The horizontal cross-sectional shape of convective plumes

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(Received 21 November 1973; revised 8 October 1974. Communicated by Dr. G. A. McBean)

SUMMARY

The horizontal cross-sectional shape of the temperature field of convective plumes at a height of 3.5m in the atmospheric surface layer was observed to be very elongated in the downwind direction with a ratio of downstream to cross-stream diameters of about 8:1. A dynamic explanation for this elongated shape is presented, based upon the minimization of the heat flux lost by a plume due to turbulent mixing around its boundary.

1. INTRODUCTION

In previous studies of the structure of convective plumes in the atmospheric boundary layer, several attempts have been made to determine the shape of the horizontal cross-section of the plumes. Warner and Telford (1963) suggested circular cross-sections on the basis of an examination of the time traces of the temperature signals from aircraft flights at various directions to the mean wind and at heights from about 15m up to near the cloudbase. However, Lenschow (1970) found a distinct elongation of the plumes in the downwind direction above 100m again using aircraft based results, over land. Frisch and Businger (1973), using aircraft measurements, found little evidence of elongation of plumes at a height of 30m over grasslands and suggested a stability dependence might be involved with circular plumes occurring only for the condition of free convection. Priestley (1959) suggested that the local 'hot spot' needed to form a plume initially would exist for some time and so the preferred type of plume would be drawn out in the downwind direction as it advected along with the wind. Clearly the shape of the horizontal cross-section of the plume has not yet been firmly established or explained.

In this study, the horizontal cross-sectional shape of the temperature field associated with convective plumes at a height of 3.5m was determined using the temperature signals from an array of five sensors. A dynamical explanation is given for the observed elongated form.

2. EXPERIMENTAL SET-UP AND DATA SET

The data to be discussed come from an intensive experimental programme undertaken in July 1971 on a slightly undulating grassland with tens of kilometres of uninterrupted fetch, at Suffield, Alberta, Canada. An outline of the experiment and some of the details of the instrumentation were given in a previous paper Davison (1974); this paper will be referred to as D.

The present paper involves an analysis of the temperature signals from four thermistors and a sonic anemometer-thermometer at a height of 3.5m on an array of five towers (Fig. 1). The data set is the run starting at 1155 local time (1955GMT) on 16 July 1971 discussed in D. On 16 July there were clear skies, a mean temperature at 3.5m of 32°C, and winds of 5-5m/s and 10.1m/s at 3.5m and 92m respectively. The mean wind direction at 3.5m as

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measured by the sonic anemometer was $236^\circ \pm 5^\circ$M. For a period of over two hours during the run the wind speed and direction and the temperature were nearly steady.

3. Experimental determination of the plume horizontal cross-sectional shape

In D the statistical values of the coherences and phases between pairs of the five temperature signals of the array at 3.5m were used to determine the average translation velocity of the convective plumes. For the run on 16 July, the translation velocity was found to be $10 \pm 2$ m/s from $229^\circ \pm 6^\circ$M. Knowing this translation velocity and the physical separations of the sensors (see again Fig. 1), the time traces of the temperature signals were appropriately spaced and delayed to effect a time-to-space transformation, with the assumption that the plumes do not change shape significantly in their passage through the array.

The temperature and velocities at 3.5m associated with the passage of a convective plume are shown in Fig. 2. As can be seen, the passage of a plume past a sensor took about 20 seconds corresponding, for a translation speed of 10 m/s, to a plume length of about 200m. Since the total length of the array used on 16 July was only 73m, the above assumption of no change in plume shape during passage through the array seems reasonable.

On this assumption, temperature contours were visually interpolated between appropriately time-shifted simultaneously recorded time traces of the temperature signals. In this way a 'snapshot' -like picture of the horizontal cross-section of the temperature field at a height of 3.5m was made in a frame of reference moving with the plumes (Fig. 3). For this procedure to produce meaningful results, the magnitude of the fluctuations due to plumes must be significantly larger than those due to random turbulence effects which may not advect at the same speed as the plume and which do not form a shape-preserving feature in time. As can be seen from Fig. 2, the temperature signal satisfies this criterion much better than the vertical velocity signal. This is why the temperature rather than the vertical velocity was used to outline the plume boundaries.

Only four sensor lines are shown in Fig. 3. Sensor 4 was sufficiently displaced crosswind from the other sensors for its temperature signal to be so poorly coherent with the other
Figure 2. Simultaneous temperature and velocity time traces at a height of 3.5 m during the passage of a convective plume during the 16 July run.

signals that temperature contours involving it could not be reliably drawn. Note that information about the temperature field exists only along the sensor lines in Fig. 3; so all the contours are visual interpolations.

Figure 3. Horizontal temperature contours of a plume at 3.5 m. The contours were visually sketched from appropriately delayed time traces of the indicated temperature signals.

There are uncertainties in the details of the temperature contours in Fig. 3 due mostly to changes in the temperature field during its passage through the array; to deviations of particular plume motions from the average plume motion used for the time–space transformation; and to simplifications in the process of visual interpolation. However, the general features of the plume cross-section are clear.

The most obvious feature of the temperature contours is the marked elongation of the plumes in the downwind direction. This had been suspected from the very sharp crosswind
fall-off of the coherences between the temperature signals which was found in D. If the plume cross-sectional shape is approximated by an ellipse with the major axis in the downstream direction, then the ratio of the major to minor axes (or the downstream to cross-stream diameters) in Fig. 3 is about 8 ± 4. The large uncertainty is due mostly to uncertainty in the cross-stream width (minor axis). For the ratio of the axes to have been close to 1, the translation speed of the plume would have had to be about 1 or 2m/s, compared with the measured average translation speed of 10 ± 2m/s.

4. DYNAMICAL EXPLANATION FOR THE OBSERVED CROSS-SECTIONAL SHAPE

(a) Approach to problem

The hypothesis was made that the cross-sectional shape of a plume is such that the heat flux lost by a plume due to mixing effects around the plume boundaries is minimized. This hypothesis means that the shape reflects the angular dependence, around the plume boundary, of the mixing. It is equivalent to maximizing the heat flux carried by the plume through the surface shear zone and so is analogous to the hypothesis used as a basis for the explanation of the observed translation speed in D.

A plume scale heat flux equation was developed and applied to a model of a plume having an elliptic horizontal cross-section. The terms which contribute to the heat flux lost by a plume due to mixing at the plume boundaries were formulated in terms of measurable quantities, and the ratio of the semi-major to semi-minor axes of the plume horizontal cross-section. The expression for the heat loss was minimized to give the required ratio. The numerical value of the required ratio was estimated by substituting temperature and velocity measurements from the 16 July run and was compared with the ratio measured from the temperature contours in the previous section.

Although there are some rather gross simplifications used in the following analysis this study shows that the observed shape can be explained in terms of a dynamic balance and indicates the parameters controlling the plume shape.

(b) Formulation of the heat flux losses

(i) Plume scale heat flux equation. Following Lumley and Panofsky (1964, pp. 59 ff.) but using three scales of motion (as was done by LeMone 1972), rather than two scales, then a plume-scale equation of motion and a heat equation can be derived. These were combined to give a plume-scale heat flux equation which when averaged over plume scales (to be defined below) becomes:

\[
- \frac{\partial [W^p T^p]}{\partial t} = [U^p (T^p W^p)]_p + [U^p T^p W^p]_p + T^p [(W^p)^2]_p - \frac{g}{T^p} [(T^p)^2]_p + \frac{[T^p P^3]}{\rho} + [T^p (U^p U^p)]_p + [W^p (T^p U^p)]_p - \kappa [W^p T^p x^p]_p - \nu [T^p W^p]_p
\]

(1)

where superscript \( p \) refers to plume scales, \( s \) to synoptic scales, and \( t \) to turbulent scales. Square brackets followed by a subscript \( p \) denote plume-scale averaging (to be defined below) and curly brackets followed by a subscript \( t \) denote turbulent-scale averaging. The \( x, y, z \) components are represented by subscripts 1, 2, 3, respectively. \( \kappa \) is the thermal diffusivity; \( \nu \) is the kinematic viscosity. Differentiation is denoted by a comma (e.g. \( \phi_{x_t} = \partial \phi / \partial x_t \)).
In the above equation it was assumed that the three scales of motion could be separated by appropriate averaging. Turbulent-scale averaging was taken in a frame of reference moving with the plumes over a period of time (say, several minutes) short compared to the plume lifetime but long enough for random turbulent effects to average to zero. Plume-scale averaging was taken over a scale large enough for variations due to plumes to average to zero.

In their influence on the amount of heat flux lost (or gained) by a plume due to shape changes, the molecular diffusion terms in (1) were considered negligible compared with the turbulence terms; and the two source terms, \((g/\bar{T})(\bar{T}''p)\) (the buoyancy term from the equation of motion) and \(T_{33} [(\bar{H}''p)]p\) (a source term if the mean temperature profile is unstable), were assumed to have no direct dependence on plume shape.

After plume-scale averaging, the convective field was assumed to be stationary and horizontally homogeneous. Thus the synoptic-scale transport term (second term on the right-hand side of (1)) and the time derivative vanished.

The plume-scale transport term and the pressure term were assumed to balance. This assumption means that the plume-scale pressure field is created by the plume-scale motions. As discussed in D, the plume-scale pressure field close to the surface was assumed to have two major features: a low pressure inside the main body of the plume caused by a region of positive vertical velocity at higher levels and a region of increased pressure at the upwind edge due to the observed convergence effects at the upwind edge. Some experimental verification of this assumption and the sensitivity of the final result to this assumption are discussed below.

With the above assumptions, the turbulence interaction terms are the only terms left in Eq. (1) which contribute to the heat flux lost by a plume and which have a possible dependence on the shape of the plume. These terms will be examined below.

(ii) Plume model. A model of the plume was adopted with an elliptic cross-section in the \(X-Y\) plane (Fig. 4). The ratio of the semi-major to semi-minor axes (or the downstream to cross-stream diameters) of the cross-section is defined as \(\gamma\) and is given by \(\gamma = a/b\) where \(a\) and \(b\) are the semi-major and semi-minor axes of the horizontal plume cross-section, respectively. The dependence of the amount of heat flux lost by a plume due to interactions represented by the turbulence terms, on the plume shape, can be found by taking a volume

![Figure 4. The model of the plume horizontal cross-section where \(\mu = \mu_b\) is the plume boundary.](image)
integral of the terms over plume scales. The turbulence terms were formulated such that only measurable quantities and the semi-axes \(a\) and \(b\) appear.

Minimization of the heat flux lost by the plume then led to an equation for \(\gamma\).

(iii) *Form of the volume integrals.* Because of the assumed elliptic cross-section of the plumes in the \(X-Y\) plane, the volume integrals were made using an elliptic cylindrical co-ordinate system. Following the notation of Morse and Feshback (1959, pp. 997 ff.) the volume element is

\[
h_1 h_2 h_3 d\xi_1 d\xi_2 d\xi_3 = -(\cosh^2 \mu - \cos^2 \phi) \lambda^2 d\mu d\phi dz .
\]  

(2)

where \(h_i\) and \(\xi_i\) are the \(i\)th scale factor and co-ordinate. In the \(X-Y\) plane this co-ordinate system consists of ellipses and hyperbolas. \(\mu = \text{constant}\) is an ellipse; the \(\mu\)-vector points outwards from the ellipse. \(\phi = \text{constant}\) is a hyperbola whose asymptote forms an angle \(\phi\) with the line between the two foci of the ellipse (i.e. the major axis or the \(X\)-axis in Fig. 4). \(\lambda\) is a length scale given by \(\lambda^2 = a^2 + b^2\) where \(a\) and \(b\) are the downstream and cross-stream radii defined earlier. The plume boundary is taken to be \(\mu = \mu_b\).

The turbulence terms in Eq. (1) will be shown to be of the form \(F(\mu)G(\phi)H(z)\) where \(F\), \(G\) and \(H\) are functions of \(\mu\), \(\phi\) and \(z\) respectively. The vertical variation \(H(z)\) will be considered only qualitatively in this study.

All the terms will be shown to involve \(\partial T^p/\partial \mu'\) or \(\partial W^p/\partial \mu'\) as the \(\mu\)-dependence, where \(\mu'\) is the \(\mu\)-vector in the \(X-Y\) plane along the normal to the ellipse perimeter. \(d\mu'\) is the line element \(h_1 d\xi_1\) in Eq. (2).

![Figure 5](image)

**Figure 5.** The approximation involved in the \(\mu\)-integrations; the form of \(\lambda^2 \cosh^2 \mu\) and \(\partial T^p/\partial \mu'\) across the plume boundary.

If the region where plume scale gradients in temperature and vertical velocity are large, is confined to a sufficiently small region near the boundary of the plume so that \(\lambda^2 \cosh^2 \mu\) can be taken as a constant where the gradients are large, see Fig. 5, then (for example)

\[
I = \iiint F(\mu)G(\phi)H(z)h_1 h_2 h_3 d\xi_1 d\xi_2 d\xi_3 \\
= \int H(z)dz \int G(\phi) h_2 d\xi_2 \int \frac{\partial T^p}{\partial \mu'} h_1 d\xi_1 \\
= - \int H(z)dz \Delta T \int G(\phi) \lambda (\cosh^2 \mu_b - \cos^2 \phi)^3 d\phi
\]  

(3)
where $\Delta T$ is the total temperature difference between the inside and the outside of the plume.

Some of the terms have no $\phi$-dependence, ($G(\phi) = 1$). For these terms Eq. (3) reduces to an elliptic integral of the second kind

$$ I \doteq \int H(z) dz \Delta T(\frac{1}{2}(a^2 + b^2))^{\frac{1}{2}} 2\pi $$

(4)

Since the perimeter of the ellipse is given by $(1(a^2 + b^2))^{\frac{1}{2}} 2\pi$, then the $\phi$-integration for terms with no $\phi$-dependence is just the distance around the perimeter of the plume cross-section.

Several terms have an angular dependence on the upwind edge (from $\phi = -\frac{1}{4}\pi$ to $+\frac{1}{4}\pi$) of the form $\cos \alpha$, where $\alpha$ is the angle between the perimeter normal in the $X$-$Y$ plane and the $X$-axis (see again Fig. 4). Equation (3), with $G(\phi)$ equal to $\cos \alpha$, can be shown to reduce to

$$ I \doteq \int H(z) dz \Delta T 2b^2/a $$

(5)

In arriving at Eq. (5), the approximation was made that $b^2/a^2 \ll 1$; for the observed $b/a$ ratio, this is a valid approximation.

With the use of Eqs. (4) and (5) the volume integrals of the turbulence mixing terms can be solved.

(iv) **Formulation of the turbulence terms.** Consider first the term in Eq. (1) describing the turbulence interaction with the plume-scale temperature, $[T^p\{U_{3}^{i},U_{j}^{j}\}]_p$. For the case of incompressibility (as used in the development leading to the Eq. (1)), this can be written: $T_{i}^{p}\{U_{3}^{i}U_{j}^{j}\}_p$, where the brackets for plume scale averaging have been omitted for clarity. This term is now clearly seen as the interaction between the vertical component of the Reynolds stress and the plume temperature gradient.

The temperature gradient at the upwind edge of the plume is assumed to be normal to the boundary. Hence, for a plume with a downwind tilt of the upwind edge of $\theta$ (from the vertical)

$$ T_{3}^{p} = \tan \theta T_{u}^{p} $$

(6)

where $\mu'$ is the $\mu$-vector in the $X$-$Y$ plane along the normal to the ellipse perimeter; $\mu'$ is associated with the line element, $h_{i}d\xi_{i}$, in (2). With the use of (6) the temperature turbulence interaction term becomes

$$ T_{3}^{p}\{U_{3}^{i}U_{j}^{j}\}_p = [\tan \theta \{(U_{i}^{j})^{2}\}_i + \{U_{i}^{j}U_{i}^{j}\}_i]T_{p}^{u}$$

(7)

An estimate of the $\phi$-dependence of $\tan \theta$ is now obtained. It is assumed that near the edges (inside and outside the plume) of the upwind edge convergence zone the components of velocity directed towards the boundary are equal and opposite; see Fig. 6. This result follows from the previous assumption that the plume scale pressure field is balanced by the plume scale inertial terms close to the surface where the turbulence levels inside and outside the plume are nearly the same.

For an upwind edge boundary with a downwind tilt of $\theta$, the components of velocity normal to the boundary are $W_{p}^{'}\sin \theta - U_{p}^{'}\cos \theta$ inside the plume, and $-W_{e}^{'}\sin \theta + U_{e}^{'}\cos \theta$ outside the plume, where primed velocities are velocities measured relative to the mean boundary motions. Subscript $p$ refers to inside the plume and $e$ to outside. In $D$, these normal components were equated to give

$$ \tan \theta = (U_{e}^{'} + U_{p}^{'})/(W_{e}^{'} + W_{p}^{'}) $$

(8)

which led to the estimate $\theta \approx 80^\circ$ at a height of 3.5m. This estimate of $\theta$ agreed reasonably well with the measured average tilt between 3.5m and 48m of $64^\circ \pm 10^\circ$. 

Figure 6. Velocity components relative to the mean velocities at the upwind boundary of the plume showing convergence towards the boundary in a direction normal to the boundary. Subscript ε refers to velocities outside the plume and p to velocities inside the plume; θ is the downwind tilt of the plume boundary.

If from Eq. (8) the tilt at the upwind edge at \( \alpha = 0 \), is approximated by

\[
\tan \theta_m = \frac{U'_\varepsilon m}{W'_p m}
\]

where subscript \( m \) refers to values at the upwind edge at \( \alpha = 0 \), and if the effect of the \( \phi \)-dependence of the tilt over the range \(-\frac{1}{2} \pi \) to \(+\frac{1}{2} \pi \) is limited to the change in the component of \( U'_\varepsilon \) normal to the boundary, then a rough estimate of the \( \phi \)-dependence of \( \tan \theta \) is

\[
\tan \theta \approx \tan \theta_m \cos \alpha
\]

Before a volume integral can be taken, the turbulent scales have to be estimated. The term \( \{(U'_3)^2\}_i \), can be evaluated from the integral under the vertical velocity spectrum from high frequencies to some lower frequency limit which is still above the plume scales, i.e.

\[
{(U'_3)^2}_i = \int \Phi_{uw}(f) df
\]

The term \( \{U'_\mu U'_3\}_i \), can be divided into a mean wind shear contribution (call it \( \{U'_\mu U'_3\}_a \)), and a contribution due to the \( W^p \) gradient across the boundary (call it \( \{U'_\mu U'_3\}_b \)). The wind shear effect will be important only on the surfaces normal to the wind (since the \( VW \) correlation is very small) and hence will depend roughly on \( \cos \alpha \), as does the turbulent vertical velocity term, (11); rough evaluations show that \( \{U'_\mu U'_3\}_a \) is over an order of magnitude smaller than the term (11) and so will be neglected. However, the time traces suggest that the angular dependence of the \( W^p \) gradient at the boundary around the plume is small, hence this term will be assumed to have no \( \phi \)-dependence. Thus its dependence on \( a \) and \( b \) will be different from (11) and so this term cannot be neglected even though it is of the same order of magnitude as \( \{U'_\mu U'_3\}_b \).

\( \{U'_\mu U'_3\}_b \) can be roughly estimated as follows. The \( U'_3 \) average value due to the \( W^p \) gradient across the plume boundary is roughly the turbulence mixing length (approximately equal to the height, \( z \)) multiplied by the \( W^p \) gradient; the average value of \( U'_\mu \) can be estimated from the integral of the velocity spectrum for turbulence scales. Thus

\[
\{U'_\mu U'_3\}_b \approx \left( \frac{\partial W^p}{\partial \mu} x z \right) \left[ \int \Phi_{uw}(f) df \right]^{\frac{3}{2}}
\]

The expression for the required ratio of the semi-major to semi-minor axes will turn out to be rather insensitive to errors in the above expression or its evaluation.
CONVECTIVE PLUMES

With the above considerations, the volume integral over a layer of thickness \( h \), of the turbulent term involving \( T^p \) becomes (to the same approximations as used earlier)

\[
\int \int \int [T^p \{ U_j^l, U_j^l \}]_r d \text{(volume)} = h\Delta T \left[ \tan \theta_m \left( (U_3^l)^2 \frac{2b^2}{a} + (U_3^l U_{\mu}^l)^2 \pi \left( \frac{a^2 + b^2}{2} \right)^2 \right) \right] \quad . \quad (13)
\]

The turbulence term involving the plume scale vertical velocity, \([W^p\{ T_j^l U_j^l \}]_p\), can be treated in an analogous way. The turbulent parts can be considered to have contributions from the overall turbulent correlations and from the effects of the plume scale gradient in \( T^p \). The effects when roughly evaluated are all very much smaller than previously considered effects except for one term. This is the term analogous to the one in Eq. (12), and is of a similar form for \( |\phi| < \frac{1}{2} \pi \)

\[
\{ T^l U_\mu^l \}_p \overset{\theta}{=} \left( \frac{\partial T^p}{\partial u^l} \times z \right) \left[ \int \Phi_{wU}(f) df \right]^{\text{turbulence scales}} \quad . \quad (14)
\]

where the subscript \( tg \) refers to the part of the term arising from the existence of the temperature gradient at the plume boundary averaged over turbulent scales. This term is analogous to the term \{ \( U_3^l U_{\mu}^l \) \}_p discussed in (12), except that (14) is applicable only for \( |\phi| < \frac{1}{2} \pi \). Hence the \( W^p \) turbulent term has a volume integral

\[
\int \int \int [W^p \{ T_j^l U_j^l \}]_p d \text{(volume)} \approx \left[ \text{expression with the same form of dependence on } a \right] + \left[ \text{and as the first term in (13) but much smaller} \right] + \\
+ h\Delta W \{ T^l U_\mu^l \}_p \pi \{ \frac{1}{2} (a^2 + b^2) \}^4 \quad . \quad (15)
\]

(c) Required ratio of the semi-axes of the plume cross-section

(i) Determination of the required ratio. In the previous subsection the plume scale heat flux losses due to the turbulent mixing terms were formulated in terms of measurable quantities and of \( a \) and \( b \), the downstream and cross-stream semi-axes of the plume cross-section. These terms were assumed to be the only terms in the plume scale heat flux equation which contributed to the heat flux lost by a plume and which had a dependence on the shape of the plume. The value of the ratio \( \gamma \), given by \( \gamma = a/b \), which minimizes the heat flux lost by the plume due to mixing at the plume boundaries, is the required ratio from the original hypothesis.

The sum of the volume integrals of the turbulence terms can be written

\[
\int \int \int [T^p \{ U_j^l, U_j^l \}]_r + [W^p \{ T_j^l U_j^l \}]_p d \text{(volume)} = h\left[ \tan \theta_m \Delta T \left( (U_3^l)^2 \frac{2b^2}{a} + 2\pi T \{ U_3^l U_{\mu}^l \} \pi \left( \frac{a^2 + b^2}{2} \right)^2 \right) \right] + \\
+ \left[ \pi \Delta W \{ T^l U_\mu^l \} \pi \left( \frac{1}{2} (a^2 + b^2) \right)^4 \right] \quad . \quad (16)
\]

If estimates are made for each term from the data of 16 July then the right-hand side (r.h.s.) of Eq. (16) reduces to

\[
h\left[ 7b^2/a + 0.44(a^2 + b^2)^4 + 0.17(a^2 + b^2)^4 \right] \quad . \quad (17)
\]

where the terms are in the same order as in (16). The estimated possible errors for the numerical values in (17) are all about 35 to 40%.

In spite of these large possible errors and the rather gross approximations for the
\( \phi \)-dependencies of the various terms, it is seen that the mixing of temperature through the upwind edge is very large and depends more strongly upon the width of the plume, i.e. on the cross-section of the plume exposed to the wind, rather than upon its length. The turbulence mixing terms without angular dependence coming from the plume scale gradients around the whole perimeter of the plume are roughly an order of magnitude smaller.

Defining

\[
\begin{align*}
C_1 &= 2\Delta T \tan \theta_m \{(U_3')^2\}_t \\
C_2 &= 2\Delta T \pi \{U_3'U_3'\}_t \\
C_3 &= \Delta W \pi \{T'U_3'\}_t
\end{align*}
\]

(18)

and writing \( b = (A/\pi \gamma)^{\frac{1}{2}} \) and \( a = (A\gamma/\pi)^{\frac{1}{2}} \) where \( A \) is the cross-sectional area of an ellipse with semi-axes \( a \) and \( b \) \( (A = \pi ab) \), then the r.h.s. of Eq. (16) becomes

\[
h[C_1(A/\pi)^{\frac{1}{2}}\gamma^{-\frac{4}{3}} + (C_2 + C_3)(A/\pi)^{\frac{1}{2}}\gamma^{-\frac{4}{3}}(\gamma^2 + 1)^{\frac{1}{3}}]
\]

(19)

The above expression represents the heat flux lost by a convective plume due to the turbulent mixing terms parameterized in terms of \( \gamma \). To minimize the heat flux loss, expression (19) was differentiated with respect to \( \gamma \) and set to zero. After some algebra, the following equation for \( \gamma \) results

\[
\gamma = (3C_1/(C_2 + C_3))^{\frac{1}{4}}
\]

(20)

For the data from 16 July, the r.h.s. of (20) was estimated to be about 5, with a range from 3.3 to 7.6 when the estimated errors for the numerical values in \( C_1, C_2 \) and \( C_3 \) are included. This value for \( \gamma \), the ratio of the semi-major to semi-minor axes of the plume horizontal cross-section, agrees reasonably well with the observed ratio, 8 ± 4, determined from the temperature contours.

It was assumed previously that the pressure and plume scale inertial terms balanced at the upwind edge; this appeared to be a reasonable assumption based on experimental evidence of the upwind edge tilt presented in D. However, calculation of the \((T' P_3')/\rho \) term shows that it is of the same form as the turbulent vertical velocity term (11) (i.e. has a \( b^2/a \) dependence) and also is just as large. Therefore, there are three large terms of the form \( b^2/a \), two of which have been assumed to nearly balance. Because of the form of (20), errors due to this assumption come in as square roots. An error of 50% in the assumption changes the \( \gamma \)-estimate by only 22%. Thus the assumption of pressure-inertial-term balance does not greatly increase the uncertainty in the calculated estimate of \( \gamma \).

(ii) Discussion of the plume cross-sectional shape. Although there are many rather gross assumptions and approximations in the development of Eq. (20), it is seen that a minimization of the heat flux lost by a plume due to turbulent mixing at the plume boundaries leads to a predicted value of the ratio of the semi-axes of the plume cross-section which agrees reasonably well with the observed ratio. The vertical shear of the mean wind greatly increases the turbulent mixing at the upwind edge of the plume compared to the turbulent mixing around the rest of the plume boundary. This is the physical basis for the elongated form of the plume cross-section.

At higher levels, where the vertical shear of the mean wind is much less, the shape which would minimize heat flux loss would be much less elongated in the downwind direction than closer to the surface where the wind shear is larger. However, if the plume cross-section, rising up to a level where the wind shear is smaller, already has an elongated form due to the wind shear at lower levels, then this shape would be expected to persist for some time.

In the case of very light winds or a low drag coefficient the elongation at a given height would be expected to be decreased due to the decreased wind shear. The fact that Lenschow's
(1970) observations of elongated plumes were taken under conditions of moderately strong winds (U at 100m was 9·1m/s) and that Warner and Telford’s (1963) observations of no plume elongation were taken under conditions of light winds, might explain the apparent discrepancy between their observations.

The above considerations indicate that in the study of convective elements by Frisch and Businger (1973), the assumption of plume cross-sectional shape remaining the same (circular) from 30m to lower levels is probably not valid. However, the way in which the plume areal densities were calculated by Frisch and Businger was independent of this assumption and so their results are unaffected except for the calculation of mean plume lengths (a calculation which also assumed Taylor’s hypothesis which the results of D indicate is not valid at low levels for plume scales).

5. CONCLUSIONS

With the use of the measured plume translation velocity, the time traces of the temperature signals from an array of five sensors at a height of 3·5m were appropriately spaced and delayed to effect a time-to-space transformation; temperature contours were visually sketched in. The plume horizontal cross-sectional shape was found to be markedly elongated in the downwind direction with a ratio of the downstream to cross-stream axes of 8 ± 4.

A dynamical explanation was given for the observed shape. The hypothesis was made that the shape of the plume would be such as to minimize the heat flux lost by the plume due to mixing around its boundary. A simplified model of this heat flux loss was formulated and led to an expression for the ratio of the plume cross-sectional axes which agreed with the observed ratio.

ACKNOWLEDGMENTS

Much of this study constitutes a portion of a Ph.D. dissertation submitted to the University of British Columbia, and was supported in part by grants from the National Research Council of Canada and the Atmospheric Environment Service of Canada. The author is indebted to Dr. M. Miyake and other members of the Institute of Oceanography at U.B.C. for guidance throughout the study.

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