The use of a multipoint filter as a dissipative mechanism in a numerical model of the general circulation of the atmosphere

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SUMMARY

A multipoint filter for use as a dissipative mechanism in a numerical model is formulated, and some of its properties discussed. The results of three numerical experiments are compared, of which one employs a non-linear diffusion term, another has the filter applied in the zonal direction only, and the third employs the filter in both zonal and meridional directions. The simulated climatology of the filtered experiments is seen to be significantly better in some respects than that of the experiment using a diffusion term.

1. INTRODUCTION

The representation of sub-grid scale atmospheric dissipation processes has been a major problem for numerical modelling groups ever since major efforts were first exerted in this field. The effect of sub-grid scale motions in the implicit flow can be represented by Reynolds stresses, which have to be evaluated in terms of model variables. The simplest assumptions lead to a linear diffusion-type term, with constant coefficients, while assumptions involving the spectrum of the eddies lead to more complicated non-linear dissipation terms (Smagorinsky 1963; Leith 1969). When put in terms of finite differences which have to be evaluated numerically on a grid, dissipative processes expressed in these forms are not sufficiently selective with respect to wavelength, with the result that energy is removed from scales which are important in the development of the general circulation. Gilchrist et al. (1973) showed that with the non-linear diffusion used in their model significant amounts of energy were removed from the ultra-long waves. A multipoint filtering process described by Shapiro (1971) has the property of being highly selective with respect to wavelength. A short discussion of the properties of the filter is given in the following section. The filter has been used in several numerical experiments and some of the results are discussed in later sections. Initially the filter was applied only in the zonal direction because its application in this case is simple and straightforward. Before applying the filter to the meridional direction decisions had to be made on how to deal with pole and equator, and for this particular general circulation model how to define the filter to operate on a staggered grid.

In later sections we shall compare the atmospheric circulations as produced in three separate experiments. Experiment I is similar to that of Gilchrist et al., having a diffusion term to effect dissipation. Experiment II has a diffusion term applied in the meridional direction and a multipoint filter in the zonal direction, while experiment III employs the filter in both the zonal and meridional directions. The experiments start from day 0 (real initial conditions) and are run to day 80. Most of the comparisons made between the experiments use the mean of the last 40 days of each experiment.

2. THE FILTERING SCHEME

Consider the filtering operator (one dimensional)

$$1 + \frac{1}{4} F^2$$

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where $F^2$ denotes the finite difference operation $\theta_+ - 2\theta_0 + \theta_-$ on a variable $\theta$ held at discrete equally spaced grid points. The symbolic operation

$$\theta' = (1 + \frac{1}{4}F^2)\theta$$

represents a smoothing operation that completely damps any 2 grid-length wave component of $\theta$, and also damps to a lesser degree any longer wavelength components. This is best illustrated by writing

$$\theta_m = A \cos(km\Delta x)$$

where $k$ is the wave number, $\Delta x$ the grid length and $m$ a designated grid point label. We then have

$$\theta'_m = (1 + \frac{1}{4}F^2)\theta_m = \theta_m + \frac{1}{4}(\theta_{m-1} - 2\theta_m + \theta_{m+1}) = A \cos(km\Delta x) \cdot D_k$$

where $D_k$ is a damping coefficient given by $D_k = 1 - \sin^2\left(\frac{k\Delta x}{2}\right)$.

For 2 grid-length waves $D_k = 0$ and for very long waves $D_k \rightarrow 1$. This simple filter is not selective enough at the shortwave end of the spectrum since for the 4 grid-length wave $D_k = 0.5$, i.e. an appreciable amplitude reduction results from one application. We now write symbolically

$$\theta = \left(1 + \frac{1}{4}F^2\right)^{-1} \theta' = \left(1 + \frac{1}{4}F^2\right)^{-1} \left(1 + \frac{1}{4}F^2\right) \theta$$

$$= \left(1 + \frac{1}{4}F^2\right)\left(1 + \frac{1}{4}F^2\right)^{-1} \theta' = \left(1 + \frac{1}{4}F^2\right)\left(1 - \frac{1}{2^2}F^2 + \frac{1}{2^4}F^4, \ldots\right) \theta$$

where $F^4 = F^2 \cdot F^2$ and denotes successive applications of the simple operator. The first term of the product on the r.h.s. of the above equation represents the simple filter already mentioned. The second term represents a series by means of which restoration of amplitude can be achieved by the remainder of the spectrum i.e. other than the 2 grid-length wave. As more terms are included in the series more points need to be included in the filter, and a more and more selective dissipation profile is obtained. The first few terms in the series are given by

3 point filter \hspace{1cm} $1 + \frac{1}{2^2}F^2$

5 point filter \hspace{1cm} $\left(1 + \frac{1}{2^2}F^2\right)\left(1 - \frac{1}{2^2}F^2\right) = 1 - \left(\frac{1}{2^2}F^2\right)^2$

7 point filter \hspace{1cm} $\left(1 + \frac{1}{2^2}F^2\right)\left(1 - \frac{1}{2^2}F^2 + \frac{1}{2^4}F^4\right) = 1 - \left(\frac{1}{2^2}F^2\right)^3$

In the experiments described in this paper we have employed a 17 point filter of the form $1 - \left(\frac{1}{2^2}F^2\right)^8$. The damping factor for a filter using $2n + 1$ points is easily shown to be

$$D_k^n = 1 - \sin^2n\left(\frac{k\Delta x}{2}\right).$$

Using this formula we have derived a few numerical values for various waves and various filters.
A MULTIPOINT FILTER

TABLE 1. VALUES OF $D_n$ FOR VARIOUS WAVELENGTHS AND FILTERS

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>$n = 1$</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3\Delta x$</td>
<td>0.25</td>
<td>0.4375</td>
<td>0.6836</td>
<td>0.90</td>
</tr>
<tr>
<td>$4\Delta x$</td>
<td>0.50</td>
<td>0.75</td>
<td>0.9375</td>
<td>0.9962</td>
</tr>
<tr>
<td>$6\Delta x$</td>
<td>0.75</td>
<td>0.9375</td>
<td>0.9962</td>
<td>1.0</td>
</tr>
</tbody>
</table>

A useful statistic is the ratio between the dissipation $(1 - D_n)$ for the 2 grid-length and 4 grid-length waves. This gives an indication of how steeply the dissipation profile changes. The ratio varies from 2 for a 3 point filter ($n = 1$) up to greater than 300 for a 17 point filter ($n = 8$). Another way to examine the effect of the filter is to evaluate the time required to reduce the amplitude of a wave by a factor of $e$ from its initial value. Assuming one application of the filter every 10 min we have the results given in Table 2.

TABLE 2. $e$-FOLDING TIMES ASSUMING ONE FILTER APPLICATION EVERY 10 MIN

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>$n = 1$</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3\Delta x$</td>
<td>7 min</td>
<td>12 min</td>
<td>26 min</td>
<td>96 min</td>
</tr>
<tr>
<td>$4\Delta x$</td>
<td>14 min</td>
<td>35 min</td>
<td>2.6 hours</td>
<td>43.2 hours</td>
</tr>
<tr>
<td>$5\Delta x$</td>
<td>24 min</td>
<td>1.3 hours</td>
<td>0.5 days</td>
<td>34.2 days</td>
</tr>
<tr>
<td>$6\Delta x$</td>
<td>35 min</td>
<td>2.6 hours</td>
<td>1.8 days</td>
<td>45.1 days</td>
</tr>
</tbody>
</table>

As can be seen from these figures, using a 17 point filter effectively limits the range of dissipation to 2-4 grid-length waves. With the resolution of the model being used set at 300 km, baroclinic waves with a real scale of 2-3 \times 10^3 km are represented by scales down to 5 or 6 grid-lengths. The selective dissipation profile of the filter is seen to be very critical when the development of such waves is required in a realistic simulation.

So far we have been considering a one-dimensional filter; according to Shapiro (1971) the extension to two dimensions is obtained simply by applying the one dimensional filter separately in both directions. Writing the filter symbolically as $1 - \left(\frac{1}{2} F^2\right)^8$, it is seen to be equivalent to 8 applications of the finite difference operator $\frac{1}{2} F^2$, subtracted from the initial field. We make use of this equivalence when applying the filter in the meridional direction. There are several possible methods of using the filter in the meridional direction on a grid superposed on a sphere. The one chosen for use here was selected because it conserves area-weighted properties of the flow and also offers a solution to the problem of using the filter near the pole. The operator $F^2$ in the meridional direction is written

$$F^2 \phi = \frac{\phi_+ - \phi_0}{\cos \lambda_0} \cos \lambda_+ - \frac{\phi_0 - \phi_-}{\cos \lambda_0} \cos \lambda_-$$

instead of the obvious form suggested by the zonal operator. At the polar row, $\lambda_+ = 90^\circ$, hence the term $\frac{\phi_+ - \phi_0}{\cos \lambda_+}$ vanishes and there is no problem in applying the operator for the required number of times. This formulation for $F^2$ brings it into line with the finite difference form of a (meridional) second spatial differential operator on a sphere. In order to apply the filter near the equator we have made the following assumptions:

$$u_3 = u_N; T_S = T_N; q_S = q_N; v_S = - v_N$$
where the subscripts $S$ and $N$ denote values south and north of the equator respectively, and $u$, $T$, $q$ and $v$ denote zonal wind, temperature, humidity mixing ratio and meridional wind. The surface pressure $P_s$ is not filtered, since earlier experiments in which $P_s$ was filtered showed large anomalous waves caused by inconsistencies between $P_s$ and the topographic height $\Phi_s$. See also Holloway, Spelman and Manabe (1973). The filter is applied to the central time level field after the physical adjustments (radiation, convection etc.) have been carried out.

3. THE MODEL

The model used in this investigation is essentially that described in Corby et al. (1972), but using a different grid and time stepping procedure. The horizontal grid is designed to give quasi-uniform resolution over the hemisphere. If $\theta$ denotes latitude and $\Delta \theta$ the latitudinal spacing then the number of rows from equator to pole is given by $N = \frac{\pi}{2\Delta \theta}$.

The rows are positioned at $\frac{1}{2} \Delta \theta$, $\frac{3}{2} \Delta \theta$, ... $\left(\frac{\pi}{2} - \frac{1}{2} \Delta \theta\right)$. Each row has the integral part of $4N \cos \theta$ points, corresponding to $4N$ at the equatorial rows. With this scheme the last three rows near the pole would have 16, 9 and 3 points respectively. In view of the need for an acceptable resolution near the pole (Shuman 1970; Grimmer and Shaw 1967; Dey 1969; Rao and Umscheid 1969) 16 points are placed on all of the last three rows. In this model there are $N = 30$ rows, i.e. $\Delta \theta = 3^\circ$, corresponding to a lateral grid spacing of about 330km in the main portions of the grid.

A ten-minute 'leap-frog' time step is used throughout the grid except near the poles where the spatial resolution is increased. To deal with this zone a variable time step is employed with a suitable interpolation in time. In order to control the computational mode a weak time-filter is applied after every time step. The filter takes the form

$$\bar{\theta}(n\Delta t) = \theta(n\Delta t) + 0.005\{\bar{\theta}(n-1\Delta t) - 2\theta(n\Delta t) + \theta(n+1\Delta t)\}$$

where $\bar{\theta}$ denotes a time smoothed value.

It is perhaps necessary here to reiterate the form of the diffusion term used in experiment I. As in Corby et al. we write

$$D_\phi = K_\phi \{\nabla \cdot P_s \nabla \phi\}$$

where the effective non-linear eddy viscosity coefficient $K_\phi$ is proportional to the modulus of the term within parenthesis. This term is evaluated using the finite difference approximations formulated in Corby et al., a forward time step being used in order to maintain computational stability. The staggered grid necessitates a special treatment of the meridional differencing (Grimmer and Shaw 1967); the fractional overlap of grid 'boxes' has to be evaluated in order to ensure the conservation properties that are required. This technique is also used when applying the filter in the meridional direction.

The model is programmed using FORTRAN and ASSEMBLER language and run on an IBM 360/195 at the Meteorological Office. Both experiments I and II require 7 minutes per day of simulated time, indicating that the use of the filter in the zonal direction is no more expensive in computing time than the more usual diffusion term; a carefully written computer programme uses 17 additions and 1 multiplication per point per application. Because of the staggered grid the meridional differencing is more expensive and the same economic programming technique cannot be used; hence the increased amount of meridional differencing required by the filter brings the required time up to 10 minutes per
A MULTIPPOSITE FILTER

day for experiment III. Another computing requirement for experiment III is the extra
core storage needed in order to accommodate the 17 latitude rows required for the application
of the meridional filter.

4. ENERGETICS OF THE EXPERIMENTS

The presentation of available potential energy values and the rates of conversion
between potential and kinetic energy is only meaningful when given on pressure levels.
Accordingly the technique employed in Gilchrist, Corby and Newson is adopted for transforming
energy values from sigma levels. For temperature the interpolation assumed a constant lapse rate with height between the sigma levels, but for other variables the interpolation was simply linear in pressure. The averages on pressure levels were taken over
points where values existed (because of topography some values were missing). The following
symbols are used:

[] denotes a zonal average, a function of pressure and latitude; [T^H] denotes a hemispheric
average, a function of pressure only. The stability factor γ which occurs in the expression
for the available potential energy was evaluated for pressure levels 900, 700, 500, 300 and
100mb from

\[ \gamma = \frac{R}{P} \frac{\Delta T^H}{\Delta P} - \frac{K[T^H]^P}{P} \]

using a centered difference and \( \Delta P = 400\)mb for the middle three levels, and a one-sided
difference with \( \Delta P = 200\)mb for the top and bottom levels of the atmosphere. The suffix \( \rho \)
denotes interpolated values. The zonal available potential energy per unit mass was then
taken as

\[ \frac{1}{2} \gamma \{(T^H_T) - (T^H_T)^2 \} \]

and the eddy available potential energy becomes

\[ \frac{1}{2} \gamma \{(T^H_T - [T^H_T])^2 \} \]

The kinetic energy of the zonal mean flow was taken as

\[ \frac{1}{2}(u^2 + v^2) \] per unit mass.

To deal with the kinetic energy of the eddies, the kinetic energies of sigma layers at each
point were first calculated and the values attributed to the 200 mb pressure layers proportionately according to the overlap in pressure between the pressure and sigma layers. The small
amounts of kinetic energy below 1000mb were neglected in this process. The eddy kinetic
energy was then taken as the difference between the total kinetic energy and the kinetic
energy of the zonal mean flow. The transformations of zonal available and eddy available
potential energy were calculated from

\[ - R \{(T^H_T) - (T^H_T)^2 \} \{(w^2) - (w^2)^2 /P \} \]

and

\[ - R \{(T^H_T - [T^H_T]) \{w^2 - [w^2]^2 \}/P \} \] respectively.

The evolution of the hemispherically averaged zonal kinetic and eddy kinetic energy,
over the period 40 to 80 days, for each experiment is shown in Fig. 1(a). The rise in the
level of zonal kinetic energy in experiments II and III is checked at day 40 by an increase
of baroclinic instability in a stronger meridional temperature gradient than existed in the
initial conditions. Because of the stronger damping in experiment I the baroclinic gradient
is not strong enough at day 40 and further differential cooling has to occur (with an
associated increase of zonal velocity) before the level of eddy activity can increase. The
Figure 1. Evolution of kinetic energy.

differences between the mean value of zonal kinetic energy in the experiments occurs mainly in the upper levels. This is illustrated in Table 3 which shows mean values of zonal kinetic energy, at all levels, over the period 41 to 80 days; i.e. values of \( \frac{1}{2}\{[\bar{u}]^2 + [\bar{\varphi}]^2\} \) where the over-bar denotes averaging in time.

**TABLE 3. LONG PERIOD HEMISPHERIC MEAN ZONAL KINETIC ENERGY VALUES, DAYS 41–80 (10^{-3} J g^{-1})**

<table>
<thead>
<tr>
<th>Level (mb)</th>
<th>January mean</th>
<th>Exp. I</th>
<th>Exp. II</th>
<th>Exp. III</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>7</td>
<td>7</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>700</td>
<td>20</td>
<td>27</td>
<td>26</td>
<td>23</td>
</tr>
<tr>
<td>500</td>
<td>66</td>
<td>85</td>
<td>78</td>
<td>72</td>
</tr>
<tr>
<td>300</td>
<td>180</td>
<td>222</td>
<td>200</td>
<td>193</td>
</tr>
<tr>
<td>100</td>
<td>117</td>
<td>288</td>
<td>240</td>
<td>234</td>
</tr>
</tbody>
</table>

January figures are extracted from Oort and Rasmusson (1971). The figures in Table 3 highlight the differences between the zonal kinetic energy values of the experiments in their upper levels. While all three experiments have excessive zonal kinetic energy levels at 100mb, experiment III has 19% less than experiment I.

At day 40 of each experiment the hemispheric average of eddy kinetic energy is just less than \( 50 \times 10^{-3} J g^{-1} \) (Fig. 1(b)). The experiments show different evolutions from then on. Experiment I maintains a level of about \( 40 \times 10^{-3} J g^{-1} \) until day 60 and by day 80 has risen to \( 60 \times 10^{-3} J g^{-1} \). Experiment II exhibits a long period oscillation (about 15–20 days) about a mean of \( 60 \times 10^{-3} J g^{-1} \) while experiment III rises up to \( 90 \times 10^{-3} J g^{-1} \).
TABLE 4. Long period hemispheric mean eddy kinetic energy values, days 41–80 (10^{-3}J g^{-1})

<table>
<thead>
<tr>
<th>Level (mb)</th>
<th>January mean</th>
<th>Exp. I</th>
<th>Exp. II</th>
<th>Exp. III</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>43</td>
<td>30</td>
<td>32</td>
<td>38</td>
</tr>
<tr>
<td>700</td>
<td>60</td>
<td>34</td>
<td>41</td>
<td>53</td>
</tr>
<tr>
<td>500</td>
<td>112</td>
<td>50</td>
<td>64</td>
<td>84</td>
</tr>
<tr>
<td>300</td>
<td>205</td>
<td>75</td>
<td>96</td>
<td>130</td>
</tr>
<tr>
<td>100</td>
<td>99</td>
<td>49</td>
<td>69</td>
<td>87</td>
</tr>
</tbody>
</table>

The values for January are taken from Oort and Rasmusson.

by day 60 and then oscillates about 80 × 10^{-3} J g^{-1}. In Table 4 is shown the eddy kinetic energy meaned over the period 41–80 days.

The figures in Table 4 show that at all levels the filtered experiments have more eddy kinetic energy than the experiment using a diffusion term. This is especially so at the topmost levels where experiment III has >70% more eddy kinetic energy than experiment I. The value at 300mb is the most deviant from the observed January mean figure. During the course of experiment III the ratio of eddy kinetic energy to zonal kinetic energy is much higher than in the other experiments. At day 62 this ratio approaches unity when the values of eddy and zonal kinetic energy are 94.7 and 96.3 × 10^{-3} J g^{-1} respectively.

Figure 2. Vertically integrated eddy kinetic energy, 41–80 day mean.

Fig. 2 shows the latitudinal distribution of zonally meaned vertically integrated eddy kinetic energy for the three experiments, and also mean January data taken from Oort and Rasmusson. The differences between the experimental results are clearly emphasized, the filtering experiments having more eddy kinetic energy at all latitudes. The maximum value for experiment III at 34\textdegree N is 240 × 10^{-3} J g^{-1} at 300mb. The maxima for experiments I and II are 128 and 161 × 10^{-3} J g^{-1} respectively, while Oort and Rasmusson indicate a value of 328 × 10^{-3} J g^{-1}. The result of experiment II is similar to that obtained in a higher resolution model of Manabe, Smagorinsky, Holloway and Stone (1970).

The mean eddy kinetic energy distribution on sigma levels was Fourier analysed for the period 41–80 days, and the results of the analysis are shown for different wave bands in Table 5. The ultra-long waves (wavenumbers 1 to 3) have similar amplitude in experiments I and II, but have much larger amplitude in experiment III. The baroclinic regime of
wavenumbers 4 to 8 shows that the filtered experiments attain a higher level of baroclinic activity. At all levels experiments II and III exhibit more eddy kinetic energy in these critical wavelengths than does experiment I. The suggested mechanism by which energy is passed into the ultra-long waves is through their interaction with baroclinic waves which develop preferentially on the eastern side of major stationary troughs. The higher energy levels in the wavenumbers 4 to 8 of experiments II and III presumably allows more of this type of energy transfer, resulting in the observed higher amplitudes of the ultra-long waves in these experiments. The amplitude of the shorter waves demonstrates how selective the filter is. Wavenumbers 9–13 have more energy in the filtered experiments while wavenumbers 19–60 have more energy in the diffusion experiment.

![Figure 3. Spectra of kinetic energy, mean for 15°-75°N and 41-80 days.](image)

The kinetic energy spectrum for each experiment, for the area 15°-75°N and averaged over days 41–80, is shown in Fig. 3. The value of $K_x$ is plotted for wavenumbers 1–30 on a logarithmic scale. The solid lines are drawn with a slope of $-3.0$. It is immediately apparent that the slope of the wavenumber-energy variation in experiment I (from wavenumber 8 onward) is much less than $-3.0$, due primarily to an excess of energy in the higher wave-

<table>
<thead>
<tr>
<th>TABLE 5. EDDY KINETIC ENERGY IN WAVEBANDS, FOR EACH LEVEL OF THE MODEL FOR ALL 15°-75°N FOR THE PERIOD 41–80 DAYS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>0.1</td>
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<tr>
<td>0.3</td>
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<td>0.5</td>
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<tr>
<td>0.7</td>
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<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>0.9</td>
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</tbody>
</table>
numbers. Experiments II and III have about half the energy of experiment I in wavenumbers 29 and 30, and between one and a half and twice the energy in wavenumbers 8–16. This redistribution of energy in the spectrum gives experiments II and III a slope more nearly in agreement with the theoretical value of $-3.0$ (Kraichnan 1967).

The daily hemispheric averages of zonal and eddy available potential energy for the days 41 to 80 are shown in Fig. 4. The level of zonal available potential energy ($P_Z$) is less in the filtered experiments than in the diffusion experiment throughout the whole period.

![Figure 4. Evolution of available potential energy.](image)

Experiments I and II exhibit slight overall decreases in the level of $P_Z$ from day 40 to day 80 whereas experiment III has a definite loss of $P_Z$, maintained through the period. The peaks of eddy kinetic energy in the filtered experiments around day 60 are also exhibited in the daily variation of eddy available potential energy ($P_E$). However, experiments I and II have similar average levels of $P_E$ over the period, which was not the case with eddy kinetic energy ($K_E$). The level of $P_E$ in experiment III is generally much higher than in the other experiments.

Using the information available on energy levels and rates of transformation we can construct energy ‘box-diagrams’ showing an energy budget over the period 41–80 days (Fig. 5). Quantities that have been directly calculated are underlined, the rates of change being inferred from values at days 41 and 80. As has already been noticed the distribution of kinetic energy and available potential energy is different in the diffusion and filtering experiments, more energy going into eddy mode when filtering is used as the dissipation process, and the level of the zonal mode also decreases. The rate of conversion from $P_Z$ to
Figure 5. Mean hemispheric energy 'box diagrams' for days 41–80. $P_E$, $P_s$ zonal and eddy available potential energy, $K_s$, $K_E$ zonal and eddy kinetic energy. Units: Energies in $10^{-3}$ J g$^{-1}$. Transformations in $10^{-3}$ J g$^{-1}$ day$^{-1}$. (Quantities underlined have been calculated, others inferred from rates of change of calculated quantities.)

$P_E$ is not shown in Fig. 5 and hence is included in the 'source' terms for these quantities.

The source of $P_E$ is obtained from both conversion of $P_Z$ and from heat of condensation and convection. The lower values of $P_Z$ in experiment III, and the continuing fall of $P_Z$ with time, indicate that the increase in the source term for $P_E$ in experiment III derives mainly from an increase in the rate of conversion from $P_Z$. The conversion of $P_Z$ to $P_E$ in experiment III is so large over this period (41–80 days) that it exceeds the generation of $P_Z$, resulting in a negative 'source' term for $P_Z$. As might be expected the increase in the level of $K_E$ in experiment III is accompanied by an increase in the rate of conversion from $P_E$ to $K_E$; the rate of dissipation of $K_E$ by friction and the explicit damping terms also increases. The conversion of $K_E$ to $K_Z$ is higher in experiment III, presumably resulting from the larger amounts of $K_E$ in the ultra-long and baroclinic waves. As a result of a stronger Ferrel cell in experiment III more $K_Z$ is transformed into $P_Z$ in middle latitudes, the net effect being a smaller transformation of $P_Z$ to $K_Z$ on a hemispheric scale. Oort (1964) gives mean values for energy levels and energy conversion rates for the atmosphere (yearly mean). The values of $P_Z$, $K_Z$ and $K_E$ in experiment III appear to be reasonable by comparison. However, the value of $P_E$ is outside the range given by Oort, i.e. $150 \pm 50 \times 10^{-3}$ J cm$^{-2}$. The value of the ratio $K_E/K_Z$ for the last forty days of experiment I is 0.38, similar to that at the end of the experiment in Gilchrist et al. Experiment III, however, averages a value of 0.73, a much more realistic figure.

5. Meridional Transports

(a) Mass

The mean meridional circulation patterns for experiments I and III are shown in Fig. 6. The isolines give the total transport of mass below the level considered. Values are given for pressure surfaces after using an interpolation process which ensures that the vertical sum is preserved. Three distinct cells are apparent in each experiment. The direct Hadley cell is too weak in all three experiments. The filtered experiment has a slightly stronger Hadley cell than the diffusion experiment, maximum values being $-9.7 \times 10^{13}$ g s$^{-1}$ (Exp. I), and $-10.6 \times 10^{13}$ g s$^{-1}$ (Exp. III). As in Gilchrist et al. the maxima are at too low a level, implying too concentrated a flow in the lower troposphere with a more diffuse return flow at high levels.

The indirect Ferrel cell is also too weak. Oort and Rasmusson give a value in excess of $5 \times 10^{13}$ g s$^{-1}$ at 40°N. The maximum value for experiment III is $4.6 \times 10^{13}$ g s$^{-1}$.
which is 40% higher than that of experiment I (3.3 \times 10^{13} \text{ g s}^{-1}). Phillips (1954) relates an increase in eddy kinetic energy due to baroclinic instability with an increase in the strength of the Ferrel cell. This correlation is observed in these experimental results. The polar cell is well positioned in all the experiments and of about the correct strength.

(b) Latent heat

The weak Hadley cells in these experiments might be expected to affect significantly the transport of water vapour by the mean meridional circulation, and such is the case. Fig. 7 shows the vertically integrated zonal mean flux of latent heat for each experiment, over the period 41–80 days, together with values derived from Oort and Rasmusson. The southward flux in the tropics due to the Hadley cell is underestimated in all the experiments, whereas the northward flux in low and middle latitudes due to eddy motions is quite well represented. The maximum northward flux (mean + eddy) for each experiment is 1.9
(Exp. I) and 2.7 (Exp. III), all units being $10^{20}$ J day$^{-1}$. The maximum given by Oort and Rasmusson is $2.2 \times 10^{20}$ J day$^{-1}$.

(c) **Momentum**

Unlike the transport of water vapour, the flux of momentum due to eddy processes is much affected by the change in dissipation process in the model. The vertically integrated zonal mean fluxes of momentum for the period 41–80 days are shown in Fig. 8. Once again the mean January values are derived from Oort and Rasmusson. The flux by the mean meridional circulation reflects the relative strength of the Hadley cells of the experiments. The maximum southward flux due to the Ferrel cell is best represented in experiment III, a value of $-11 \times 10^{16}$ g cm s$^{-2}$ compared with the observed value of $-13 \times 10^{16}$ g cm s$^{-2}$.

![Figure 8. Momentum flux, 41–80 day mean.](image)

The flux due to eddy motions shows the most difference between the experiments. The total eddy northward flux in middle latitudes has the following maximum values, 47 (Exp. I), and 68 (Exp. III), all units $10^{16}$ g cm s$^{-2}$. The value given by Oort and Rasmusson for January is $70 \times 10^{16}$ g cm s$^{-2}$. The separation of the eddy flux into standing and transient parts shows a distribution different from that observed in the atmosphere. In the data of Oort and Rasmusson the northward maximum of eddy transport is mainly due to transient eddies (77%) whereas the opposite holds for all the experiments, i.e. the standing eddy flux dominates. This is probably due to the prolonged January-type forcing conditions that are applied in the model, different time scales are then involved in separating out what are essentially monthly and seasonal variations. The total eddy flux in high latitudes is directed equatorward and is too extensive in latitudinal extent as well as being too large. The southward eddy flux in the models is equally attributable to standing and transient eddies whereas in Oort and Rasmusson's data standing eddies alone contribute to the flux in this region.

(d) **Sensible heat plus potential energy**

Finally in this section we examine the transport of sensible heat and potential energy ($C_p T + \phi$). The zonally meaned vertically integrated fluxes for the period 41–80 days are shown in Fig. 9. In this case it is not obvious how to separate mean and eddy components, hence only the total flux is shown. The observed January data is from Oort and Rasmusson. The tropical maximum is due to the mean meridional circulation and consequently is not reproduced well in any of the experiments. The mid-latitude maximum is due to eddy
motions and reflects the degree of baroclinic instability present in the flow. Only experiment III reproduces the mid-latitude maximum to any degree, although positioned too far south.

6. Features of the General Circulation

(a) Rainfall and evaporation

The equatorial rainfall belt in all the experiments is very concentrated because of the model boundary at the equator. The areas for which rainfall exceeds evaporation correspond well with actual data (Newell, Vincent, Dopplick, Ferruzza and Kidson 1969). The filtered experiments have less evaporation in mid-latitudes, giving a better excess of rainfall than is observed in experiment I. This effect is due to the ascent in the northern half of the Ferrel cell, this being stronger in the filtered experiments. The moisture balance is maintained by the greater eddy transport northward in these experiments. Table 6 gives the hemispheric mean values for rainfall (large-scale and convective) and evaporation for the period 41–80 days.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Large-scale rain</th>
<th>Convective rain</th>
<th>Total rain</th>
<th>Evaporation</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.984</td>
<td>2.368</td>
<td>3.352</td>
<td>3.337</td>
</tr>
<tr>
<td>II</td>
<td>1.093</td>
<td>2.323</td>
<td>3.416</td>
<td>3.384</td>
</tr>
<tr>
<td>III</td>
<td>1.144</td>
<td>2.336</td>
<td>3.480</td>
<td>3.448</td>
</tr>
</tbody>
</table>

Experiment III has 16% more large-scale rainfall than experiment I, an indication of the increase in eddy activity due to baroclinic instability.

(b) Temperature and zonal wind component

The differences between the zonal mean latitudinal cross sections of the temperature fields for experiments III and I are given in Fig. 10. As would be expected from the greater northward transports of both potential energy, sensible heat and latent heat, the higher latitudes of experiment III are warmer than those of experiment I, and there is also a cooler area in the tropics. The maximum difference in the temperature fields occurs in the polar middle troposphere where the experiments have mean temperatures of 226 (Exp. I) and 233 K (Exp. III); the latter figure would appear to be more realistic (Newell et al. 1969).
The ability of baroclinic disturbances to amplify more easily in experiment III obviates the need for the stronger temperature gradients found in experiment I and the results of Gilchrist et al. As a consequence of the weaker temperature gradient the jet stream in the filtered models is slightly weaker than in the diffusion experiment. Maximum values of the 41–80 day mean sub-tropical jets are 38-4m s⁻¹ (Exp. I) and 36-5m s⁻¹ (Exp. III). The reduction in jet maxima gives the observed decrease in zonal kinetic energy values at high levels that has already been mentioned. Other than the warmer high latitudes and lower jet maxima the overall features of the zonal mean cross-sections of temperature and zonal velocity are similar to those in Gilchrist et al. (Fig. 11). There is a larger area of polar easterlies, and a stronger shear on the warm side of the jet.

(c) Pressure at mean sea level (PMSL)

The 41–80 day mean PMSL chart for experiment III is shown in Fig. 12. The same
basic features of the observed January mean sea level pressure distributions are represented as in Gilchrist et al., but in general a more realistic picture is presented. The Icelandic low pressure area is too deep and too far south, as in Gilchrist et al., but the North Pacific low pressure area is deeper and more extensive than in previous experiments with this model (1001mb off Alaska, 996mb off Kamchatka). Crutcher and Meserve (1970) give a North Pacific low pressure minimum of between 995mb and 1000mb for a January mean. The high pressure cells over North America and North Africa have smaller areas in excess of 1024mb and the Asiatic high, although too far south, has a central pressure in excess of 1040mb.

7. SUMMARY AND CONCLUSIONS

The introduction of a multipoint filter into a general circulation model to replace the more usual dissipative terms leads to a more realistic simulation of the atmospheric northern hemisphere January circulation. The more selective damping of short wavelengths that characterizes the filter allows higher levels of eddy available potential and eddy kinetic energy in the model. The increased eddy activity transports more latent heat, sensible heat and potential energy northwards out of the tropics, ensuring a warmer polar and high latitude troposphere. The Ferrel cell is strengthened as a result of more baroclinic instability in middle latitudes, while the Hadley cell is also strengthened. The large scale rainfall in middle latitudes increases as a result of more cyclonic activity associated with the increase in baroclinic instability; this is seen on the mean PMSL chart as a deepening and enlarging of the North Pacific low pressure area.
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