Mesoscale air motions derived from wind finding dropsonde data: the warm front and rainbands of 18 January 1971

W. T. ROACH and M. E. HARDMAN
Meteorological Office, Bracknell

(Received 2 October 1974; revised 27 January 1975)

SUMMARY

A diagnostic dynamical study is presented of the three-dimensional field of motion based on data from dropsondes released through the warm-frontal rainband situation of 18 January 1971, and is complementary to an earlier radar-synoptic study of this situation.

Principal features of interest are:
(i) The identification of two low level jets. The first (axis below 1km) lies just ahead of and parallel to the surface warm front. The second (axis near 2km) overrides the first at an angle of about 50° and may be a continuation of the cold front jet overriding the warm front just ahead of the point of occlusion. The flow in the lower jet appears to be supergeostrophic; in the upper jet slightly subgeostrophic.
(ii) A distinct mesoscale perturbation of the synoptic scale wind field associated with the rainbands above the warm front. Large negative values of the vertical component of absolute vorticity were observed in the main rainband.
(iii) The mesoscale fields of motion produce a marked ‘peak’ in the spectrum of horizontal and vertical kinetic energy.

These and other features are discussed in relation to the results of a model of frontogenesis by B. J. Hoskins. Possible causes of the circulation associated with the rainbands and the interaction with other scales of motion are considered.

I. INTRODUCTION

Since about 1966 a study has been in progress in the Meteorological Office with the aim of understanding how dynamical interactions on various scales control the release of precipitation in frontal systems. This phase of the study has concentrated on the fundamental properties of the frontal systems unmodified by topography, and so the measurement programme has been largely over the sea. A second stage of these studies is to understand how their fundamental properties are modified by topography.

The studies so far published (Harrold and Nicholls 1968; Browning and Harrold 1969; Harrold and Browning 1969; Browning and Harrold 1970; Harrold 1973; Browning, Hardman, Harrold and Pardoe 1973 – hereinafter referred to as BHHP) have confirmed that the distribution of warm frontal precipitation is often organized into bands of order 100km separation which are roughly aligned with the surface warm front, or cold front, or both. The association of these rainbands with the principal synoptic-scale precipitation producing flows has also been defined. The use of dropsondes (Hardman, James and Goldsmith 1972 – hereinafter referred to as HGJ) has provided a glimpse of the mesoscale wind and vertical velocity fields associated with the rainbands; there appears to be a broad correspondence between mesoscale areas of precipitation and areas of ascent.

In the warm-frontal case study of 18 January 1971 described by BHHP, the dropsonde data box was more extensive than hitherto. This has made it feasible to attempt a more thorough dynamical analysis of the three-dimensional motion field associated with the rainbands on 18 January 1971 than presented in HGJ, so that this paper is to be regarded as a companion paper to BHHP.

The plan of this paper is to describe the main features of the mesoscale wind field, its differential properties and its fluxes of momentum and kinetic energy on various scales.
In the final section, the significance of the results is discussed under two headings: (i) observed frontal structure and its relationship to a theoretical model of frontogenesis due to Hoskins (1971); (ii) possible sources and sinks of the energy of motion fields associated with the rainband circulation and its interaction with other scales of motion.

2. Sources of wind data

Data for this case study were obtained from dropsondes, pulsed Doppler radar, serial radiosonde ascents and an instrumented Canberra aircraft. The details of the disposition of this data are shown in Fig. 2 of BHHP and the area covered by the dropsonde network in relation to the (moving) frontal system is shown in Fig. 2 of the present paper.

(a) Dropsondes

The techniques of the development and deployment of dropsondes are described in HJG. These provide a distribution of wind soundings with a typical spacing of 30km – just sufficient to resolve the major variations of wind field within the rainbands (typical spacing of 100km). This largely bridges the gap between the synoptic upper air network, with a typical spacing of order 300km, and the detailed cross-sections of wind fields within precipitation obtained from pulsed Doppler radar (Catón 1963). The dropsonde, of course, has the advantage over the Doppler system of providing wind field data both in and out of precipitation.

The principal assumption made in the analysis of the dropsonde data is that the wind field is steady-state in a frame of reference moving with a constant velocity known as the pattern velocity. This pattern velocity is determined from the dropsonde data by a statistical method described in HJG. It so happens that this pattern velocity is not significantly different from the overall movement of the rainband system as estimated from radar observations (Fig. 6 of BHHP). Since one of the main themes of this paper is a study of the wind field associated with the rainband system, the pattern velocity is henceforth referred to as the 'system velocity', S.

Of course, the pattern of motion in and below the warm front was found to be moving at a different velocity, so that the use of S distorts the derived motion fields in these regions. This distortion is relatively slight however, and its effect, and the effect of other sources of error on the representation of the wind field, is assessed in an appendix to this paper.

A total of 53 dropsondes were released from two aircraft – the Varsity aircraft of the Meteorological Research Flight and an Argosy aircraft from the Air Transport Development Unit, Abingdon. The Varsity dropsondes were similar to those described in HJG. The Argosy targets were of a different type, the construction of the aircraft allowing them to be ejected in a fully open configuration. Two radars were used allowing a nominal dropping interval of 6 minutes. For this experiment, a modification of the flight plan was made to provide a more even resolution of the dropping pattern. The Argosy flight plan was displaced 10km upstream from the Varsity pattern to allow for a necessary time interval between the two aircraft in the dropping zone. Both aircraft were used to provide comparison drops downstream of the Isles of Scilly.

(b) Pulsed Doppler radar

A mobile 3cm pulsed Doppler radar was located on St. Mary's, Isles of Scilly. Vertical cross-sections of wind, divergence, deformation and vertical velocity were obtained within the main precipitation regions using the conical scanning mode (Catón 1963). The vertical
extent of the data was restricted within band III (see BHHP) by the limited vertical and horizontal extent of precipitation cells within this band.

(c) **Camborne radiosonde ascents**

Serial ascents from this station provided a time cross-section of the wind field throughout the period of the dropsonde study with a spacing of approximately 90 minutes. These data were used to infer the vertical circulation in the plane of the radiosonde time cross-section outside the dropsonde data region.

(d) **Canberra aircraft**

Measurements of wind, temperature and turbulence were made within the high level regions of the rainband structure (at 5.5 and 6.8km) from the Canberra aircraft of the Meteorological Research Flight. The wind-finding technique was based on a combination of Doppler navigational aids with an inertial-platform/nose-probe system and some results from it have been described by Axford (1968, 1971, 1972).

3. **THE WIND FIELD**

The dropsonde observations covered a volume about 3.5km in depth, 80km in a direction parallel to the surface warm front, and about 300km in a direction parallel to the system velocity vector, S.

The speed and orientation of the warm front (estimated using conventional synoptic techniques) and system velocity are shown on a polar diagram in Fig. 1, which also demonstrates the principal characteristics of the wind profile. The basic co-ordinates used

---

**Figure 1.** Hodogram demonstrating some principal features of the wind field.

- **F** velocity vector corresponding to warm frontal motion
- **S** system velocity vector
- **V(%)** hodogram of wind at \(x_F = 160\text{km}, y_F = 0\text{km}\)
- **L_1, L_2, U_1, U_2** approximate boundaries of the lower and upper sections of the main warm frontal zone
- **WF WF** orientation of warm front
- **RB RB** orientation of rainbands
- **u_f, U_f** axes defining front-parallel and front-transverse velocity components
- **v_f, U_v** axes defining band-parallel and band-transverse velocity components

**Heights are in hundreds of metres**
in representing horizontal sections of meteorological parameters discussed in this paper are $x_f$, distance from the surface warm front in the direction of S; and $y_f$, distance from the Isles of Scilly radar parallel to surface warm front ($y_f$ positive towards the north). As will be seen from Fig. 6, $x_f$ and $y_f$ are not quite orthogonal although the sections (apart from Fig. 6) have been drawn with $x_f$ and $y_f$ orthogonal for convenience. Due allowance has been made for this departure from orthogonality in deriving differential fields of velocity and kinetic energy budgets. The co-ordinates for deriving front-relative and rainband-relative velocities are orthogonal and are defined in Fig. 1.

(a) The synoptic environment

Fig. 2 (adapted from Fig. 4(b) of BHHP) shows the plan location of the surface fronts and rainband system in relation to the dropsonde data box. Also shown are the 700mb isotherms for 12 GMT 18 January 1971. The rainbands are also drawn schematically on all

![Figure 2. Schematic plan presentation of the location of the dropsonde data box (parallelogram) in relation to the surface fronts, rainband system (——) and 700mb isotherms (-----) labelled in °C. SS is the orientation (080°) of the system velocity vector and passes through the Scilly Isles.](image)

horizontal and vertical sections of the various properties of the motion field derived from the dropsonde data. The upper and lower boundaries of the warm frontal zone are also drawn on all vertical sections. The location of these boundaries were based partly on time cross-sections of temperatures, drawn from the Camborne serial radiosonde ascents, and partly on a few dropsonde temperature profiles. There was evidence of a distinct double structure from both temperature and wind profiles. Fig. 1 shows lower ($L_1L_2$) and upper ($U_1U_2$) sections of the frontal zone containing respectively south-east and southward wind shear vectors.

(b) The mesoscale horizontal wind field

The horizontal wind vector is defined by $V = ux + vy$ in conventional fashion. Fig. 3 shows cross-sections of wind speed and direction in the $x_f - z$ plane at $y_f = 0$. The dropsonde and Doppler observations are superimposed. The dropsonde observations were made some 30km (25 minutes) upstream of the Doppler observations, but they have both been reduced to the $x_f$ co-ordinate. The agreements between the sections is quite good, considering that completely different methods of sampling on different scales were used to measure the wind field.
Figure 3. Vertical cross-section of wind magnitude (\( u' \) in m s\(^{-1} \)) and direction (\( \alpha' \) in degrees) in \( x_f - z \) plane at \( y_f = 0 \) km.

- Dropsonde winds
- Doppler winds
- Frontal boundaries and approximate rainband boundaries (shown on all vertical cross-sections)

The main features of the wind field are:

(i) The sloping band (slope about 1 in 200), about 1 km deep, of wind direction shear, which is closely identified with the lower part of the warm frontal zone.

(ii) An elongated maximum of wind speed of just over 30 m s\(^{-1} \) straddling the direction shear belt in the western half (small \( x_f \)) of the section.

Figure 4. Vertical cross-section of \( u \) and \( v \) wind components (m s\(^{-1} \)) in \( x_f - z \) plane at \( y_f = -10 \) km.

(iii) The cellular structure with a characteristic scale of about 100 km of the wind speed and direction fields above the frontal zone. Fig. 5 of BHHP demonstrates a tendency for winds to be backed in rainbands and veered in-between.

(iv) All these features are present with little change in intensity but with some lateral shift (towards increasing \( x_f \) with increasing \( y_f \)) in \( x_f - z \) sections at different values of \( y_f \).

The rather curious structure of the low level jet in which maximum shear appears to coincide with maximum wind speed can be interpreted as the transition region between a
predominantly westerly 'conveyor belt' current and a predominantly southerly airstream of cold air ahead of the warm front (see Fig. 6 of Harrold 1973). This is best shown by $x_f-z$ sections at $y_f = -10\text{km}$ of westerly ($u$) and southerly ($v$) components shown in Fig. 4 in which a westerly 'jet' of $27\text{m s}^{-1}$ between 1-5 and 2km lies above a southerly 'jet' of similar maximum speeds at 0-6km or even lower.

(c) System (or band) relative velocity fields

Reference of the horizontal wind field to co-ordinate axes moving with the rainbands and orientated parallel ($v_\lambda$) and transverse ($u_\lambda$) to the bands (defined in Fig. 1) reveals with greater clarity the mesoscale flows associated with the bands.

The behaviour of the band-relative wind, $V_\lambda (= V - S)$ (Fig. 5) is of some interest. The flow at all levels appears to be dominated by the band II circulation. At 2-6km, the impression of an airflow into band II over the whole section is strong. $V_\lambda$ tends to be strongest and band-parallel near the band centres.

![Diagram](image)

Figure 5. Horizontal sections of band-relative velocity vector, $V_\lambda$ at 2-6, 3-2 and 3-8km. The approximate boundaries of the main rainbands are indicated, as defined in Fig. 6.
At higher levels the circulation resolves into two branches; a strongly anticyclonic flow which appears to involve most of band II and part of band I, and a generally northward drift in band III which also acquires marked anticyclonic curvature in the upper levels.

This suggests a general organization of rainband flow on a scale of order 200km, in contrast to that (100km) suggested by the veering-backing pattern of ground relative winds illustrated in Fig. 5 of BHHP.

(d) The vertical velocity field, \( w \)

The technique of deriving vertical velocity fields from an upward integration of the divergence field has been described in HJG, and an approximate, essentially two-dimensional, representation of the vertical velocity field has been presented in Fig. 7(e) of BHHP.

The fields of \( w \) presented in this paper are mean values over areas of about \( 10^3 \text{km}^2 \) and are based on integration upwards from a level of 500m.

Fig. 6 shows a horizontal plan section of \( w \) at 3-5km superimposed upon the distribution of echo intensity based on Fig. 4(c) of BHHP. It is fairly representative of the pattern of vertical velocity above the frontal zone. While the association of rising air with band II is quite definite, the corresponding associations are less clear in the other bands – the maximum echo in band I appears near a region of descent. The cell of upward motion in the north-west corner appears to be largely dissociated from band III, and some doubt attaches to its reality as it is on the edge of the data region.

The horizontal pattern of vertical velocity demonstrates no preferred orientation in either the rainband or surface warm front directions, and reflects the 'loss' of two-dimensional organization normally associated with fronts as the scale-of-motion studied drops below the synoptic scale.

The vertical velocity field, however, shows definite persistence in a vertical plane (Fig. 7) which extends well down into the frontal zone. Since \( w \) is derived from the horizontal wind field, it naturally reflects the rather large scale (200km) of organization noted in Fig. 5.

The range of values in \( w \) of \( \pm 0.3 \text{m s}^{-1} \) appears to swamp the synoptic scale velocity field, but the latter is clearly seen (Fig. 8) from vertical profiles of \( w \) averaged over three 100km square sections of the dropsonde grid. The classical transition from upward motion above the frontal zone to weak downward motion below it is well shown. The magnitudes
of the mean vertical velocities shown in Fig. 8 are, however, about twice those estimated from the adiabatic method (motion in wet bulb potential temperature surfaces) derived from the conventional radiosonde network.

(e) Relative streamline flow

The assumption of a steady-state motion field relative to the system allows the use of the identity

$$\frac{D}{Dt} = (\mathbf{V} - \mathbf{S}) \cdot \mathbf{V} + w \frac{\partial}{\partial z}$$

(1)

Figure 8. Profiles of vertical velocity averaged over the eastern (I), middle (II) and western (III) thirds of the dropsonde grid. Dotted lines represent corresponding profiles from Hoskins' model. U and L denote the upper and lower boundaries of the main frontal zone. The symbols I, II, III are chosen to denote that the averaging areas of each section are mainly within the corresponding rainbands.
Applying this operator to the system-relative position vector \( r_s \) gives
\[
\delta r_s = (iu_s + jv_s + kw) \delta t \tag{2}
\]

The integration of Eq. (2) in system-relative co-ordinates results in the construction of streamlines which are also trajectories relative to the system by virtue of the steady-state assumption. A value of \( \delta t = 500 \) seconds was used.

Fig. 9 shows four sets of streamlines drawn with origins within or below band II. In each set, parcels of air were started at equal intervals along a horizontal datum line DD, and tracked forward and backwards for 4 hours, or until the streamlines ran out of the data box. The datum lines lie vertically above each other at altitudes of 1.5, 2, 2.5 and 3 km.

These diagrams, in conjunction with Fig. 5, give further insight into the nature of flow in the main band. The main feature is the fundamental change of character of the flow as the region of interest shifts from within the frontal zone upwards into the rainbands.

In the frontal zone, the flow is uniform and rapid – a given streamline taking only 2 hours to pass through the data box with little change in altitude. At 2 km, the flow is still fairly uniform, but the time taken to traverse the box has doubled, and there is an ascent of 0.5–1 km during transit. At 2.5 and 3 km, the relative flow continues to decrease and to acquire a three-dimensional character. The streamlines become very steep, particularly in regions where \( V_s \) falls below 1 m s\(^{-1}\). At 3 km there appears to be a centre of anticyclonic circulation, C (also apparent in Figs. 5(b) and (c)) close to which the vertical displacement becomes almost comparable with the horizontal displacement. Some of the air entering this circulation appears to have descended from band I. Similarly, some of the air entering the western boundary of band II may have come from band III. Thus the transition from a
predominantly two-dimensional frontal regime to a predominantly three-dimensional convective regime is apparent.

4. The Differential Properties of the Wind Field

The characteristic magnitude of the measured differential coefficients of the wind vector, $V$, is given approximately by $\delta V/l$ where $\delta V$ is the characteristic magnitude of the vector change of wind over a grid length $l$.

In an atmosphere in which fluctuations of $V$ occur on all scales, $V$ itself tends to increase with $l$—roughly as $l^4$ (see section 5) — so that $V/l$ varies roughly as $l^{-1}$. On the synoptic scale, $V/l$ is generally small compared with the Coriolis parameter, $f$, and this assumption forms the basis of many studies of hydrodynamic stability on this scale. On the mesoscale, $V/l$ is the same order of magnitude as $f$, i.e. inertial forces are comparable with Coriolis forces; the Rossby number is of order unity.

Thus, reducing grid spacing from synoptic to mesoscale brings us into a different dynamical regime of particular interest.

The probable errors in the representation of differential fields on this scale can be rather large (see appendix), but not so large as to obscure the principal features of interest.

![Figure 10.](image)

**Figure 10.** Vertical section of absolute vorticity in units of $f$, and divergence in units $10^{-4}s^{-1}$.

(a) **Divergence**

Fig. 10(b), shows a cross-section of divergence in the $x_f-z$ plane at $y_f = -10\text{km}$. The main features shown here are roughly representative of those observed in other sections in this plane.

There is a belt of convergence lying more or less in the frontal zone, with the characteristic cellular structure superimposed and extending through the depth of the section. Above the frontal zone the main region of convergence occupies band II and is flanked by regions of divergence extending into bands I and II. Beneath the front, the flow tends to be weakly divergent, except in the vicinity of band II, where it is weakly convergent.

(b) **Vorticity**

The vertical component of absolute vorticity, $\zeta_v$, normalized by the Coriolis parameter,
is shown in Fig. 10(a). The cold air flow below the front appears to be generally anticyclonic \((\zeta_a < 0)\) and a belt of cyclonic vorticity is embedded in the frontal zone under band II. Above the frontal zone there is a marked structure of wavelength about 150km, and of particular interest is the large area of negative vorticity occupying most of band II, and reflected in Figs. 5 and 9. The magnitude of this negative vorticity approaches \(-2f\), a value which comfortably exceeds two standard deviations (see appendix), even in convective regions.

Figure 11. Vertical section of components of vertical wind shear in units of \(10^{-2}\)s\(^{-1}\).

The dynamical implications of this negative vorticity are considered later, but we may inquire how it may have been generated. The vorticity equation can be written

\[
\frac{D\zeta_a}{Dt} = V \cdot \nabla \zeta_a + w \frac{\partial \zeta_a}{\partial z} = -\zeta_a \nabla \cdot V + \frac{\partial u}{\partial z} \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \frac{\partial w}{\partial x} + \text{friction + solenoid term} \tag{3}
\]

The first expression on the r.h.s. is simply the application of Eq. (1) to \(\zeta_a\), the second represents the source terms for vorticity. Both expressions can in principle be estimated (except for the friction and solenoidal terms) from the velocity fields derived from the dropsonde data. An attempt was made to do this. The errors were so large that in general it was not possible to make any sensible comparison, but it was noted that both expressions produced large negative values of \(D\zeta_a/Dt\) of order \(5 \times 10^{-8}\)s\(^{-2}\) (2f h\(^{-1}\)) near the base of band II due largely to the tilting terms. One can see by inspection that large horizontal gradients of vertical velocity co-exist with large vertical wind shear in this region.

(c). Vertical wind shear

Vertical cross-sections of the components of the vertical wind shear in the \(x_f - z\) plane at \(y_f = -10\)km are shown in Fig. 11. A double peak in both components within the frontal zone is apparent, mainly below band II. There is some cellular structure in the shear pattern above the frontal zone which is reflected by a weakness in the frontal belt of shear under band II.

Fig. 12 demonstrates the systematic change in the field of \(\partial V/\partial z\) with height.
The principal feature is the progressive backing of the shear vector from left to right in the lowest four levels. This horizontal change is reflected in the vertical direction in Fig. 1 and occurs principally because the shear field is tilted in sympathy with the frontal zone. The main significance of this behaviour is its relationship to the thermal pattern. The synoptic scale thermal field (Fig. 2) suggests that the geostrophic wind shear vector should run parallel to the surface warm front, whereas the actual shear vector tends to be at 45° to the front, implying ageostrophic flow.

(d) Acceleration and ageostrophic flow

The equation of motion may be written in the form:

\[
\frac{1}{f} \frac{D\mathbf{V}}{Dt} = (\mathbf{V} - \mathbf{G}) \times \mathbf{k} + \mathbf{F}
\]

\[
= \frac{1}{f} \left[ (\mathbf{V} - \mathbf{S}) \cdot \nabla \mathbf{V} + w \frac{\partial \mathbf{V}}{\partial z} \right]
\]

\[
(4)
\]

The geostrophic wind, \( \mathbf{G} \), cannot be measured directly, but \( \frac{D\mathbf{V}}{Dt} \) may be estimated using Eq. (1), and on the assumption that friction, \( \mathbf{F} \), can be neglected, \( \frac{1}{f} \frac{D\mathbf{V}}{Dt} \) will be a direct measure of the ageostrophic wind. Since \( \mathbf{V} \) is also known, it follows that Eq. (4) could be used to give an indirect indication of the mesoscale horizontal pressure gradient. Differentiation of Eq. (4) with respect to height gives the thermal wind equation

\[
\frac{1}{f} \frac{\partial}{\partial z} \left( \frac{D\mathbf{V}}{Dt} \right) = \frac{\partial \mathbf{V}}{\partial z} \times \mathbf{k} - \frac{g}{fT} \nabla T + \frac{\partial \mathbf{F}}{\partial z}
\]

\[
(5)
\]

Eq. (5) can be used in principle – again neglecting friction – to estimate the horizontal temperature gradient which may then be compared with the synoptic scale estimate of \( \nabla T \) derived from Fig. 2.

Unfortunately, local estimates of \( \frac{D\mathbf{V}}{Dt} \) are subject to unacceptable error (see appendix). However, by averaging \( \frac{D\mathbf{V}}{Dt} \) over each of three 100km square sections of the dropsonde data, a coherent pattern of the profile of mean acceleration is obtained. These are displayed as polar diagrams in Fig. 13(a). The sections are labelled III, II and I to correspond roughly to bands III, II and I. For comparison, hodograms of \( \mathbf{G} \) (derived from Eq. (4)) and \( \mathbf{V} \) are shown in Fig. 13(b).

The following features of interest are noted.

(i) The transition, within the frontal zone, from highly supergeostrophic flow (in III)
Figure 13. (a) Polar diagram of acceleration averaged over three sections of the dropsonde network and expressed as an equivalent geostrophic wind in m s\(^{-1}\). (b) Polar diagram of wind, \(V\), and geostrophic wind, \(G\), estimated using Eq. (4). All values are entered at 0.2km intervals. Figures on curves are heights in km. Component resolved along (\(\theta\)) and across (\(\phi\)) front.

to subgeostrophic flow (in I) with increasing \(x_f\). In III, the acceleration vector is directed roughly perpendicular to \(V\), thus imparting an anticyclonic curvature to the flow, whereas in II and I, the acceleration vector is directed roughly parallel to \(V\) such as to increase it.

(ii) The change of the acceleration vector with height (Fig. 13(a)) appears to be directed roughly eastward in the lower part of the frontal zone (I and II) and roughly south-westward in the upper part of the frontal zone.

(iii) The tangent to the hodogram of \(G\) represents geostrophic wind shear and runs parallel to the isotherms. The backing and weakening of \(\partial G/\partial z\) from I to III (with decreasing \(x_f\)) is consistent in direction and magnitude with the synoptic scale pattern of isotherms at 700mb, as shown in Fig. 2. It reflects the approach of the occlusion and associated thermal ridge to the western edge of the dropsonde pattern.

(iv) While ageostrophic winds in the rainbands are generally less than 5m s\(^{-1}\), and rather erratic, a marked westward trend or ‘kink’ in the hodogram of \(G\) is apparent in I and II near the base of the rainbands, and this may reflect a local perturbation of the temperature field in which \(VT\) is directed towards the main ascent region of band II.

(e) **Internal friction**

Roach (1970) suggested that the Richardson number of a shear layer was limited by dissipation of turbulent energy working against the deformation fields trying to reduce Richardson number. Browning, Harrold and Starr (1970) produced evidence of the operation of this limiting mechanism in frontal zones similar to the one discussed here.

An expression for the rate of energy dissipation, \(\varepsilon\), was given as

\[
\varepsilon = \frac{(\Delta V)^2}{24} D
\]
where \( \Delta V \) = velocity difference across the shear layer and \( D \) = total deformation. Both \( \Delta V \) and \( D \) are scale dependent, so that the absolute value of \( \varepsilon \) is indeterminate unless some criterion can be found for the choice of layer thickness. It has been suggested that since the deformation fields and turbulence tend to be strongest within the frontal zone, the thickness of the latter should be taken, while \( D \) is measured from wind fields with a suitably matched horizontal resolution. In this case, layer thickness was taken as 400m to match the horizontal resolution of order 30km. This may result in an underestimate of \( \Delta V \) and an overestimate of \( D \), resulting in an underestimate of \( \varepsilon \) (in view of \( \Delta V^2 \) relationship) but probably not by a large factor.

Figure 14. Vertical sections of total deformation, \( D \), in units of \( 10^{-4} \text{s}^{-1} \), and turbulence index, \( \varepsilon \) (defined by Eq. (6)) in units of cm\(^2\)s\(^{-3}\).

The fields of \( D \) and \( \varepsilon \) obtained, Fig. 14, show a maximum of order 15cm\(^2\)s\(^{-3}\) in the vicinity of the low level jet core (Fig. 4). The corresponding order of magnitude of shear stress can be estimated from the scale equations

\[
\varepsilon \sim K \left( \frac{\partial u}{\partial z} \right)^2 \quad \text{dissipation} \sim \text{generation by Reynold stresses}
\]

\[
F = \frac{1}{\rho} \frac{\partial \tau}{\partial z} \sim K \frac{\partial^2 u}{\partial z^2}
\]

\[
\therefore |F| \sim \varepsilon/\Delta V \sim 2 \times 10^{-4} \text{m s}^{-2} \quad \text{for } \Delta V \sim 8 \text{m s}^{-1}
\]

This corresponds to an ageostrophic wind of 2m \( \text{s}^{-1} \) which is small compared with observed values (Fig. 13).

Thus it seems unlikely that internal friction will affect the estimates of ageostrophic flow discussed earlier.

5. ENERGY AND MOMENTUM TRANSFER

Estimates of fluxes and budgets of energy and momentum on all sub-synoptic scales is a major objective of this project. In this section the evidence from two sources is discussed: the aircraft inertial platform measurements of velocity fluctuations on scales up to about 20km (section 2(d)); and the dropsonde data which gives information on scales approximately in the range 50–200km.
(a) Kinetic energy budget

Writing the equation of motion in the form

\[ \rho \frac{DV}{Dt} = f \rho k \times (G - V) + F \rho \]  \hspace{1cm} (7)

then taking the scalar product of \( V \) with Eq. (7) and using Eq. (1), we obtain

\[ \rho V_s \cdot \nabla K = f \rho k \cdot G \times V + \rho V \cdot F \]  \hspace{1cm} (8)

where \( K = \frac{1}{2} V \cdot V \) = kinetic energy per unit mass.

Now

\[ \rho V_s \cdot \nabla K = \nabla \cdot (\rho K V_s - K V_s \rho) \]  \hspace{1cm} (9)

But \( \nabla \cdot \rho V_s = - \frac{\partial \rho}{\partial t} = 0 \) in steady state relative flow. \hspace{1cm} (10)

Combining Eqs. (8), (9) and (10), integrating over a volume and using Gauss’s divergence theorem, we obtain

\[ \frac{1}{V} \int_{S} \rho K V_s \cdot n dS = f \rho k \cdot G \times V + \rho V \cdot F \]  \hspace{1cm} (11)

where \( n \) is a unit vector perpendicular to the surface \( S \) enclosing the volume \( V \), and the bar notation on the r.h.s. indicates the volume mean.

Eq. (11) expresses the total rate of change of kinetic energy within a given volume in terms of the rate of generation by pressure forces and of destruction by friction without having to use estimates of the differential velocity fields.

In practice, the dropsonde data were divided into 15 boxes and the flux of kinetic energy across the six surfaces of each box was evaluated.

The alternative method of obtaining energy budget is based on Eq. (4) which is used to obtain an estimate of \( G \) (Fig. 13(b)) and hence \( k \cdot G \times V \). The comparison between the two methods is shown in Fig. 15 and is seen to be quite good. This gives further confidence in the estimation of differential fields from dropsonde data.

The presence of significant internal friction will not affect this comparison, since this term appears in both energy budget equations, but the estimated value of \( G \) may be affected. However, it does appear from section 4(e) that internal friction is probably negligible in this case.

Figure 15. Profiles of \( k \cdot G \times V \) estimated from the acceleration field (solid line) and the kinetic energy budget (dashed line) for the three 100km square sections of data corresponding to Fig. 13. U and L denote upper and lower boundaries of frontal zone.
The profiles of the energy budget are similar with respect to the frontal zone but with a lateral shift towards positive values (air accelerating) with increasing $x_f$. Significant deceleration is apparent near or just above the upper frontal boundary.

(b) Scales below 20km

Observations of the three components of velocity fluctuations were made using the inertial platform of the Canberra aircraft of the Meteorological Research Flight during a pass through the rainband system at 5–5km. The horizontal components were resolved along and transverse to the rainbands ($u' + ve$ towards 019°, $v' + ve$ towards 289°).

Spectra of sections of this run were obtained as follows:

I Overall run of $8\frac{1}{2}$ minutes (85km).
II 2-minute section of section I where convective activity was greatest (25km).
III 1-minute section of section I in a 'quiet' area (12km).

The time resolution used was 0.25s, corresponding to a flight distance of about 50m.
The resultant spectra are displayed in Fig. 16. The wavelength range over which useful information is obtained ranges from about 100m (the Nyquist value) up to about 20% of the length of the sample.

A $-\frac{5}{3}$ slope in $nS(n)$ fits quite well to all the data up to a wavelength of at least 1km. At longer wavelengths all the $w$ spectra and all the section III spectra tend to flatten out. In section I, the $-\frac{5}{3}$ slope persists up to about 5km for the $w$ component and to at least 20km for the $v$ component.

If this slope is identified with the inertial subrange, then the Kolmogorov relationship gives an estimate of the rate of energy dissipation

$$S(n) = C \varepsilon^{2/3} n^{-5/3}$$

where $S(n)$ is the spectral density at wave number $n$ ($= 1/\lambda$)

$C$ is a universal constant ($\sim 0.15$) (Panofsky and Pasquill 1963)

$\varepsilon$ is the rate of energy dissipation.

![Figure 16. Spectra of three components of velocity fluctuations from aircraft inertial platform measurements. In this diagram only, $u$ is positive along 019°, $v$ positive along 289°. Axes are linear in $\log nS(n)$ and $\log n$, where $n = (\text{wavelength})^{-1}$, $S(n)$ is spectral density.](image)

Then the values of $\varepsilon$ resulting are about $10 cm^2 s^{-3}$, overall, $30 cm^2 s^{-3}$ in the convective section and $3 cm^2 s^{-3}$ in the 'quiet' section.

The three components in general are comparable in magnitude up to a scale of 1–2km, but with the lateral ($v$) and vertical ($w$) components systematically higher than the longitudinal ($u$) component (a ratio of 4/3 is predicted for isotropic turbulence).

Looking now at the characteristics of individual spectra, perhaps the main features
are the marked minimum in all three components of \( nS(n) \) at wavelengths 5–7km adjacent to a corresponding maximum near 3km (also apparent in section II). This may reflect a modal size of individual convective elements (within the bands) of horizontal scale comparable with the depth (3–5km) of the cells.

The momentum fluxes, \( \overline{u} \overline{w'} \), \( \overline{u'} \overline{w'} \), are estimated by integrating the co-spectra over wavelength. There appears to be a net downward transfer of westerly momentum at this level (5-5km) of order \( 0.1 \text{m}^2\text{s}^{-2} \) by scales of 2–5km and a downward transfer of southerly momentum of similar magnitude by scales of 15–20km. However, the magnitude of these fluxes is small (typical values of momentum flux in the atmospheric boundary layer are \( 0.5 - 1 \text{m}^2\text{s}^{-2} \)) and probably not significant.

(c) Scales above 20km

For any given horizontal section, one may form quantities of the type
\[
\overline{u_i u_j} = \overline{u_i u_j} + \overline{u_i u_j}
\]
(12)
i.e. 'mean' and 'eddy' fluxes of the various components of momentum \((i \neq j)\), or the velocity variance \((i = j)\) in a wavelength band.

The result of this averaging is shown in Fig. 17. The largest values in all parameters occur in the frontal zone primarily due to leakage of synoptic scale contribution into the wavelength band 200–50km, and manifested by the frontal slope. In this situation the assumptions made in deriving Eq. (12) are no longer valid.

![Figure 17. Profiles of components of velocity variance, \( u'^2 \), \( v'^2 \), \( w'^2 \); co-variance, \( \overline{u'w'} \), \( \overline{v'w'} \); and mean velocities \( \overline{u}, \overline{v} \) averaged over complete horizontal section of the dropsonde box.](image)

Above the frontal zone, the synoptic scale contribution to the energy appears to be much smaller, but the problem of aliasing of high frequency energy into this band arises. It is estimated (see appendix) that the contribution to variance of kinetic energy centred near 100km is of order \( 2 \text{m}^2\text{s}^{-2} \) in the convective regions decreasing to less than \( 1 \text{m}^2\text{s}^{-2} \) in 'quiet' regions.

It is apparent (Fig. 17) that the total horizontal kinetic energy variance, \( \frac{1}{2}(u'^2 + v'^2) \), varies between about 3 and 7\( \text{m}^2\text{s}^{-2} \) in the rainbands (above about 2.5km). Allowing for aliasing reduces this value to a range of about 2–5\( \text{m}^2\text{s}^{-2} \) within a logarithmic spectral interval of about unity. This is appreciably higher than the few available estimates of spectral density in this region. Kao and Woods (1964), investigating mesoscale fluctuations in wind along and across jet streams, observed a peak in \( nS(n) \) of about \( 1 \text{m}^2\text{s}^{-2} \) at a wave-
length of about 100 km. Axford (1972) observed a value of 1–2 m² s⁻² at 100 km in a flight across an upper trough.

The vertical velocity variance is less affected by long wave contributions and increases progressively to about 0.03 m² s⁻² at a height of 4 km – as might be expected from the increasing amplitude of the w-field with height.

The momentum flux was of order 0.1 m² s⁻² at the rainband levels for both components, indicating (rather surprisingly in view of Fig. 5 of BHHP) little correlation between horizontal and vertical velocity fluctuations on these scales. This momentum flux is of similar order to that for wavelengths less than 20 km and probably also not significant.

The contribution of this flux to the ageostrophic wind is given by

\[ \frac{1}{f} \frac{\partial}{\partial z} (u'w') \sim \frac{0.2}{fH} \text{ m s}^{-1} \sim 0.5 \text{ m s}^{-1} \]

where \( H \) is a characteristic depth of convection.

An attempt was also made to derive the horizontal flux of momentum \( \overline{u'v'} \) (where \( u' = \) perturbation of westerly component, \( v' \) = perturbation of wind component perpendicular to \( x_f \) or S) in the rainband region. The averaging was over the wind field between 2.5 and 4 km in the \( x_f - z \) plane. Table 1 shows the result as a function of \( y_f \).

<table>
<thead>
<tr>
<th>( y_f (\text{km}) )</th>
<th>( u (\text{m s}^{-1}) )</th>
<th>( \overline{u'v'} (\text{m}^2 \text{s}^{-2}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>20.6</td>
<td>-0.8</td>
</tr>
<tr>
<td>20</td>
<td>20.9</td>
<td>-0.3</td>
</tr>
<tr>
<td>0</td>
<td>21.4</td>
<td>-0.2</td>
</tr>
<tr>
<td>-20</td>
<td>21.7</td>
<td>-0.3</td>
</tr>
<tr>
<td>-40</td>
<td>21.6</td>
<td>( { &lt;0.1 } )</td>
</tr>
<tr>
<td>-60</td>
<td>21.0</td>
<td></td>
</tr>
</tbody>
</table>

There is a small counter gradient eddy flux of momentum, but its horizontal gradient (\( \sim 1 \text{ m}^2 \text{s}^{-2} \) in 40 km) would only contribute \( \sim 0.2 \text{ m s}^{-1} \) to the ageostrophic wind.

There is, however, some indication that the mean values of vertical velocity averaged over 100 km squares (Fig. 8) were significantly greater than estimated from the adiabatic method, suggesting that the vertical transfer of heat might be significant and that the rainband system might 'short-circuit' the normal baroclinic ascent to some extent in respect of heat, but not momentum transfer.

In conclusion, it does appear that eddy fluxes of momentum in rainbands are rather small within the range of scales investigated. This is consistent with the general view that the role of convection is to transfer heat rather than momentum (except perhaps in severe storms driven by wind shear – Moncrieff and Green 1972).

6. Discussion

There are two principal themes emerging from this study:

(i) The motion fields within the frontal zone and its associated low level jets.
(ii) The motion fields associated with the rainbands.

(a) Frontal structure

Hoskins (1971) investigated the development of a front within a fluid initially uniformly (but weakly) baroclinic, and subjected to a uniform deformation field. By assuming
geostrophic balance across but not along the front and employing a Lagrangian approach based on the Kelvin circulation theorem, the mathematical difficulties normally associated with a highly non-linear problem were largely circumvented.

In spite of the idealized nature of this model a frontal zone developed having some features very similar to those observed in the atmosphere, e.g.:

(i) a sloping hyperbaroclinic zone extending throughout the troposphere;
(ii) a folding tropopause and associated high level 'westerly' jet;
(iii) a low level 'easterly' jet with its core on the surface in the cold air immediately beneath the frontal zone.

Hoskins (private communication) later modified the model to include surface friction and latent heat release, which had the effect of raising the core of the low level jet to about 800m and increased the vertical velocity of air ascending the front.

We have attempted to compare the wind field associated with this case study with the 'prediction' of Hoskins' model. We refer to Hoskins' model as $H$ and this case study as $S$.

![Figure 18](image)

Figure 18. Comparison of hodograms of front-relative velocity from $H$ (solid lines) and $S$ (dotted lines). $U$ and $L$ denote upper and lower boundaries of frontal zone in $H$ ($\Delta$) and $S$ ($\bullet$).

Fig. 18 shows a comparison of the hodograms of front-relative velocities of $H$ and $S$ as a function of $x_f$ — distance ahead of surface warm front. The comparison of vertical velocities is shown in Fig. 8.

(i) For $x_f \geq 100$km, the magnitudes of $u_f$ are comparable in $H$ and $S$. $u_f$ reaches a maximum of 20–25m s$^{-1}$ at a height of 0.5–1km, then decreases with height, changing sign at 2–2.5km in $S$ and about 1km in $H$.

(ii) The behaviour of $u_f$ is very different in $H$ and $S$. In $H$, $u_f$ increases from about -5m s$^{-1}$ at low levels to 3–5m s$^{-1}$ above 1.5km with little further increase with height. There is a slight bodily shift of the $H$ hodogram towards negative $u_f$ values with increasing $x_f$. In $S$, $u_f$ also increases with height, but to a well-defined maximum of about 10m s$^{-1}$ between $x_f = 100$ and 200km.

The height at which $u_f$ changes sign increases with $x_f$ in both $H$ and $S$ — in $S$ from 0.5km near $x_f = 0$ to 2.5km at $x_f = 300$km; in $H$ the change is about 0.5km lower.

(iii) As a consequence of the large $u_f$ values in $S$, the vertical shear vector is backed about 45° on the frontal orientation in $S$, but only about 10–20° in $H$.

(iv) The profiles of vertical velocity in $S$ (averaged over 100km squares) and $H$ are similar, but the magnitudes in $S$ are about twice those in $H$ (Fig. 8).
(v) In H, scale analysis shows that the cross-front accelerations are small compared with along front accelerations. In S (Fig. 13), it is seen that the components of acceleration are comparable.

Thus the low-level front parallel wind maximum is common to both H and S but the cross front flows and (consequently?) vertical velocities though similar in sign are much greater in magnitude in S than in H.

These differences may arise from one or both of two obvious differences between H and S:

(i) Following close on the heels of the warm front in S is the cold front, and the maximum cross-front flow at about 2km may be associated with the cold front jet riding up the warm frontal zone as part of the warm sector 'conveyor belt' (Harrold 1973, Browning and Pardoe 1973). It is well known that vertical velocity maxima tend to occur near the point of occlusion, which was not far to the NW of the area covered by S.

(ii) In H, the front is stationary; in S it is moving eastwards at about 16m s⁻¹.

This comparison therefore suggests that the warm front parallel wind maximum in S is produced by a large scale deformation associated with baroclinic development in a low Rossby number regime similar to that predicted by H, but that the overriding cold front parallel jet cannot be so explained; it is flowing in the opposite direction in relation to the thermal field across the cold front, and may in fact be an inertial boundary layer (Rossby number of order unity) induced by the eastward progress of the cold front and modified by surface friction.

(b) Rainband structure

It has been suggested in BHHP and elsewhere that the rainbands are due to the release of potential instability produced by the overrunning at middle levels of relatively dry air. BHHP says: ' It is not clear whether the organization of the convection into bands was the result of pre-existing banded structure within the potentially cold air aloft or to other (unknown) dynamical factors leading to a banded pattern of large-scale ascent within the underlying conveyor belt flow. Of the two explanations, that involving pre-existing bandedness within the pattern of potential instability isfavoured by the finding that the bands lost their identity when the potential instability became exhausted. Alternatively it is possible that the existence and spacing of the bands is some function of the cumulonimbus convection itself rather than the result of streakiness previously introduced. . . . If this is the case, then it would be necessary to think of the band development as leading to the disturbance in the field of wind and baroclinicity rather than vice versa.'

The analysis presented in this paper inclines us to the latter view because of the following considerations:

(i) A scale analysis of the equations of motion and continuity suggests that small differences in density between air in and between the bands would produce enough hydrostatic pressure difference to induce (through the resultant perturbation of the horizontal pressure field) wind perturbations of the characteristic dimensions and magnitudes observed.

(ii) Hoskins (1974 a, b) has suggested that the rainbands may be initiated by what he calls 'conditional symmetric instability'. 'Conditional' implies that the instability is dependent upon latent heat release – a possibility foreshadowed by Sawyer (1956) – and 'symmetric' implies a roll type of perturbation of amplitude which varies in the cross-flow, but not the along-flow, direction (as in baroclinic and Rayleigh instability). It appears that instability can occur if the θ-surfaces (and in saturated ascent, the θ₀ surfaces) are steeper than the absolute vorticity vector.
MESOSCALE AIR MOTIONS

If the vorticity vector lies between the $\theta$ and $\theta_v$ surfaces, there is conditional symmetric instability, the criterion for which depends in any particular situation on continuity requirements, i.e., whether unstable saturated ascent can 'drive' stable dry descent.

7. CONCLUSIONS

It is suggested on the basis of the observations presented that:

(i) The observed warm frontal structure can be accounted for in terms of Hoskins' (1971) model of frontogenesis modified by the proximity of the cold front circulation and its associated low level jet stream.

(ii) The mesoscale motion field associated with the rainbands is induced by the persistence for a sufficient period of a perturbation pressure field associated ultimately with the release of latent heat within the bands. The characteristic spacing of the bands may result from a type of symmetric instability proposed by Hoskins (1974 a, b).

(iii) The horizontal and vertical transfer of eddy flux of momentum on scales up to about 100km appears to be unimportant to the local dynamics of the situation.

The main shortcomings of the observations were:

(i) The dropsonde network was too restricted in depth and extent.

(ii) Temperature and humidity observations were lacking.

It is planned to remedy these defects in a future field exercise. It is also planned to construct a numerical model of rainband formation in the vicinity of a frontal zone (taking into account the release of latent heat) based on a scheme formulated by Hoskins (1974b).

ACKNOWLEDGMENTS

The results of this paper were obtained as part of Project Scillonia, led by Mr. P. Goldsmith. We are grateful to all those who took part in the investigation, particularly the aircrew and staff of the Meteorological Research Flight, Farnborough, the Air Transport Development Unit RAF Benson, the radiosonde staff at Camborne, Crawley, Aberporth and Larkhill, and our colleagues of the Cloud Physics Branch of the Meteorological Office. We are grateful for useful discussions with Dr. B. J. Hoskins of the Department of Meteorology and Geophysics, University of Reading.

REFERENCES


1970 'Richardson number limited shear zones in the free atmosphere,' Ibid., 96, pp. 40-49.

1973 'Structure of low level jet streams ahead of mid-latitude cold fronts,' Ibid., 99, pp. 619-638.
APPENDIX

The problem of the representation of mesoscale motion fields derived from a dropsonde data network has been fully discussed by the authors in an unpublished note. The main findings are summarized here.

The analysis of the dropsonde data depends upon the determination of a system-velocity and upon the assumption that the wind field is steady-state (or 'frozen') in a frame of reference moving with this system. This principle is equivalent to the use of the Taylor assumption in turbulence measurements in which the mean wind speed represents the system velocity and allows a time scale to be converted to a space scale.

1. THE WIND FIELD

Incorrect representation of the wind field may arise from four sources:

a. instrumental error;

b. aliasing of sub-grid scale motions;

c. system-velocity error;

d. non steady state wind field.

Of these, the first is the only source of error over which we have control. The other three are controlled by the natural behaviour of the atmosphere and are significantly interrelated.

(a) Instrumental error

This has already been discussed in Hardman, James and Goldsmith (1972) – (HJG). Briefly the wind error depends upon the error of radar determination of the position of the sonde, which is recorded once per second. The winds are first derived from the observed horizontal shift in the sonde positions over 60 seconds, and then smoothed by a numerical
filter which considerably reduces the scatter and produces a wind representative of the mean wind over a depth of about 500m.

The following errors are quoted in HJG:
wind \( \pm 0.3 \text{m s}^{-1} \)
divergence \( \pm 2 \times 10^{-5} \text{s}^{-1} \)
vertical velocity \( \pm 1 \text{cm s}^{-1} \).

These errors, as will be seen, are generally small compared with those introduced by the atmosphere.

\(b\) Aliasing

 Aliasing can affect the wind observations in two ways:
(i) by increasing the uncertainty of individual wind observations;
(ii) by contaminating spectral estimates of mesoscale kinetic energy. The grid spacing of dropsonde observations is about 30km (I) in the horizontal and 0.5km (h) in the vertical. A given wind observation, however, is averaged over a vertical interval \(W\Delta t\) and horizontal distance \(V\Delta t\), where \(W = \) fall speed of dropsonde \((\sim 8 \text{m s}^{-1})\), \(\Delta t\) is averaging interval (1 minute). Thus the vertical interval is roughly matched to \(h\), but the horizontal interval is two orders of magnitude less than \(I\).

Thus the problem is to decide the extent to which a 'point' observation of wind can be considered as representative of an area of \(\sim 10^3\text{km}^2\). Away from the convective areas, the atmosphere has a stratified structure and identifiable features of a wind profile have horizontal dimensions of perhaps two orders of magnitude greater than their vertical dimensions. Thus vertical smoothing of the wind profile must have an indirect horizontal smoothing over an area comparable to \(I^2\). This is not true in convective rainbands, where the physical dimensions of horizontal and vertical fluctuations are probably comparable.

An attempt to estimate the magnitude of the aliasing effect was made using the results of the inertial platform measurements (section 5(b)). This led to the conclusion that wind observations in convective regions will be representative of the mean value over a grid square with a probable error of not less than 2m s\(^{-1}\), but away from convective regions should be less than 1m s\(^{-1}\).

These measurements were also used to infer that aliasing may contribute a total of 2m s\(^{-2}\) to the value of \(nS(n)\) – where \(n\) is frequency and \(S(n)\) spectral density – at 100km, estimated from dropsonde data.

\(c\) Errors in system velocity

Estimates of system velocity will be affected by:

(i) Aliasing. The comparison drop may fall through a convective region, thus causing an unrepresentative bias of the wind profile, which will alter the field of variance (see HJG) on which the estimate of system velocity is based.

(ii) Wind shear. A considerable contribution to total variance will come from the frontal zone. This (HJG) may bias the estimate of system velocity toward the movement of the frontal system.

(iii) Variations in system velocity over the depth of a given profile. For instance, it was noted in BHHP that the rainband system was overtaking the warm frontal system at about 5m s\(^{-1}\).

(iv) Development. The method of determining system velocity automatically minimizes development in relation to a particular comparison drop, but may not apply elsewhere in the pattern.
Errors in the system velocity have no direct effect on errors in the wind measurement, but they will strain the system-relative co-ordinates. The effects of (i) and (ii) above will be to compress or stretch the horizontal co-ordinates; (iii) will shear and (iv) will deform these co-ordinates. This will result in errors in the differential properties of the wind field and in the computation of vertical velocity.

(d) Development

Development is, of course, occurring on all scales. On the synoptic scale it is reflected in changes of system velocity. System velocity itself may be regarded as the velocity of a basic flow on which mesoscale perturbations are superimposed; this basic flow is, in turn, part of a larger (baroclinic) eddy superimposed on a basic flow of planetary scale. Conversely, $V_s$ is the basic flow for sub-mesoscale motions, the development of which gives rise to aliasing problems.

Thus the term 'development' is used in a fairly restricted sense here to refer to changes of the motion fields on the particular scale of the rainbands during the period required to complete a data set, and is thus separated from the problem of development on smaller (aliasing problem) and larger (system-velocity changes) scales. The synoptic scale motion field may deform the mesoscale fields of motion appreciably in a few hours. Thus as for system velocity, the effect of development is to strain the system-relative co-ordinates, and not to increase errors in wind determination.

2. The differential wind field

The interpretation of the differential wind field formed a central part of the diagnostic analysis discussed in the main paper. It was found that the potential of this analysis was considerably limited by the wind field errors discussed above.

If the observations of a parameter $x$ are made at two points $A$ and $B$ a distance $L$ apart, then the quantity $(x_A - x_B)/L$ is a measure of the mean gradient of $x$ along $AB$. If $\sigma_x$ is the standard deviation in $x$ due to the various causes discussed in section 2, the error in the determination of this gradient will be $\sqrt{2} \sigma_x/L$.

If $A$ and $B$ form part of a three-dimensional network of observations of $x$ of which some form of objective analysis is made, then it is reasonable to suppose that the error in the mean gradient along $AB$ will be reduced as the result of surrounding additional information. The actual factor of reduction cannot be objectively assessed as it will depend upon the method of analysis, but it will not be much less than unity, as the method of smoothing by numerical filter used on the vertical wind profiles is not available in the horizontal. An error of $\sigma_x/L$ is probably a conservative estimate and estimates of errors in the differential wind field are based on this figure.

It has been estimated that $\sigma_x$ will vary from rather less than $1 \text{m s}^{-1}$ in 'quiet' air to $2-2.5 \text{m s}^{-1}$ in convective activity. The corresponding errors in differential quantities such as divergence, vorticity and deformation will be of order $\sigma_x/L$, and will range from $\pm 2-3 \times 10^{-3} \text{s}^{-1}$ in 'quiet' air to $\pm 6-8 \times 10^{-4} \text{s}^{-1}$ in rough air. Since the overall variation in these quantities is $\pm 2-3 \times 10^{-4} \text{s}^{-1}$, this represents a signal-to-noise ratio in the range of $3-10$ to $1$. Similar considerations apply to error in vertical wind shear which will range from about $\pm 2-5 \times 10^{-3} \text{s}^{-1}$ depending upon the conditions, and this compares with magnitudes of wind shear of up to $3 \times 10^{-2} \text{s}^{-1}$ in frontal zones. Thus the general patterns of these fields will be well defined in 'quiet' air and only roughly defined in regions of convective activity.
(a) **Vertical velocity**

Errors in vertical velocity ($\sigma_w$) are estimated using the divergence equation. Contributions to $\sigma_w$ arise from:

(i) **Aliassing.** It is estimated that $\sigma_w$ varies from 1–2 cm s$^{-1}$ in 'quiet' air to 3–5 cm s$^{-1}$ in rough air.

(ii) **System velocity errors.** An error in system velocity will stretch the co-ordinates leading to errors in divergence and vertical velocity of order 5% (i.e. $\sim$ 1 cm s$^{-1}$ in vertical velocity) and can therefore be neglected.

A variation of system velocity with height will cause a lateral shift of one part of the wind pattern relative to another part at a different level during the time taken ($\sim$ 30 min) to complete a divergence square. The resultant error in vertical velocity is unlikely to exceed 3 cm s$^{-1}$.

(iii) **Development.** This will also deform the co-ordinates, but at a slower rate, and its effect on vertical velocity is probably negligible.

Thus it is concluded that the error in vertical velocity due to all atmospheric causes is unlikely to exceed $\pm$ 5 cm s$^{-1}$.

(b) **Lagrangian derivatives**

Attempts to measure the Lagrangian derivative of the system relative position vector, $r_s$; the velocity, $V$; and absolute vorticity, $\zeta_a$ were discussed in sections 3 and 4.

(i) **Position vector, $r_s$.** This was used to derive streamlines (system relative trajectories) from

$$\Delta r_s = (i\nu_s + jv_s + kw)\Delta t.$$  

The probable error in $r_s$, $\sigma_s$, will be of the order

$$\sigma_s \Delta t \text{ in the horizontal}$$
$$\sigma_w \Delta t \text{ in height}$$

after one time step $\Delta t$ and respectively $N^4 \sigma_s \Delta t$ and $N^4 \sigma_w \Delta t$ after $N$ time steps. For $N \sim 30$, $\sigma_s \sim 2$ m s$^{-1}$, $\Delta t \sim 500$s, this gives a probable error of $\pm 5$ km in horizontal location and $\pm 0.1$ km in height. These are small compared with the total displacements observed during the period $N\Delta t$ ($\sim$ 4 hours) and can be neglected.

(ii) **Velocity, $V$.** We have

$$\frac{DV}{Dt} = (V - S) \cdot VV + w \frac{\partial V}{\partial z},$$

and

$$\delta \frac{DV}{Dt} = VV \cdot \delta V_s + V_s \delta (VV) + \frac{\partial V}{\partial z} \cdot \delta w + w \delta \left( \frac{\partial V}{\partial z} \right),$$

and

$$\sigma_{acc}^2 \sim (VV \cdot \sigma_v)^2 + \left( V_s \cdot \frac{\sigma_v}{f} \right)^2 + \left( \frac{\partial V}{\partial z} \right)^2 \sigma_w^2 + \left( w \frac{\sigma_v}{n} \right)^2$$

Each standard deviation has a multiplier the magnitude of which will vary over the field. Using some characteristic values from the main paper gives an acceleration error equivalent to an ageostrophic wind of order 10 m s$^{-1}$.

In the rainband region, the vertical velocity terms dominate $\sigma_{acc}$; in the frontal zone and below, values of $V_s$ become large and of $w$ small so that the horizontal advection terms dominate here. In either case, the resultant errors are so large that the signal-to-noise ratio is of order unity over most of the field.
It was thus found impracticable to attempt to delineate acceleration fields in any detail, although averaging acceleration over squares of approximately 100km produced coherent results which are discussed in the main paper.

(iii) Vorticity, $\zeta$. It is quite impracticable to evaluate $D\zeta/Dt$, since the signal-to-noise ratio here is much less than unity.

(c) Second differential fields of velocity

Since fairly coherent fields of vorticity, divergence, deformation and wind shear were obtained, this implies that it is possible to identify principal areas of change of these quantities. Analysis of the error fields suggests that the signal-to-noise ratios will be rather near unity, and it is considered that it will only be possible in general to glimpse the approximate direction of second differential fields where these are large.