude of the mixing effect itself be stability dependent, to produce a working empirical hypothesis to account for the discrepancy illustrated in Fig. 2, at least in the approximate range $0.02 > Ri > -0.12$.

The discrepancy between the Thetford (and other) results for $F$ and the currently accepted semi-empirical formulations can be expressed as

$$F_D = F / F_s$$

where, from Eqs. (6), (7) and (12),

$$F_s = (1 - 16Ri)^4 \quad Ri - ve$$

and

$$F_s = (1 - 5.2Ri)^2 \quad Ri + ve$$

are the 'smooth surface' values of $F$, appropriate to large enough values of $\zeta$. In general let us postulate that the 'wake diffusivity factor' $F_D$ be a function of $Ri$ and $\zeta$ such that $F_D > 1$ for $\zeta < \zeta_*$, the upward limit of the suggested wake mixing zone, itself a function of $Ri$, and that $F_D = 1$ for $\zeta \geq \zeta_*$. If we write

$$\zeta_* = \zeta_{*0} + ARi,$$

where $\zeta_{*0}$ is the value of $\zeta_*$ applicable to neutral stability, then the simplest empirical form of relationship is

$$F_D = 1 + B Ri + C(\zeta_{*0} - \zeta).$$

Values of $B$ and of $C(\zeta_{*0} - \zeta)$ can be found immediately from free-hand curves drawn through the experimental points in Fig. 2, as follows (in interpreting the data in Fig. 2
it is assumed that five regimes exist: LM for \( Ri > +0.14 \), gravity wave and radiative exchange processes \( (F \sim 0.07) \); MN for \( 0.14 \geq Ri \geq 0.02 \), the 'Webb regime'; NO for \( 0.02 \geq Ri > 0 \), damped wake mixing; OP for \( 0 > Ri \geq -0.12 \), amplified wake mixing; and PQ for \( Ri < -0.12 \), essentially free convection \( (F \sim 6.5) \):

(i) at \( Ri = 0 \), \( F_{D} = 2.2 \) so that \( C(\zeta_{*0} - \zeta) = 1.2 \) (for all \( Ri \) with the same \( \zeta \), namely \( \zeta_{R} = 9 \) here);

(ii) at \( Ri = +0.02 \), \( F_{D} = 1 \) so that \( B = -60 \) for \( 0 < Ri \leq 0.02 \); and

(iii) at \( Ri = -0.12 \), \( F_{D} = 2.92 \) (i.e. \( F/F_{s} = 6.5/2.23 \)), which gives \( B = -6 \) for \( 0 > Ri \geq -0.12 \).

To find values for \( A \) and \( C \) is not possible from the Thetford data alone, but representative values can be obtained by judging from previous publications that detectable effects from wake mixing (or whatever) do not exist beyond \( \zeta = 25 \), or so. Although proposed wind-tunnel investigations may show otherwise, let us assume, if only to acquire explicit relationships here, that such a value of \( \zeta \) is appropriate to neutral stability, i.e. that \( \zeta_{*0} = 25 \). As \( \zeta = \zeta_{R} = 9 \), it then follows that \( C = 0.075 \). Thus

\[
F_{D} = 1 - 60Ri + 0.075(\zeta_{*0} - \zeta) \quad 0 < Ri \leq 0.02
\]

and

\[
F_{D} = 1 - 6Ri + 0.075(\zeta_{*0} - \zeta) \quad 0 > Ri \geq -0.12
\]

Also, as \( A = B/C \),

\[
\zeta_{*} = \zeta_{*0} - 800Ri \quad 0 < Ri \leq 0.02
\]

and

\[
\zeta_{*} = \zeta_{*0} - 80Ri \quad 0 > Ri \geq -0.12
\]

These equations suggest, for example, that when \( Ri = -0.1 \), \( F_{D} = 2.8 \) and \( \zeta_{*} = 33 \), but that when \( Ri \) is positive and as small as 0.01, \( F_{D} \) and \( \zeta_{*} \) are reduced to 1.6 and 17, respectively. We emphasize, however, that the very effective damping down of the magnitude and vertical extent of wake mixing by relatively small degrees of boundary-layer stability thus implied, is in response to the observed distribution of the limited number of points in Fig. 2 and may require substantial future revision.

Mukammal et al. (1966) noted that aerodynamic estimates of evaporation from a corn field were close to lysimeter values when the corn was short at the beginning of the season, but fell to 40% of the measured values at the end of the season. However, the instruments used to obtain the aerodynamic estimates were fixed relative to the ground, so that their distance from the corn canopy, effectively expressed by the quantity \( (\zeta_{R} - d) \) decreased steadily as the seasonal growth progressed. Values of \( F/F_{s} \) deduced from the data tabulated by Mukammal et al., are plotted against the corresponding values of \( d \), and of \( (\zeta_{R} - d) \), in Fig. 4.

With the current wake diffusion postulate particularly in mind, it is perhaps justifiable to interpret the distribution of experimental points in Fig. 4 as indicating two regimes:

(i) \( (\zeta_{R} - d) \geq 270 \)cm, in which \( F/F_{s} \) is effectively constant at about 1.3, indicating consistent agreement with the lysimeter measurements, apart from systematic error; and

(ii) \( (\zeta_{R} - d) < 270 \)cm, where \( F/F_{s} \) rises steeply with decreasing \( (\zeta_{R} - d) \); as demonstrated by the two straight lines constructed on the figure. If the distance \( (\zeta_{R} - d) = 270 \)cm is taken to indicate the depth of the postulated wake mixing zone at the corresponding stage in the growth of the crop, when \( z_{0} \) was about 12cm, the implied value of \( \zeta_{*} \) is about 23 (for \( Ri \sim -0.02 \)), similar to the threshold assumed earlier.

Unfortunately, nothing is gained from the data in Fig. 4 by plotting \( F/F_{s} \) against \( \zeta \) itself: the tabulated values of \( z_{0} \) exhibit not only the usual amount of scatter but also a net downward trend with crop growth, which together serve to obscure any meaningful inter-
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Figure 4. Four-hourly mean values of \( F/F_s \) versus \( d \), and \( z_R - d \), for corn, from data obtained by Mukamal et al., during a 7-week period of growth \( (z_R = 360 \text{cm}) \). \( F \) is calculated by dividing lysimeter measurements of evaporation by uncorrected aerodynamic estimates (as from Eq. (11)); and \( F_s \) is from Eq. (17).

\( R_i \) was in the range \(-0.005 \) to \(-0.091 \). \( F_D \) here is equated to \( (F/F_s) \div 1.3 \).

relationship. Nevertheless, we have indicated on the right-hand ordinate axis on Fig. 4 a scale of \( F_D \) obtained by dividing the left-hand ordinate scale of \( F/F_s \) by the factor 1.3, treated as a systematic discrepancy. With this adjustment, rough agreement with the Thetford data is illustrated; e.g. from Fig. 4, \( F_D \approx 2 \) when \( d \approx 140 \text{cm} \) which corresponds to \( \zeta = 12 \), from Eq. (22), or to \( z_0 \approx 20 \text{cm} \). Perhaps the latter is a more realistic roughness parameter for the corn canopy at that particular stage of growth than the corresponding tabulated value of about 7cm. However, any such absolute interpretation of these data should be avoided; they are reproduced here merely as apparent qualitative support for the wake diffusion postulate.

6. CONCLUSION

Large discrepancies have been shown to exist between the aerodynamic and energy budget estimates of fluxes over Thetford Forest (and elsewhere). Except under moderately stable conditions, the ' \( F \)-factor ' is in substantial disagreement with equations currently in use. It seems that the aerodynamic method of flux determination cannot be used over rough surfaces where measurements have to be made within a relatively few \( z_0 \) of the tops of the roughness elements. If measurements over the forest were taken at higher levels better agreement might be found, but this approach is impracticable owing to the prohibitive amount of equipment required, and to likely difficulties in satisfying enhanced fetch requirements.

Although wake diffusion has been shown to be a likely possible cause of the observed discrepancy this is far from being proven and even farther from being properly quantified. We must therefore conclude that the complexity and uncertainty of obtaining reliable flux estimates over tall vegetation by the aerodynamic method preclude its use in a fully
independent context, although there is no related reason why such techniques should not be used in a purely empirical way to extend an experimental procedure based on the energy-balance cum Bowen-ratio approach. A similar conclusion is reached by Mukammal et al. (1966).

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