The effects of vertical wind shear on the evolution of convective clouds

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SUMMARY

The effects of ambient wind shear $U_0$ on the evolution of an isolated three-dimensional convective cloud are studied. Sixteen cases are considered which differ from one another both in the value of $U_0$ and in the initial value of the energy of atmospheric instability per unit height of the unstable layer, $E_0/\Delta H$. It is found that convective clouds may be grouped into two types, "weak" and "strong": if $E_0/\Delta H < 0.65 \times 10^5 \text{cm s}^{-2}$ then convection is "weak"; if $E_0/\Delta H > 0.65 \times 10^5 \text{cm s}^{-2}$ then convection is "strong". For "weak" convective clouds there is a critical value of shear, $U_{0cr}$. For $|U_0| < U_{0cr}$ there is an inhibiting effect on the development of convection; and for $|U_0| > U_{0cr}$ its evolution is completely suppressed. The essential new result is that "strong" convective clouds are intensified under the effect of $U_0$. In this case there is a resonance value of shear, $U_{0res}$, at which the degree of the intensification of convection has a maximum (30–40%).

1. INTRODUCTION

It is known from direct observation and from theory that the ambient wind, and especially its vertical shear, plays an important role in the dynamics of convective clouds (see, for instance, Browning and Ludlam 1962; Newton 1967; Mason 1969; Sulakvelidze et al. 1970). In particular, the prohibition, as well as the maintenance, effects of shear on the development of cumulus convection is widely accepted from both observation and theory. None the less, although both observational and theoretical data demonstrate the fact of shear influence on the development of convective clouds, neither include, as a rule, any specification of the meteorological conditions under which shear appears to change its influence. As a consequence it is still not yet quite clear what combinations of meteorological situations and values of wind shear are required for the existence, maintenance, prolongation, inhibition or intensification of convective systems. Neither is it clear whether or not it is the case that the stronger the vertical wind shear, the more favourable it is for the maintenance and development of a convective system.

The primary reason for this is apparently that comparatively little attention has been paid to investigating the joint influence of wind shear and energy of atmospheric instability on the development of convective clouds. One exception is a recent theoretical paper by Moncrieff and Green (1972) who have expressed the relative influence of these parameters on a steady convective overturning in terms of a Richardson number.

Another point is that practically all previous mathematical cumulus models have been two dimensional and no serious attempts towards the construction of a full three-dimensional model, including wind shear, have been made. (Recent references include Gutman 1969; Takeda 1969; Pastushkov and Shmeter 1972.)

The main purpose of the present study was to treat the joint effects of vertical ambient wind shear and initial energy of atmospheric instability on the development of an isolated three-dimensional large convective cloud. This is achieved by means of a numerical integration of the hydrodynamic and thermodynamic equations. Here it is important to point out that the model does not include the ice phase and hence more closely simulates warm cumulus congestus. It is also worth noting that the model under consideration is not fully three-dimensional in that the ambient shear is still taken to be two-dimensional.
\[ \mathbf{V} = iU + jV + kW \]

Vector of air velocity, \(i, j, k\), being along wind (\(x\)), cross wind (\(y\)) and vertical unit vectors

\[ U = U_0(z) + u, V, W \]

Components of air velocity, \(U_0\) being the ambient wind

\[ U_0 = dU_0/dz \]

Vertical shear of ambient wind

\[ t \]

Time

\[ \rho, p, q, \theta \]

Deviations of air density, pressure, specific moisture content (saturation specific humidity plus liquid water content) and 'virtual temperature'\(^*\) from initial reference values: \(\rho_0(z), P_0(z), Q_s(z), T_0(z)\)

\[ \Theta \]

Initial vertical average of 'virtual temperature'

\[ T_0(z) \]

'T Parcel curve' temperature, defined by dry adiabatic lapse rate from the ground up to the level of condensation and wet adiabatic above

\[ T_{00} = T_0(0) \]

Temperature of the ground

\[ q_s = Q_s - Q_0(z) \]

\(Q_s\) being the saturation specific humidity

\[ m = m_e + m_r \]

Specific liquid water content, \(m_e, m_r\) being the water content in cloud droplets and raindrops

\[ \Pi = R \Theta p/P_0 \]

Gravitational acceleration

\[ g \]

when \(q < q_s\)

\[ \alpha_1 = \Gamma - \Gamma_d \]

\[ = \Gamma - \Gamma_w \]

when \(q \geq q_s\), \(\Gamma_d, \Gamma_w\) being the dry and wet adiabatic lapse rates, and \(\Gamma = -dT_0/dz\)

\[ \alpha_2 = -dQ_0/dz \]

\[ \sigma = -\frac{1}{\rho_0} \frac{d\rho_0}{dz} \]

\[ W_r \]

Eddy diffusion coefficient

\[ K \]

Characteristic average scale of turbulence (mesh size)

\[ \Theta^2 \]

Square magnitude of the deformation tensor

\[ L \]

Latent heat of condensation

\[ R, R_e \]

Gas constants of dry air and water vapour

\[ \theta_0 \]

Initial temperature disturbance

\[ X_L, Y_W, Z_H \]

Length, width, height of the domain

\[ z = 0 \]

Level of the ground (lower boundary of unstable layer)

\[ \Delta H \]

Thickness of unstable layer (its upper boundary height)

\[ E_0 = g \int_0^{\Delta H} \frac{T(z) - T_0(z)}{T_0(z)} dz \]

Initial value of energy of atmospheric instability

\[ Ri \]

Richardson number

Angular brackets mean operator of specific mean field, for instance

\[
\frac{g}{\Theta} \left< W\theta \right> \equiv \frac{g}{\Theta} \int_{X_L}^{X_U} \int_{Y_W}^{Y_H} \int_{Z_L}^{Z_H} \rho_0 W\theta dx dy dz \]

\[
\int_{X_L}^{X_U} \int_{Y_W}^{Y_H} \int_{Z_L}^{Z_H} \rho_0 dx dy dz
\]

\(*\) 'Virtual temperature' deviation as defined here includes deviations of molecular temperature and all water substance. This accounts for the appearance of factors 1.51 and 0.61 in Eqs. (1), (4) and (6).
3. ** Governing equations. Boundary and initial conditions **

The model includes:
the three equations of motion
\[
\frac{\partial V}{\partial t} + (V \cdot V)V = -\nabla \Pi + \nabla K \nabla V + kg \left( \frac{\theta}{\Theta} - 1.61m \right)
\] (1)
the continuity equation
\[
\nabla \cdot V = \sigma W
\] (2)
the thermodynamic equation
\[
\frac{\partial \theta}{\partial t} + (V \cdot V)\theta = \alpha_1 W + \nabla \cdot K(\nabla \theta - \kappa \alpha_1)
\] (3)
the equation of thermal expansion of moist air
\[
\rho = -\frac{\rho_0}{\Theta} (\theta - 0.61 \Theta m)
\] (4)
the equation of conservation of all water substance
\[
\frac{\partial q}{\partial t} + (V \cdot V)q = \alpha_2 W + \nabla \cdot K(\nabla q - \kappa \alpha_2) + \frac{\partial}{\partial z} \left( W_r m_r \right)
\] (5)

Eqs. (1) to (5) are completed by relations for:
the determination of saturation specific humidity
\[
q_s = Q_0(z) \exp \left( \frac{L(\theta - 0.61 \Theta m)}{R_e \Theta^2} \right) - Q_0(z)
\] (6)
the liquid water content
\[
m = \begin{cases} 
0 & \text{in the unsaturated region (} q < q_s \text{)} \\
q - q_s & \text{in the saturated region (} q \geq q_s \text{)}
\end{cases}
\] (7)
the separation of liquid water into cloud water droplets and rain drops
\[
m_c = m \quad \text{when } m < 10^{-3}; \quad m_c = 10^{-3} \quad \text{when } m \geq 10^{-3}
\] (8)
the determination of the terminal fall velocity of raindrops relative to the air (in m s\(^{-1}\))
\[
W_r = \begin{cases} 
4.5 \times 10^4 m_r^4 & \text{when } m_r \leq 4 \times 10^{-3} \\
9 & \text{when } m_r > 4 \times 10^{-3}
\end{cases}
\] (9)
and the calculation of eddy diffusion coefficients* (m\(^2\)s\(^{-1}\))
\[
K = K_0 + h^2 |\mathcal{D}|
\]
\[
K_0 = 0.2 \times 10^{-43/3} h^{4/3}
\]
\[
|\mathcal{D}| = \nabla u \cdot \nabla u + \nabla V \cdot \nabla V + \nabla W \cdot \nabla W
\] (10)

* Forms of relations (8) and (9) are after Takeda (1969); and form of (10) after Lilly (1962) and Smagorinsky (1963). The assumptions adopted in formulating the system of basic equations are well known and have been discussed elsewhere (Asai 1964; Gutman 1969; Takeda 1969).
The following boundary and initial conditions have been adopted:

at \( x = 0 \) and \( X_L \):
\[ U = U_0(z); \ V, W, \theta, q, \frac{\partial \Pi}{\partial x} = 0 \]

at \( y = 0 \):
\[ U = U_0(z); \ V, W, \theta, q, \frac{\partial \Pi}{\partial y} = 0 \]

at \( y = \frac{1}{4} Y_w \):
\[ \frac{\partial U}{\partial y}, V, \frac{\partial W}{\partial y}, \frac{\partial \theta}{\partial y}, \frac{\partial q}{\partial y}, \frac{\partial \Pi}{\partial y} = 0 \]

at \( z = 0 \) and \( Z_H \):
\[ U = U_{0|z = 0, Z_H}; \ V = W = 0 \]
\[ \frac{\partial \theta}{\partial z} = \alpha_1 \bigg|_{z = 0, Z_H}; \ \frac{\partial q}{\partial z} = \alpha_2 \bigg|_{z = 0, Z_H} \]
\[ \frac{\partial \Pi}{\partial z} = \frac{g}{\Theta} \left( \frac{\Theta}{\Theta} - 1 \cdot 61m \right) \bigg|_{z = 0, Z_H} \]

at \( t = 0 \):
\[ U = U_0(z); \ V, W, q = 0; \ \theta = \theta_0(x, y, z) \]

(11)

(12)

4. COMPUTATIONAL SCHEME

The dimensions of the domain under consideration are a height of 12km, a width of 14km and a length of 22km. The co-ordinate system is moving with the speed of the ambient flow at the 1.5km level. Mesh sizes, \( h \), are 1km in all directions; the time interval is 20 seconds. The initially disturbed region is much larger than a thermal and corresponds more closely to a cumulus cloud. The value of the initial temperature disturbance is taken to be 1 degC, in an initial volume of 1km³. The height of the base of the initial disturbance as well as the width and depth is assumed to be 1km.

The computational scheme is conservative and of second-order accuracy (for details see the appendix and Pastushkov and Shmert 1972). Calculations were terminated 30 to 60 minutes after the initialization of convection.

5. RESULTS OF NUMERICAL EXPERIMENTS

Four series of calculations were performed, comprising sixteen runs with different values for the ambient wind shear, \( U_0^\prime \); initial value of energy of atmospheric instability, \( E_0 \); height of unstable layer, \( \Delta H \) (or energy of instability per unit of \( \Delta H \)); temperature of the ground, \( T_{0g} \); and time of termination of simulation. The values taken for these quantities are summarized in Table 1.

| Table 1. Main Characteristics of the Atmosphere Used for Simulation of Convective Clouds |
|-----------------------------------------|-----|-----|-----|-----|
| Number of series | I   | II  | III | IV  |
| Number of experiment | 40  | 41  | 42  | 43  |
| \( U_0^\prime (m \text{ s}^{-1} \text{km}^{-1}) \) | 0   | 2   | 4   | 6   |
| Simulated time of termination (min) | 40  | 40  | 30  | 30  |
| \( E_0 (\text{cm}^2 \text{ s}^{-2}) \) | 2.8 \times 10^7 | 1.7 \times 10^7 | 1.9 \times 10^7 | 1.5 \times 10^7 |
| \( E_0/\Delta H (\text{cm} \text{ s}^{-2}) \) | 0.8 \times 10^2 | 0.7 \times 10^2 | 0.55 \times 10^2 | 0.6 \times 10^2 |
| \( T_{0g} (\text{°C}) \) | 25  | 15  | 25  | 20  |
| Type of convection | 'Strong' | 'Weak' |
In each series the values of $E_0$ and $\Delta H$ (and hence $E_0/\Delta H$) were constant and the value of $U_0$ varied from 0 to 3 or 6 m s\(^{-1}\) km\(^{-1}\).

It is found that as far as the effects of $U_0'$ and $E_0/\Delta H$ are concerned, all cases of simulated convection may be grouped into two types. As can be seen from Table 1, convection with $E_0/\Delta H < 0.65 \times 10^2$ cm s\(^{-2}\) has been classified as 'weak' and all cases with $E_0/\Delta H \geq 0.65 \times 10^2$ cm s\(^{-2}\) as 'strong'.

An example of the development of a simulated convective cloud may be seen in Fig. 1.

![Figure 1. Patterns of cumulus convection in the longitudinal plane of symmetry of the cloud. Solid lines: liquid water content (g kg\(^{-1}\)); arrows: velocity vectors (m s\(^{-1}\)); chain lines: deviations of $\Pi = R \Theta \frac{p}{P(r)}$ from its horizontal average values (m\(^2\) s\(^{-2}\)).](image)

Fig. 2 shows the initial stage of the time variations of the rates of conversion of the potential energy of moist air into kinetic energy. As seen from this figure, for 'weak' convective clouds there exist critical values of shear, $U_0'$, virtually independent of the values of $E_0$, $E_0/\Delta H$ and $T_0$:

- for $|U_0'| < U_0'$, there is, in agreement with previous numerical studies of other authors, an inhibiting effect of $U_0'$ on the development of 'weak' convective cloud;
- for $|U_0'| \geq U_0'$, its development is completely suppressed.

To be sure that convection would not regenerate, the calculations, as can be seen from Table 1, were carried on for rather longer than the time for complete dissipation of 'weak' convective clouds under the effect of shear.

On the other hand, as seen from the upper part of Fig. 2, there is no evidence for the existence of a critical value of shear for 'strong' convection. These results imply that the influence of vertical shear on a vigorous ('strong') convective cloud may not be the same as on 'weak' cumulus convection. This has been repeatedly emphasized in the literature.

The essential new result of the present study is that the development of 'strong' convective clouds is intensified under the effects of shear. This is made manifest first of all by the formation of new convective cells on the down-shear side of the cloud (Fig. 3; shaded area) and subsequently in the increase of kinetic energy of vertical motions and rate of various energy transformations.
Here it is worth remarking that the assumed lateral boundary conditions, Eq. (11), imply reflection of gravity waves. Hence the question can be raised: could this reflection be a contributory factor in the development of the new convective cells? A precise answer to this requires special investigations. Nevertheless, we may hope that there is no such influence in the present model, basing our hope on the smoothness of the calculated fields of motion near the boundaries, under the effects of turbulence; and the sufficiently large distance of the boundaries from the core of the simulated convective cloud.

The upper part of Fig. 4 shows the rates of conversion of latent heat into potential energy \((g/\Theta)(a_1 W^2)\) and of potential energy into kinetic energy \((g/\Theta)(W\theta)\) for 'strong' convection. For comparison, the lower part of Fig. 4 demonstrates analogous characteristic for 'weak' convection. The most remarkable result is that for 'strong' convective clouds there is a 'resonance' value of shear, \(U_{\text{crest}}\), which also depends on the other model parameters, at which the degree of intensification of convection has its maximum (30 to 40%). This is most clearly illustrated by Fig. 5, which shows time variations of the rates of conversion of kinetic energy of vertical motions into kinetic energy of horizontal motions,
Figure 3. Vertical (solid lines, m s$^{-1}$) and horizontal (arrows) components of velocity in the zone of development of 'strong' convection (exp. No. 41, $E_0/\Delta H = 0.8 \times 10^6$ cm s$^{-2}$, $T_{00} = 25^\circ$C, $U_0' = 2$ m s$^{-1}$ km$^{-1}$). Shaded area: cross-section of a new convective cell on the down-shear side of the cloud.

$\langle W \partial \Pi / \partial z \rangle$. We show again, for comparison, values of $\langle W \partial \Pi / \partial z \rangle$ for 'weak' convection.

It may be seen from this and previous figures that, for reasonable values of $E_0$, $E_0/\Delta H$ and $T_{00}$, the values of $U_{0,cr}$ and $U_{0,cr}^*$ are close to 2–3 m s$^{-1}$ km$^{-1}$. 
Figure 4. Time variation of the rates of conversion of the latent heat of condensation into potential energy of the air (left) and potential energy into kinetic (right) for various values of ambient wind shear.

6. CONCLUDING REMARKS

It has been found that ambient wind shear can not only inhibit the development of convective clouds and prolong their life cycles, as has been shown by a number of authors (see, for instance, Asai 1964 and Takeda 1969) but can intensify convective clouds. All these features are in good qualitative agreement with observations mentioned in the introduction.

In conclusion it is worth pointing out that the present results do not completely agree with those of Moncrieff and Green (1972), who have shown that steady persistent two-dimensional convection can occur at sufficiently small values of the Richardson number. This, in the terminology of the present paper, implies that, for any given value of energy of instability, the development of persistent steady convection requires a sufficiently large value of shear. For instance, for $E_0 = 2.8 \times 10^7 \text{cm}^2 \text{s}^{-2}$, and $E_0/\Delta H = 0.8 \times 10^2 \text{cm s}^{-2}$
"STRONG" CONVECTION

\[
E_\Delta H = 0.8 \times 10^2 \text{ cm s}^{-2}, T_w = 25^\circ \text{C}
\]

\[
E_\Delta H = 0.7 \times 10^2 \text{ cm s}^{-2}, T_w = 15^\circ \text{C}
\]

*\(U_0^*\)*
- \(\bigcirc = 0\)
- \(\bigcirc = 2\)
- \(\bigcirc = 2.5\)
- \(\bigcirc = 4 \text{ m s}^{-1} \text{ km}^{-1}\)

"WEAK" CONVECTION

\[
E_\Delta H = 0.55 \times 10^2 \text{ cm s}^{-2}, T_w = 25^\circ \text{C}
\]

\[
E_\Delta H = 0.6 \times 10^2 \text{ cm s}^{-2}, T_w = 20^\circ \text{C}
\]

Figure 5. Time variation of the rates of conversion of the kinetic energy of vertical motions into kinetic energy of horizontal motions for various values of ambient wind shear.

(first simulation series, 'strong' convection) the development of convection according to Moncrieff and Green's results will be steady \(( Ri \ll 1)\) at \(U_0^* \geq 15 \text{ m s}^{-1} \text{ km}^{-1}\).

Nevertheless, both the present results and numerous results of other authors testify the essential non-steady, or quasi-steady, development of convective clouds especially under the presence of vertical wind shear. Besides this, the quantitative disagreement between the present results and those of Moncrieff and Green may be accounted for by other factors: different treatment of the latent heat of condensation; different treatment of conversion of kinetic energy of mean motions into kinetic energy of turbulence; different geometry of the region under consideration - two-dimensional in the Moncrieff and Green paper and three-dimensional in the present one.

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APPENDIX

The set of equations (1) to (12) was solved by the method of splitting with the use of a conservative corrector.

In the first half of the time step, $\frac{1}{2}\Delta t$, with corresponding boundary and initial conditions, the non-divergent set of split equations (predictor) was solved:

\[
\frac{\partial \phi_n^{m+1/5}}{\partial t} + W_m \frac{\partial \phi_n^{m+1/5}}{\partial z} = \frac{\partial}{\partial z} K_m \frac{\partial \phi_n^{m+1/5}}{\partial z} \tag{1A}
\]

\[
\frac{\partial \phi_n^{m+2/5}}{\partial t} + V_m \frac{\partial \phi_n^{m+2/5}}{\partial y} = \frac{\partial}{\partial y} K_m \frac{\partial \phi_n^{m+2/5}}{\partial y} \tag{2A}
\]

\[
\frac{\partial \phi_n^{m+3/5}}{\partial t} + U_m \frac{\partial \phi_n^{m+3/5}}{\partial x} = \frac{\partial}{\partial x} K_m \frac{\partial \phi_n^{m+3/5}}{\partial x} \tag{3A}
\]

where

$\phi_n^m \equiv (U, V, W, \theta, q)^m$

and $m$ denotes the time step.

The divergent corrector then follows:

$\phi_n^{m+4/5} = \phi_n^m - \left( \frac{\partial}{\partial x} (U \phi_n)^m + 3/5 \phi_n^{m+1/5} + \frac{\partial}{\partial y} (V \phi_n)^m + 3/5 \phi_n^{m+2/5} + \frac{\partial}{\partial z} (W \phi_n)^m + 3/5 \phi_n^{m+3/5} \right)$
EVOLUTION OF CONVECTIVE CLOUDS

\[- \frac{\partial}{\partial x} K^m \frac{\partial \phi^m + 3/5}{\partial x} - \frac{\partial}{\partial y} K^m \frac{\partial \phi^m + 3/5}{\partial y} - \frac{\partial}{\partial z} K^m \frac{\partial \phi^m + 3/5}{\partial z} \right \} \Delta t \]

Finally the set which describes the adaption of \(\phi^m + 4/5\) fields was solved.

\[U^{m+1} = U^m + 4/5 - \frac{\partial \Pi^m + 1}{\partial x} \Delta t \]

\[V^{m+1} = V^m + 4/5 - \frac{\partial \Pi^m + 1}{\partial y} \Delta t \]

\[W^{m+1} - \frac{\Delta t}{\Theta} \theta^{m+1} = W^m + 4/5 - \frac{\partial \Pi^m + 1}{\partial z} \Delta t - 1.61 gm^m \Delta t \]

\[-\alpha^m \Delta t W^{m+1} + \theta^m = \theta^m + 4/5 \]

\[-\alpha^m \Delta t W^{m+1} + q^{m+1} = q^m + 4/5 + \frac{\partial}{\partial z} (Wm^m)^m \Delta t \]

\[\frac{\partial U^{m+1}}{\partial x} + \frac{\partial V^{m+1}}{\partial y} + \frac{\partial W^{m+1}}{\partial z} = \sigma W^{m+1} \]

This set is equivalent to:

\[\frac{\partial^2 \Pi^m + 1}{\partial x^2} + \frac{\partial^2 \Pi^m + 1}{\partial y^2} + \left( \frac{\partial}{\partial z} - \sigma \right) \frac{1}{\delta^m} \frac{\partial \Pi^m + 1}{\partial z} = F^{m+1} \]

\[F^{m+1} = \frac{1}{\Delta t} \left[ \frac{\partial U^m + 4/5}{\partial x} + \frac{\partial V^m + 4/5}{\partial y} + \left( \frac{\partial}{\partial z} - \sigma \right) \frac{1}{\delta^m} \left( W^m + 4/5 - \right. \right. \]

\[- 1.61 gm^m \Delta t + \frac{\Delta t}{\Theta} \theta^m + q^{m+1} \left. \right] \]

\[\delta^m = 1 - \frac{\Delta t^2}{\Theta} \alpha^m \]

together with equations for \(U^{m+1}, V^{m+1}, W^{m+1}, \theta^{m+1}\) and \(q^{m+1}\) which are readily derived from Eqs. (5A) to (9A).

Thus Eqs. (1) to (12) are reduced to the successive solving of Eqs. (1A) to (3A), (4A), (11A) with (12A), (13A) and equations for \(U^{m+1}, V^{m+1}, W^{m+1}, \theta^{m+1}\) and \(q^{m+1}\).