On the eddy structure of hurricanes

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SUMMARY

Given the general conditions present in a mature hurricane, this paper seeks to show by observational and theoretical means that the circulation regime within a hurricane, beyond a certain radius, must be of the eddy type rather than axisymmetric. This is done by first pointing out, following recent results of Starr, that mean radial circulations cannot transport angular momentum across surfaces of constant mean values of this quantity. A diagram is constructed showing such momentum surfaces obtained from data for a mean hurricane, and it is demonstrated that there must be a flux of momentum across these closed surfaces. This flux, therefore, can only be produced by an eddy type circulation regime.

1. INTRODUCTION

In recent years there has been a growing interest in the dynamics of hurricanes, including their numerical simulation. However, there has been a general lack of suitable data describing the large-scale circulations associated with hurricanes, thus hampering a complete understanding of the entire dynamical picture. This paper, utilizing the best available data, seeks to show that the large-scale circulation around a hurricane, beyond a certain radius, must be of the eddy type rather than axisymmetric. This is one basic feature of the hurricane circulation which can be surmised from existing data, even in the face of their acknowledged crudity. Thus, axisymmetric hurricane models which ignore the eddy structure may neglect an important feature of the natural system.

2. THEORETICAL ASPECTS

A schematic diagram illustrating the salient mean properties of a hurricane can be plotted approximately on a diagram like Fig. 1. The centre of the hurricane is at \( r = 0 \), where \( r \) is the radial distance from the central axis of the hurricane. We shall refer to motion in an inward and outward direction as radial motion, and motion around the hurricane as tangential motion, represented by \( v_T \), positive values being in the counter-clockwise sense. Thus the total angular momentum \( M \) of a parcel of air about the central axis of the hurricane is given by

\[
M = r^2 \Omega \sin \phi + rv_T
\]

Figure 1. Schematic diagram of a mean hurricane, \( r \) being the radial distance from the hurricane axis. AB represents a surface of constant mean angular momentum also averaged zonally around the hurricane.
where $\Omega$ refers to the earth's angular velocity ($7.292 \times 10^{-5}$ $s^{-1}$) and $\phi$ is the latitude of the hurricane centre. At any given radius, we assume that there is no net mass transport in the radial direction across any complete cylindrical surface defined by the curve AB in Fig. 1. This is the case for a mature hurricane which is neither weakening nor deepening (i.e. with constant central surface pressure), and for which the inward transport of moisture, falling as rain near the centre, is negligible. This effect of moisture has been calculated and shown to be small for the present purposes.

We suppose temporarily, for purposes of exposition, that the hurricane flow is steady and strictly symmetrical. The closed surface of revolution defined by the curve AB in Fig. 1 may now be taken as one over which $M$ has the same value at all points. It is obvious that here too there can be no net mass transport across this $M$-surface under steady conditions, save for the small unimportant moisture effects. Inward symmetrical mass flows at some levels are cancelled by outward such flows at other levels. That is to say, what we call (symmetrical) mean radial circulations can transport no net amount of mass across $M$-surfaces. The important point here to be made is that since each unit of mass crossing the $M$-surface has necessarily the same value of $M$, it follows that no angular momentum can be transported by the mean radial circulations across the $M$-surfaces. If such transports must take place across each such surface, which in fact isolates completely an inner region, they must be due to other mechanisms, such as unsymmetrical components of motion on some scale or scales.

Still considering the steady, strictly symmetrical flow regime, what can we say about the momentum flux across vertical walls, i.e. across surfaces of constant $r$? This question might well derive from a more conventional way of viewing hurricane dynamics. Since the state is steady, then our demonstration that no angular momentum can be transported across $M$-surfaces by the symmetric flow necessarily implies that neither can such transport take place across vertical walls. The argument follows closely along lines given originally by Starr (1974a) to deal with comparable questions concerning the atmospheric general circulation. Essentially, the result obtains because under the steady state assumption, no momentum can accumulate in the volume between a given vertical wall and the $M$-surface which intersects it at the ground. No net flux across the $M$-surface therefore implies no net flux across the vertical wall. (The existence of intersections between the two surfaces poses no limitation on the argument.) Of course, were eddies present to serve as another momentum transport mechanism in a more general flow regime, a different sort of picture could exist.

The next problem is then to deal with the more general case where no restriction as to symmetry about the central axis is made. Also various possible temporal oscillations are to be included (see in this connection also Starr 1974b). To this end the procedures used by Starr (1974a) can again in a rather straightforward way be applied to our system. The details of the development are not repeated here, the reader being directed to the above references. The results for our case are moreover simple enough to be stated directly. Instead of $M$-surfaces we now use surfaces of $\overline{M}$, where the overbar refers to a temporal average over a suitable period, and the brackets represent an average at a given height around the length of a complete circle centred at the hurricane's axis (cf. Starr's comparable usage of brackets as the average around a latitude circle).

The result again follows as before that if there is to be an inward flux of $M$ across surfaces of constant $\overline{M}$ it must be produced by an eddy type circulation involving, at a given level, temporal and spatial covariances between $M$ and the components of motion normal to these surfaces. This mean flux, $T$, attributable to the eddy motions, may be written as

$$T = \int [\overline{M} \cdot (\rho u_n)] dS + \int [\overline{M} \cdot (\rho v_n)] dS$$

(2)
or

$$T = \int_{S} [M'(\rho u'_N)]dS + \int_{S} [M''(\rho v'_N)]dS$$

(3)

where, if \( x \) is any given variable of time and space,

$$x' = x - [x] = \text{spatial deviation}$$

$$\bar{x} = \frac{1}{\tau} \int_{0}^{\tau} x dt = \text{time average}$$

$$\tau = \text{time period considered}$$

$$x^* = x - \bar{x} = \text{temporal deviation}$$

$$S = \text{surface of constant} [\bar{M}]$$

\( v_N \) = component of wind normal to \( S \)

\( \rho \) = density

With regard to the flux of \( M \) across vertical walls in the eddy flow regime, circumstances do now permit the mean symmetric component of the motion field to contribute a non-zero value. Consider the volume enclosed by a given vertical wall and the \([\bar{M}]\)-surface which intersects it at the ground. Now that eddies are permitted, they can serve as additional momentum transport mechanisms for this volume, and so the mean symmetric component of the flow can transport \( M \) across the vertical wall (though still not across the \([\bar{M}]\)-surface, of course). It becomes obvious though, in conjunction with our previous considerations of the steady, strictly symmetrical case, that any net momentum flux across a vertical wall by a mean symmetric component of motion must depend for its existence upon the simultaneous presence of eddy components of flow.

In the next sections, we attempt to make use of available data to demonstrate that surfaces of \([\bar{M}]\) are closed and that there must be an inward flux of \( M \) in our model storm. Once this is accomplished, it will follow from our discussion above that, except for the central regions, gross eddy motions, and not mean axisymmetric ones, must be responsible. It is to be noted that (2) includes a symmetric time-eddy term.

3. DATA ANALYSIS

Fig. 2 depicts a vertical cross-section through the surfaces of constant \([\bar{M}]\) about a hurricane. The radii are measured in length units of latitude degrees. For the larger scales (beyond 1° of latitude from the centre), a paper by Miller (1958) was used for a data source.

Figure 2. Surfaces of constant mean angular momentum \([\bar{M}]\) at latitude 25°. Units are \( \times 10^{19} \text{cm}^2 \text{s}^{-1} \). Data used in preparing this figure are from Hawkins and Rubsam for \( 0° < r < 1° \) and from Miller for the rest of the figure (see text).
This appears to be the most recent comprehensive collection available for the large scale circulation around a hurricane. These data represent the composite winds from a large number of hurricanes measured up to 1958 by rawinsonde. It is estimated that the average centre of the hurricanes in this data set is at $\phi = 25^\circ$. For the smaller scales (i.e. within a radius of $1^\circ$) the detailed analysis by Hawkins and Rubsam (1968) of hurricane Hilda in 1964 was used. On the day these data measured, Hilda was at $\phi = 25^\circ$. These two sets of data mesh quite well. This shows, a little surprisingly, that the combination of data from an actual hurricane to represent the smaller scale picture and mean values from the composite of many hurricanes for the larger scale is feasible. In drawing Fig. 2, it was assumed that $v_T$ approaches zero at the surface and at the top of the atmosphere. It was also assumed that direct surface frictional effects were restricted to the lowest 50 mb, and that, for simplicity, the surface pressure was 1000mb. It is important to observe that the $[M]$-surfaces depicted in Fig. 2 each have the property of completely isolating an inner portion of the hurricane from the rest of the atmosphere. Thus, any fluxes of momentum which may be required to maintain the hurricane must be across these surfaces.

4. Calculation of frictional flux

As noted above, there can be no transport of angular momentum across the closed $[M]$-surfaces by the mean radial circulations. However, if winds are to be zero at the surface there must be a large frictional drag on the winds at the surface in the inner part of the hurricane, producing a downward, into the surface, transport of angular momentum. Since, in this part of the storm, the $[M]$-surfaces are nearly horizontal near the ground (see Fig. 2), we can assume that there is a turbulent, boundary layer (eddy) transport of momentum across them.

The downward flux of $M$ into the surface was calculated for each $1^\circ$ zone about the storm, thus giving the total $M$-flux into the ground for each unit annulus. The flux may be estimated from the following traditional formulation:

$$\text{Frictional flux} = \tilde{r}\bar{p}c_dd^2r(r_0^2 - r^2)$$

(4)

where $\bar{p}$ = mean density in lowest 50 mb, $1.15 \times 10^{-3}$ g cm$^{-3}$

$c_d$ = drag coefficient, assumed to be $1.5 \times 10^{-3}$ from data of Hawkins and Rubsam

$v_{Ts}$ = surface tangential wind (at 950mb, assumed to be level below which turbulence takes place)

$r_0$ = radius of outer edge of annulus

$r_i$ = radius of inner edge of annulus

$\tilde{r}$ = mean radius of annulus = $(r_0 + r_i)/2$

The results of the calculations are presented in Fig. 3. The value for the innermost annulus, $r = 0^\circ$ to $r = 1^\circ$, is taken from a figure drawn by Hawkins and Rubsam. All the flux values are somewhat questionable, considering the rather crude parameterization used in the calculations and the uncertainty of the value of $c_d$.

As the radius, $\tilde{r}$, is increased, the two competing effects – the increasing area of each annulus and the increasing value of $\tilde{r}$ on the one hand; and the decreasing wind speed $v_{Ts}$ on the other – combine to produce a maximum frictional removal in the annulus between $3^\circ$ and $4^\circ$ radius. Beyond this, the additional flux from each annulus decreases slowly as the decrease in $v_{Ts}$ becomes dominant.

We see from Fig. 3 that, by continuity, the inward transport of $M$ across the $r = 10^\circ$ cylindrical surface must be about $300 \times 10^{22}$ g cm$^2$s$^{-2}$ to make up for the frictional losses.
These frictional losses must be balanced in the steady state by an inward momentum flux also across the \( (\mathcal{M}) = \text{constant} \)-surface meeting the lower boundary at \( r = 10^9 \). This flux comes from the environment around the storm and must arise from eddy motions, on the basis of our theoretical discussion.

5. Discussion of circulation regime

Close to the centre of the hurricane the \( (\mathcal{M}) \)-surfaces are much deformed by the strong winds and become almost horizontal near the surface (see Fig. 2). This corresponds to a high rotational Rossby number, \( N_R = |v_2|/r\Omega \sin \phi \), adapting ideas formulated by Starr (1974a). At \( 1^\circ \) radius, \( N_R > 19 \) near the surface and \( N_R > 3.5 \) even at 140mb, where the hurricane circulation is very weak. The total downward frictional flux in this region of the storm is small. Most of the balancing eddy flux across the \( (\mathcal{M}) \)-surfaces probably occurs as a high wavenumber phenomenon of small-scale turbulence in the boundary layer which transports momentum downward across the nearly horizontal portions of the \( (\mathcal{M}) \)-surfaces in the boundary layer. Above the boundary layer, where the surfaces are nearly vertical, the hurricane circulation may be nearly axisymmetric, as little eddy flux is necessary at these levels.

In the outer region of the hurricane, however, the \( (\mathcal{M}) \)-surfaces are almost vertical, even in the lower levels, and the rotational Rossby number is small. At a radius of \( 10^9 \), \( N_R < 0.2 \) below 400mb and \( N_R < 0.5 \) above 400mb. The total eddy flux across the \( (\mathcal{M}) \)-surfaces must be large in this region to balance all the frictional losses occurring, say within \( r = 10^6 \), and the boundary layer flux undoubtedly accounts for only a small part of this total. Most of the eddy flux across the \( (\mathcal{M}) \)-surfaces in the vicinity of \( r = 10^6 \) must therefore occur in the region above the boundary layer. Below about 400mb, where the \( (\mathcal{M}) \)-surfaces are essentially vertical, horizontal eddy processes would necessarily provide

![Figure 3. Frictional flux of angular momentum into the surface (lower curve). The surface frictional flux in each 1° annulus about the storm was calculated from the same data as shown in Fig. 2. Inward flux (upper curve) obtained by integration of frictional flux.](image-url)
part of the required flux, and in the upper part of the atmosphere where the $[\mathcal{M}]$-surfaces become more horizontal, vertical eddy processes would also contribute. With the present data, the proportion of the flux which is horizontal cannot be measured, but Pfeffer (1956) and others have shown that indeed observations of hurricanes do indicate large horizontal eddy structures of wavenumber 2 or 3, these being associated with the phenomenon of spiral bands. Macdonald (1968) has suggested that these horizontal eddy-type circulations associated with spiral bands do transport angular momentum inward to maintain the hurricane circulation.

The transition from a high rotational Rossby number regime near the centre of the hurricane, characterized by approximately axisymmetric flow, to one of low rotational number and horizontal eddy-type flow, is a gradual one, and is not marked by a sharp boundary. Fig. 3 however, suggests that at a radius of about 3° the transition occurs more rapidly than elsewhere. This is because up to a radius of about 3°, the required total inward flux is very close to the incremental boundary layer flux, and almost all of the eddy flux across the $[\mathcal{M}]$-surfaces is in the boundary layer. Beyond about 3°, however, there is a large difference between the boundary layer flux and the total inward flux, requiring an increasingly larger horizontal eddy flux above the boundary layer.

6. Concluding Remarks

As a result of our considerations from theory and from data, the following general statements seem to be indicated. (1) Mean radial circulations cannot transport net total angular momentum across mean angular momentum surfaces in a steady regime hurricane. (2) Such transports are needed to make up for surface frictional losses. (3) The transports must then be due to space and time eddy processes. (4) Near the centre such eddy effects are probably provided by very large wavenumber disturbances constituting the turbulent boundary layer. (5) Near the centre the more gross motion may therefore approach axial symmetry. (6) The cross-sectional distribution of mean total angular momentum obtained from hurricane wind statistics shows this to be possible. (7) At radii outside the strong circulation, these data show a necessity for much more gross eddy actions, which probably can be provided by departures from axial symmetry of rather low wavenumber. (8) At these larger radii in the higher levels, where the momentum surfaces again approach a horizontal orientation, vertical eddies may aid in producing an inward momentum flux. (9) We may note with interest the extensive anticyclonic cap which the total momentum surfaces show at higher levels above the lower cyclonic circulation. (10) More and better data would be of great value in studies such as the present one. (11) Much better data would be needed to ascertain any contribution that might be present from the time-eddy term in Eq. (2). (12) The method of analysis utilized in this work has been applied recently to studies of the dynamics of the tornado vortex, with results similar to those achieved here (see Starr 1974c and Starr et al. 1974). (13) For some purposes, one might try to parameterize the effects of the eddies, but this raises the question of where the resulting coefficients should be positive or negative.

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