Evaporation and advection II:
evaporation downwind of a boundary separating regions
having different surface resistances and available energies

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SUMMARY

Using the model and methods developed in Part I of this study, it is shown that steady-state evaporation,
downwind of a sharp boundary separating uniform regions with constant but different surface resistances
and available energies, can be written as

\[ \text{LE} = (s/(s+\gamma))(Rn - G) + (s/(s+\gamma))^2 r_1 r_2 (Rn' - G') - r_2 (Rn - G) \Phi_e \]

where \( \Phi_e \) is a dimensionless 'exchange function' that decreases from unity to zero as distance increases
downwind of the boundary. The symbols have their conventional meanings and the primes signify upwind
values. The form of \( \Phi_e \) depends on the profiles of wind speed and effective diffusivity, and on the downwind
surface resistance and temperature via the parameter \( \gamma \).

Empirical expressions for \( \Phi_e \) are obtained from a known solution of the atmospheric diffusion equations
assuming power law forms of the wind speed and effective diffusivity profiles and from a simple model
assuming perfect vertical mixing and constant wind speed beneath an impermeable inversion base. These
may give some indication of the form and magnitude of \( \Phi_e \) at small and at large distances respectively.

1. INTRODUCTION

The original aim of this investigation was to find a plausible explanation for the close
agreement between measured evaporation rates and the 'equilibrium' evaporation rate
in several experiments. In the course of this research a simple atmospheric advection
model was constructed, and this is described in the first part of this paper (McNaughton
1976, hereafter referred to as Part I).

It was shown that, in terms of the model, a general expression for evaporation can be
written as

\[ \text{LE} = s/(s+\gamma)(Rn_m - G_m) + \Psi_2/(s+\gamma) \] (1)

where \( \Psi_2/(s+\gamma) \) is an advective enhancement term that decreases to zero as distance
increases downwind of any variation in surface properties. At large distances the latent
heat flux density approaches its equilibrium value of \( s(Rn_m - G_m)/(s+\gamma) \). Definitions of
the symbols used and a discussion of terminology are given in Part I. Additional symbols
introduced in this paper are defined as they arise.

The model of Part I provides an opportunity to extend the study beyond the original
aims and to investigate the phenomena of 'advection' in a more general situation. In this
paper the model is explored further by examining the relatively simple case of steady
evaporation downwind of a boundary separating regions having different surface resistances
and available energies.

For the present the various assumptions and idealizations of the model are accepted,
together with the consequent equilibrium evaporation result, and further critical discussion

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will be postponed till the latter part of this paper. It is worth while to notice though, that the assumptions of constant available energy and surface resistance are more significant physical idealizations here than when the same assumptions were used previously only in the discussion of the limiting behaviour of the advective enhancement term.

2. Theory

This paper is intended to be read in conjunction with Part I, and the various assumptions employed and results obtained there will not be reiterated. Reference to equations in Part I are given in the form (I, n) where n is the equation number in Part I.

The problem to be discussed is the solution of the diffusion equation (I, 29), viz.

\[ u \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial z} \left( K \frac{\partial \psi}{\partial z} \right) \]  

(2)

with the lower boundary condition (I, 30) viz

\[ x > 0, \quad z = 0, \quad \psi_x = -\rho c_p K \frac{\partial \psi}{\partial z} \]

\[ = - (s + \gamma) r_s \rho c_p (\psi_x - e_m^* + s \theta_m) - s (R_n - G) \]  

(3)

together with (I, 35) – the condition that \( r_a \to \infty \) at sufficiently great height. The subscripts 2 and \( m \) have been omitted from \( \psi \) and \( \psi_x \), and \( R_n \) and \( G \) respectively. This will not lead to ambiguity here.

In Part I the initial conditions for the advection were taken as arbitrary. Here the upwind area is assumed to be extensive and uniform. Fetch is not taken to be infinite since that would imply, in terms of the model, infinite temperature and humidity also. Instead fetch is assumed to be sufficiently large that the equilibrium evaporation condition (I, 1) is closely approximated and \( \psi_0 = 0 \), where the subscript 0 refers to the value immediately upwind of the boundary \( (x = 0) \).

By applying (3) to the upwind surface the initial condition is found to be

\[ x = 0, \quad z > 0, \quad \psi = \psi_0 = - \left( \frac{s}{s + \gamma} \right) r_s \rho c_p (R' - G') + e_m^* - s \theta_m \]  

(4)

where the primes denote the upwind values of the various parameters. At large distances downwind \( \psi \) again approaches zero at all \( z \) and therefore

\[ x \to \infty, \quad z \geq 0, \quad \psi = \psi_\infty = - \left( \frac{s}{s + \gamma} \right) r_s \rho c_p (R_n - G) + e_m^* - s \theta_m \]  

(5)

Combining (4) and (5) gives

\[ \psi_\infty - \psi_0 = (s + \gamma) ((s + \gamma) (1/\rho c_p)) [r_s (R_n - G') - r_s (R_n - G)] \]  

(6)

Introducing the new variables

\[ \phi = (\psi - \psi_0)/(\psi_\infty - \psi_0) \]  

(7)

and

\[ \Phi = - (\gamma/(s + \gamma)) r_s K \frac{\partial \phi}{\partial z} \]  

(8)

the problem of solving (2) subject to (3) and (4) can be restated as that of solving

\[ \left( \frac{\gamma}{s + \gamma} \right) r_s u \frac{\partial \phi}{\partial x} = \frac{\partial \Phi}{\partial z} \]  

(9)
with the boundary conditions

\[ x = 0, \quad z > 0, \quad \phi = 0 \quad \text{and} \quad (10) \]

and

\[ x > 0, \quad z = 0, \quad \Phi_x + \dot{\phi}_x = 1. \quad \text{and} (11) \]

In terms of the new variables, (1) can be rewritten as

\[ LE_x = \left( \frac{s}{s + \gamma} \right) (Rn - G) - \frac{s\gamma}{(s + \gamma)^2} \left[ r'_s(Rn' - G') - r_s(Rn - G) \right] \frac{\partial \phi}{\partial z} \bigg|_{z=0} \quad (12) \]

or

\[ LE_x = \left( \frac{s}{s + \gamma} \right) (Rn - G) + \left( \frac{s}{s + \gamma} \right) \frac{1}{r_s} \left[ r'_s(Rn' - G') - r_s(Rn - G) \right] \Phi_x \quad (13) \]

The dimensionless concentration \( \phi \) increases from zero to unity as distance from the boundary increases, while the dimensionless flux \( \Phi_x \) decreases from unity to zero as distance increases.

The factor \( s\gamma[r'_s(Rn' - G') - r_s(Rn - G)]/(s + \gamma)^2 \) in (12) represents the amount of energy that must be transformed from sensible to latent heat, at constant enthalpy, to bring a unit volume of air initially in equilibrium with the upwind surface into equilibrium with the downwind surface. The function \( \Phi_x \) describes the rate at which this exchange takes place at the downwind surface. This function \( \Phi_x \) could be called a ‘normalized non-dimensional advective exchange function’ or, more simply, an ‘exchange function’. This usage will be adopted here. At the leading edge, \( x = 0^+ \), \( \Phi_x = 1 \) so that (13) gives

\[ LE_{0^+} = \left( \frac{s}{s + \gamma} \right) \left( r'_s/r_s \right) (Rn' - G') \quad (14) \]

and evaporation is dominated by advective influences, since the impressed temperature and humidity are characteristic of the upwind area alone. Evaporation is strongly regulated by the downwind surface resistance, displaying a simple inverse relationship. As \( \Phi_x \) decreases downwind (as \( \phi_x \) becomes increasingly modified) the available energy assumes greater importance and surface resistance control of evaporation diminishes. Available energy becomes the dominant control.

Very near the surface, or at moderate heights in near neutral stability conditions, frictionally generated turbulence predominates. The relevant parameters for scaling the \( K \) and \( u \) profiles are the friction velocity \( u_* \) and the surface roughness length \( z_0 \). For some small distance downwind of the boundary \( \Phi_x \) must be a function of \( x, z_0, u_* \) and \( \gamma r'_s(s + \gamma) \) alone. The exchange function must take the form

\[ \Phi_x = \Phi_x^* \{ x / z_0, \gamma / (s + \gamma) r_s u_* \} \quad (15) \]

Similar proportional changes in \( u_* \) and \( \gamma r'_s(s + \gamma) \) produce equivalent changes in \( \Phi_x \). At greater heights buoyant forces will usually be important in determining the profiles of \( K \) and \( u \).

The wind speed and effective diffusivity profiles in the lowest 100m or so of the atmosphere can be reasonably well represented by empirical profiles of the form

\[ u = au_* (z/z_0)^m \quad (16) \]

and

\[ K = bu_* z_0 (z/z_0)^n. \quad (17) \]

The appearance of the physically meaningful parameters \( u_* \) and \( z_0 \) in (16) and (17) does not alter their basically empirical nature. However, some guidelines are available for selecting the constants \( a \) and \( b \) for equations in this form (Brutsaert and Yeh 1970).
Using these profile forms (8) and (9) give
\[ \frac{m+n}{z_1^n} \frac{\partial \phi}{\partial z} - \frac{\partial^2 \phi}{\partial z^2} = 0 \]  
(18)
if \( \xi \) and \( \zeta \) are defined by
\[ \xi = x/\{abz_0[b(1-n)]^{m+n}\} \]  
(19)
and
\[ \zeta = u_*r_a = (z/z_0)^(1-n)/b(1-n) \]  
(20)
respectively.
The solution of (18) subject to (10) and (11) is given by Sutton (1943) and \( \Phi_x \) can be written as
\[ \Phi_x = \sum_{r=0}^{\infty} (-\omega \xi^n)^r / \Gamma(1 + r\eta) \]  
(21)
where
\[ \omega = \frac{(s+\gamma)\eta^{1-2\eta} \Gamma(\eta)}{yr_u u_* \Gamma(1-\eta)} \]  
(22)
and
\[ \eta = (1-\eta)/(2 + m - n) \]  
(23)
with the restrictions that \( n < 1 \) and \( 2 + m - n > 0 \). The restriction that \( n < 1 \) corresponds to the more general restriction that the aerodynamic resistance must become indefinitely large as \( z \) increases (cf I, 35).

Although the series (21) is absolutely convergent for all values of \( \omega \xi^n \), values of \( \Phi_x \) cannot be satisfactorily computed from it if \( \omega \xi^n \) is significantly greater than one. When \( 1/\eta \) is an even integer an alternative calculation scheme can be used (see appendix) and \( \Phi_x \) can be conveniently tabulated throughout its range.

To select suitable values of the various constants in (16) and (17) it is customary to assume a similarity between the effective diffusivities for heat and vapour and that for momentum transfer. This gives two further relationships among the constants in (16) and (17): \( abm = 1 \) and \( m+n = 1 \).

Now only the wind profile parameters \( a \) and \( m \) need be found. Brutsaert and Yeh (1970) have compared (16) with stability-corrected logarithmic wind profile expressions. They find that \( a = 6.2/(7m) \) is a suitable choice of the parameter \( a \) for \( m \) in the region of \( m = 1/7 \), which value is appropriate for neutral conditions. With these values \( \Phi_x \) can be evaluated as a function of \( x/z_0 \). In view of the relative ease of evaluating \( \Phi_x \) for \( m = 1/6 \) and \( m = 1/8 \) rather than \( m = 1/7 \). \( \Phi_x \) is plotted against \( x/z_0 \) for the former two values of \( m \) in Fig. 1. Curves are shown for various values of the parameter \( \eta r_u u_*/(s+\gamma) \). The exchange function, \( \Phi_x \), decreases very much less rapidly for larger values of \( \eta r_u u_*/(s+\gamma) \).

For a wet downwind surface \( (r_a = 0) \) the lower boundary condition (11) reduces to the simple constant-concentration boundary condition
\[ \xi > 0, \quad \zeta = 0, \quad \phi = 1 \]  
(24)
The solution of (18) with (11) and (24) is well known (e.g. Philip 1959) and is given by
\[ -K \frac{\partial \phi}{\partial z} = -u_* \frac{\partial \phi}{\partial \xi} = \frac{u_* \xi^{1-\eta}}{(1-\eta) \Gamma(\eta)(1-2\eta)} \]  
(25)
which, with (12), provides a complete solution for \( LE_x \).

The solutions (21) and (25) are appropriate to less than 1km downwind. The curves in Fig. 1 are plotted to \( x/z_0 = 10^{10} \) in order to show the form of the curves and to provide a useful test case, rather than to claim physical verisimilitude. Over the useful part of the
curves where $x/z_0$ ranges from 10 to $10^5$ say, it may be possible to approximate the relationship between $\Phi_x$ and $x/z_0$ by logarithmic functions which would be best fit straight lines on Fig. 1.

Some idea of the behaviour of $\Phi_x$ at larger distances can be gained by assuming that evaporation occurs beneath an impermeable inversion at height $h$. Integrating (9) over all $x$ and $z$ gives

$$
\frac{s + \gamma}{\gamma r_s} \int_0^\infty \Phi_x \, dx = \int_0^h u \, dz \quad . \quad . \quad . \quad (26)
$$

The integral on the right-hand side represents the total volume of air crossing the boundary that is to be modified at the downwind surface. The exact shape of the $\Phi_x \text{ v. } x$ relationship will depend on the details of the profiles of $K$ and $u$ developed in the downwind region.

This crude model for advection at very large distances can be further developed by assuming that wind speed is constant with height and that there is perfect vertical mixing beneath the inversion. Growth of the mixed layer by erosion of the inversion base and entrainment of warmer air from above is ignored. With this model $\Phi$ decreases linearly with height, so that (9) can be written

$$
\frac{(s + \gamma)r_s u}{h} \frac{d\Phi_x}{dx} = -\Phi_x/h. \quad . \quad . \quad . \quad (27)
$$

Substitution for $\phi_x$ in (27) using (11), and subsequent integration yields

$$
\Phi_x = \exp\left\{-\frac{x(s + \gamma)}{h \gamma r_s u}\right\} \quad . \quad . \quad . \quad (28)
$$
and the exchange function decreases exponentially with distance. If the depth of the mixed layer is 1000m, say, and \( \gamma r_0 \mu/(s + \gamma) \) has the reasonably low value of 10, say, then the distance constant is 10km. Advecitive exchange will be substantial to 20km or more.

In this ‘mixed layer’ model equivalent temperature increases linearly with distance. If wind speed is 4m/s, say, and available energy is 200W/m², say, then the equivalent temperature increases at about 1K each 20km. As equilibrium is approached the potential temperature will increase at less than half this rate, except in cold conditions.

3. DISCUSSION

It is appropriate here to review the present work, to discuss some of the new results and to comment on some limitations that are already apparent. Before doing this, it may be useful to give a more descriptive summary of the ideas underlying the formal development.

On the basis of the present study a new conceptual scheme for regarding the evaporation process can be proposed: energy exchange at the earth’s surface can be thought of as the sum of two processes. First, there is an amount of energy available locally at the surface which is dissipated into the atmosphere. This convective flux is an enthalpy flux and is associated with (‘diffuses down’) the gradient of equivalent temperature. This flux can be considered to be partitioned between sensible and latent heat in the constant ratio \( \gamma/s \). Superimposed on this is a second flux of energy, being both a sensible heat flux upwards and a latent heat flux downwards, that is associated with (‘diffuses down’) the gradient of \((s\theta - e)\) which is approximately related to the vapour pressure deficit gradient. Conceptually, there is an exchange of energy from latent heat to sensible heat at the surface. This flux can conveniently be called an ‘exchange’ flux. The important assumption implicit in this scheme is that \( K_s \) and \( K_p \) are equal everywhere.

An ‘equilibrium’ surface vapour pressure deficit can be defined by combining (I, 1) and (I, 8). If a fixed volume of air, having a larger vapour pressure deficit than the equilibrium value, is brought into contact with a ‘leaf’ surface it will gain latent heat and lose sensible heat by exchange at the surface. The process will proceed until the vapour pressure deficit of the air parcel is reduced to the equilibrium value. The obverse process will also occur for a volume of air with a lower-than-equilibrium initial vapour pressure deficit. Since the concomitant enthalpy flux is partitioned in the ratio \( \gamma/s \) between sensible and latent heat, it will have no effect on the vapour pressure deficit of the air.

In turbulent heat transfer from vegetated surfaces, parcels of air, with transient identity, come into brief contact with the surface. These air parcels gain in enthalpy and also heat is transformed from the sensible to the latent form (or vice versa) by exchange at the various canopy leaf surfaces. They are then swept away to blend back into the turbulent air stream above. The exchange flux within a control volume, resting on the surface, must rapidly diminish to zero unless air with a non-equilibrium vapour pressure deficit is continuously introduced and partly modified air displaced. In the model developed it is assumed that it is possible to draw the upper boundary of the control volume high enough that transfer through the lid can be ignored (cf. (I, 35)) and horizontal displacement or ‘advection’ is considered.

The vapour pressure deficit is appropriate to quantify atmospheric dryness only near the surface. Over a greater range of heights the correct parameter is the potential vapour pressure deficit, which is the vapour pressure deficit that a parcel of air would have if it were brought down adiabatically to the surface and its temperature adjusted to the surface temperature by adding sensible and latent heat in the ratio \( \gamma/s \). The exchange flux, as
defined, 'diffuses down' the potential vapour pressure deficit gradient. Using $\chi$ to denote
the potential vapour pressure deficit, the defining equation is

$$\chi = e^*_{m} + s(\theta - \theta_{m}) - e$$

(29)

and $\chi$ bears a simple linear relationship to the parameter $\psi_{z}$ used in the formal development.
The exchange flux can then be written

$$H_x = -(1/(s + \gamma))\rho c_p K \frac{d\chi}{dz}.$$  

(30)

The exchange flux is written as $H_x$ since it represents a sensible heat flux upwards. The
subscript $x$ now denotes exchange as well as signifying position dependence as previously.
(Notation has been changed from previous usage and $H_x$ now refers to only the exchange
portion at the total heat flux.) The potential vapour pressure deficit differs from other
atmospheric properties (potential temperature, vapour pressure, equivalent temperature etc.) in
that its magnitude as well as its gradient is important in determining the exchange
flux. Thus

$$H_x = [-L.E_{s}] = \left(\frac{s}{s + \gamma}\right)(R_n - G) - \frac{\rho c_p}{\gamma r_s} \chi_s.$$  

(31)

A surface $\chi$-value greater than the equilibrium value implies a negative $H_x$ value and, by
(30), $\chi$ increasing with height. A value of $\chi$ greater than the equilibrium value at any height
in the profile implies a value of $\chi$ greater than the equilibrium value at all other points
and a negative value of $H_x$ at all heights. This, of course, assumes that the profile of $\chi$
changes monotonically. The sign of the exchange flux can therefore be determined from
measurements of $r_s$, $(R_n - G)$, $\theta_x$ and of $\chi$ at, say, screen height level.

A horizontal surface can be characterized by its equilibrium potential vapour pressure
deficit, and the sign of any advective enhancement of the evaporation rate depends on
whether the advected air is drier or wetter ($\chi$ greater or less) than the equilibrium value.
The magnitude of the advective enhancement term depends on the surface resistance as
well as wind speed and the efficiency of the vertical mixing, which brings parcels of air
down into temporary contact with the surface. If the vertical mixing is ultimately restricted
so that the surface air becomes completely adjusted, the result can be expressed as in
Eq. (13), which is the main result of this paper.

Eq. (13) gives a more comprehensive account than previously available of the role of
stomatal resistance in regulating evaporation losses from vegetated areas. According to (13),
effects of changes of stomatal resistance, and hence of surface resistance, will be minimal
in homogenous regions. Where advective effects are large the surface resistance plays a
decisive role in regulating water loss.

Rider et al. (1963) noted that changes in available energy between adjacent areas as
well as changes in surface wetness could give rise to advective effects. Eq. (13) gives mathemati-
cal expression to this observation. Alteration of the available energy of a small area
by shading (by a cloud bank for example) or by alteration of the surface albedo (by dusting
the surface with an inert reflectant powder for example) will induce an advective enhance-
ment term that will tend to smooth out evaporation differences.

If the surface resistance is large, then the predicted 'equilibrium' potential vapour
pressure deficit may be so high that it could not be realized in nature. The difficulty is
resolved by noticing the rate at which 'equilibrium' is approached. For large $r_s$ the rate of
decrease of the exchange function is very small and it remains near unity for large distances.
This is illustrated in the particular case of assumed power law profiles of wind speed and effective eddy diffusivity, in Fig. 1. As $r_s \to \infty$, $\Phi_x \to 1$ for all $x$ and $LE \to 0$ for all $x$, by (13). The desert situation is correctly approached as the limiting dry surface. The conceptual ‘equilibrium’ situation is not altered but it is approached infinitely slowly.

A novel feature of (13) is that air temperature and humidity do not appear as arguments. They are dependent variables of the model except that an initial temperature is taken to be synoptically determined somewhere far upwind of the boundary, so that $s$, though approximated as a constant, is set by the prevailing synoptic conditions. Evaporation is written in terms of boundary conditions and atmospheric transport properties. A change in boundary conditions, by natural or man-made alteration of surface cover type or condition, will produce a change in the evaporation rate which is, in principle, predictable using advection theory. Eq. (13) is a prototype of such equations.

Some limitations of the theory are also apparent. The model used does not embody a complete description of all atmospheric processes affecting the surface energy balance. It has been developed on the premise that, in studying evaporation from vegetated surfaces, the central problem is the partitioning of the available energy into sensible and latent heat. Thus available energy at the surface has been assumed independent of surface temperature, rather than determined simultaneously by solving a more complex system of equations, including governing equations for soil heat flow and radiation exchange between the surface and the atmosphere. Though this is not a major limitation of the present 24-hour-averaged theory, it would cause greater distortion in the results if applied in a diurnally varying model.

Another idealization in the present model is the assumed constancy of the surface resistances. In very many cases, especially when the vegetation experiences water stress, plants respond quite rapidly to variations in their rate of water loss, by varying stomatal apertures. Knowledge of plant responses is expanding rapidly, yet it is not currently possible to specify generally valid relationships between stomatal resistance and the evaporation rate in a way that is suitable for inclusion in advection models.

Another limitation of the theory is that the effective diffusivity and wind speed fields are assumed known rather than determined as part of the solution of an expanded model. In fact the profiles of $K$ and $u$ cannot be known independently but must be found by simultaneous solution of the heat and mass transfer equations with those governing the dynamics and transport properties of the atmosphere. Coupling is by way of buoyant forces associated with the heat and moisture fluxes and their attendant potential temperature and vapour pressure gradients. Though (13) has the appearance of a similarity form, it is not strictly one. The exchange function will depend on $r_s/(Rn' - G')$, $r_s$, $Rn - G$ and $s$ since each set of these will produce different heat flux distributions downwind and consequently different modifications of the $K$ and $u$ profiles over the downwind area. Eq. (13) is a pseudo-similarity form only.

The particular ‘power law’ and ‘mixed layer’ models used to illustrate the more general aspects of the theory and to make rough estimates of magnitudes, are of limited value. A more faithful representation of natural evaporation would entail proper dynamical modelling of atmospheric turbulent exchange processes. Recently, such a model has been developed and applied to the ‘leading edge’ advection problem by Rao et al. (1974) though, in that model, a constant relative humidity condition was used at the lower boundary rather than the ‘leaf’ equation (1, 8). It is clear that further development will be along similar lines using more comprehensive physical models and complex numerical methods for their solution. The value of the present treatment is not in producing a very accurate numerical prediction methods, but in giving simple predictions for limiting cases in a form that can easily be recognized.
4. Concluding remarks

Most meteorological methods for determining evaporation used by agrometeorologists involve measurements in the lowest ten metres of the atmosphere. Micro-meteorological theory has been developed to the extent that measurements of the fluctuations or gradients of temperature, humidity and windspeed can be used to calculate the surface fluxes on the assumption that vertical flux divergences are negligible. There is no claim that the various parameters measured are physically independent. They are simply measured and used in formulae to compute evaporation. In this sense the theory used is theory of evaporation measurement. A theory of the evaporation process must relate to the independent variables of the system.

The present work is intended as a contribution towards the development of a theory of the evaporation process. In that the effective diffusivity and windspeed distributions have been assumed known, rather than determined by the model, it represents something of a half-way stage between a 'measurement' and a 'process' theory.

A complete theory of evaporation must ultimately take account of all of the processes occurring within the convective planetary boundary layer. Viewed as a PBL model, the model developed here has obvious limitations. Important processes such as large scale subsidence, horizontal flow divergence, long-wave radiation divergence, and growth of the PBL by erosion of an inversion base and entrainment of warmer, drier air from above, have all been ignored. While limited experimental evidence supports the equilibrium evaporation result (I, 1), caution demands that the model be regarded as only a first order approximation applicable to an incomplete range of atmospheric circumstances.

REFERENCES


APPENDIX

The series

\[
\Phi_x = \sum_{r=0}^{\infty} \left( -\omega \xi^n \right)^r / \Gamma(1 - r\eta)
\]

is absolutely convergent for all \(\omega \xi^n\). If \(\omega \xi^n\) is significantly greater than one the magnitude of successive terms initially increases very rapidly and a large number of terms ranging over a great many orders of magnitude must be accurately summed for direct calculation of \(\Phi_x\). Values of \(\Phi_x\) less than about 0.45 cannot be easily found from (A1).

For \(\eta = 1/\mu\), where \(\mu\) is a positive integer, Sutton (1943) shows that
\[ \Phi_x = \begin{cases} \ 
eq \\
\left( e^x \left[ 1 + \sum_{s=1}^{\mu} \frac{(-1)^s}{\Gamma(s\eta)} \int_0^x e^{-t} t^{s-1} dt \right] \right) (\mu \text{ even}) \\
\left( e^{-x} \left[ 1 + \sum_{s=1}^{\mu} \frac{(-1)^s}{\Gamma(s\eta)} \int_0^x e^t t^{s-1} dt \right] \right) (\mu \text{ odd}) \end{cases} \] (A2)

where \( \chi = \omega^n \varphi. \)

To evaluate (A2) for \( \mu \) odd entails evaluating the integral by numerical means. For \( \mu \) even, (A2) can be rewritten as

\[ \Phi_x = \sum_{s=1}^{\mu} \frac{(-1)^{s+1}}{\Gamma(s\eta)} \int_\chi^\infty e^{s-t} t^{s-1} dt \]

and an asymptotic expansion for the integral can be obtained by successive integration by parts, so that

\[ \Phi_x = \sum_{s=1}^{\mu} \frac{(-1)^{s+1}}{\Gamma(s\eta)} \left\{ -\chi^{s-1} \left[ 1 + \frac{s\eta - 1}{\chi} + \frac{(s\eta - 1)(s\eta - 2)}{\chi^2} \right. \\
\left. + \frac{(s\eta - 1)(s\eta - 2)(s\eta - 3)}{\chi^3} + \cdots \right] \right\} \] (A3)

\( \Phi_x \) can be conveniently calculated from (A3) when \( \chi \) is sufficiently large, say \( \chi > 10 \). In Fig. 1 \( \Phi_x \) is calculated from (A1) for \( \chi \leq 10 \) and from (A3) for \( \chi > 10 \).