Correspondence and Notes

Statistics of vertical motion over land and water

By M. Merry* and H. A. Panofsky

Summary

Observations from many sources have been brought together to study the relationship between the ratio of standard deviation of vertical velocity to friction velocity, and \( z/L \) where \( z \) is the height and \( L \) the Monin–Obukhov length. A good compromise for this relationship is:

\[ \sigma_w/u_* = 1.3[\phi_m - 2.5z/L]^b \]

where \( \phi_m \) is the normalized wind shear.

This equation has been combined with theoretical expressions for the wind profile to derive a nomogram for the standard deviation of vertical angle as function of \( z/L \) and \( z/z_0 \), where \( z_0 \) is the roughness length.

1. Introduction

The standard deviations of vertical velocity and vertical angle are of considerable interest in the study of diffusion and of flight bumpiness.

It is well known, other things being equal, that turbulent vertical motions are weaker over water than over land. For example, the ratio of the standard deviations of vertical velocity to mean velocity is, typically, 0.1 over land and 0.05 over water. If this difference is entirely due to the difference in roughness, it can be explained completely in the context of Monin–Obukhov similarity theory; however, Davidson (1974) has suggested that another factor enters these statistics over water which is absent over land: the properties of the ocean waves. Clearly, such an effect would be outside the similarity as ordinarily used.

To clarify the behaviour of vertical-velocity statistics, it was decided to bring together vertical-velocity measurements from as many recent sources as possible. As it happens, many observations over land and over water have been published in the last few years. But each investigator, with very few exceptions, only deals with his own measurements without reference to any agreement or disagreement with those of others.

2. Theory

According to Monin–Obukhov theory, the ratio of the standard deviation of vertical velocity to the friction velocity is given by \( \sigma_w/u_* = \phi_3(z/L) \), where \( L \) is the Monin–Obukhov length and where \( \phi_3 \) is a 'universal' function of \( z/L \). If Davidson is correct, \( \phi_3 \) may also depend on the ratio of wind speed to wave speed.

There is some question as to whether the friction velocity and \( L \) are to be interpreted as local or surface values. Most high-level measurements refer to local values; in contrast, surface-layer statistics are usually given in terms of friction velocities and \( L \)'s extrapolated to height zero. However, the variation through the surface layer is so small that we will interpret all measurements of these quantities as local measurements.

The principal purpose of this exercise is to determine \( \phi_3(z/L) \) and, if possible, to suggest an empirical expression for this function which fits the well-confirmed theoretical limit for 'free convection' at large negative Richardson number. Previous attempts to do this were summarized by Lumley and Panofsky (1964), but this summary is obsolete.

Once a formula for \( \phi_3(z/L) \) has been established, we can then derive an expression for the standard deviation of vertical angle, which will be assumed to be equal to \( \sigma_w/V \), by introducing the expression for the wind profile (see, e.g. again Lumley and Panofsky):

\[ \sigma_w = \frac{\sigma_w}{V} = \frac{k \phi_3(z/L)}{\ln(z/z_0) - \psi(z/L)} \]  

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Here \( \psi \) is another universal function which has been derived from wind profile measurements. There is not complete agreement concerning the exact behaviour of this function, but the uncertainty is not great. In stable air we have, according to several independent sources,

\[
\psi(z/L) = -4.7z/L
\]  

(2)

In unstable air, there are a number of hypotheses concerning \( \psi \), all quite similar. That used here is based on a hypothesis for the normalized wind profile proposed by Carl et al. (1973): \( \phi_M = (1 - 15z/L)^{-\frac{1}{2}} \) and

\[
\psi = \int_{0}^{1/\beta} 1 - \phi_M(\xi) \, d\xi
\]

Then, given (1) and an expression for \( \phi_3 \), we will prepare a nomogram for \( \sigma_\infty \) as function of \( z/z_0 \) and \( z/L \). However, if Davidson is right, different nomograms will be needed for different (wind speed)-(wave speed) ratios.

3. TREATMENT OF THE OBSERVATIONS

Table 1 lists the particulars of the observations used, including height of observations, averaging period, sampling interval, availability, and instrumentation.

Often, Richardson numbers rather than \( L \) were available. In that case, \( z/L = R_i (\text{unstable air}) \) and \( z/L = R_i / (1 - 4.7 R_i) \) (stable air).

Sampling periods and averaging periods differed considerably; no attempt was made to reduce all data to a common system. This fact leads to systematic discrepancies between different sets of observations, which become particularly pronounced at the larger heights, e.g. above the surface layer. Also random variations become large, as pointed out by Wyngaard (1973).

In some cases only the standard deviation of vertical angle was given, along with a measure of stability. In those cases, Eq. (1) was solved for \( \phi_M(z/L) \).

4. RESULTS

Figs. 1 to 3 summarize all the observations over land and water, respectively. When the number of measurements was small in a given study, individual points were plotted for each period. When the observations were too numerous, mean values for certain values of \( z/L \) are given, along with ± one standard deviation of each mean.

Generally, agreement between different sets of observations is reasonably good but deteriorates with increasing elevation. The solid curve in the figures represents the expression

\[
\sigma_\infty/u_* = \phi_3 = 1.3[\phi_M - 2.5z/L]^\frac{1}{2}
\]  

(3)

where \( \phi_M \) is the 'normalized' wind shear, defined by \( (kz/u_*)/(\partial V/\partial z) \). In unstable air, we have

![Figure 1. \( \sigma_\infty/u_* \) as function of \( z/L \), land observations.](image-url)
used (3), the expression for $\phi_M$ recommended by Carl et al. In stable air, the same authors, as well as Businger et al. (1971) recommend $\phi_M = 1 + 4.7z/L$, an equation consistent with (2). As is seen from Fig. 1 to 3, (3) fits all observations passably well, except Davidson's data in stable air over water. In contrast, in unstable air there appears no significant difference between $\phi_3$ over land and $\phi_3$ over water. For large negative $z/L$, (3) overestimates the observations.

The fit in stable air is somewhat doubtful, even though measurements shown in Fig. 1 agree quite well with the equation. The difficulty is that, in stable air, both $\sigma_w$ and $u_*$ become small, so that ratio $\sigma_w/u_*$ is subject to considerable error. Further, the uncertainty in $u_*$ is larger than that in $\sigma_w$. If the error distribution of $u_*$ is symmetrical, the error distribution of $\sigma_w/u_*$ will show considerable positive skewness. Hence, if the relation of $\sigma_w/u_*$ to $z/L$ is inferred from plots such as Figs. 1 to 3, it is likely to be biased toward high values. The larger the uncertainty of $u_*$, the larger this bias is. Therefore, it is quite possible that the increase of $\sigma_w/u_*$ with $z/L$ in stable air is actually much slower than that suggested by Eq. (3).
<table>
<thead>
<tr>
<th>Author</th>
<th>Location</th>
<th>Surface</th>
<th>Stable or unstable</th>
<th>Dates</th>
<th>Instrumentation</th>
<th>Height (metres)</th>
<th>Readings per sec.</th>
<th>Duration (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Busch</td>
<td>S. W. Kansas</td>
<td>Land</td>
<td>Unstable</td>
<td>June to Sept. 1968</td>
<td>Hot-wire anemometer</td>
<td>5-66</td>
<td>≻1</td>
<td>Unknown</td>
</tr>
<tr>
<td>2. Caughey and Readings</td>
<td>England; Florida</td>
<td>Land</td>
<td>Unstable</td>
<td>1969 to 1973</td>
<td>Yawmeter (angle wire)</td>
<td>3-1000</td>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td>3. Davidson</td>
<td>E. of Barbados</td>
<td>Water</td>
<td>Stable</td>
<td>May 1969</td>
<td>Orthogonal hot films</td>
<td>2, 3, 6, 8</td>
<td>25</td>
<td>80</td>
</tr>
<tr>
<td>4. Donelan and Miyake</td>
<td>E. of Barbados</td>
<td>Water</td>
<td>Unstable</td>
<td>May 1969</td>
<td>Sonic anemometer</td>
<td>19, 46, 150, 300</td>
<td>200</td>
<td>20km flight</td>
</tr>
<tr>
<td>5. Elder</td>
<td>Lake Ontario</td>
<td>Water</td>
<td>Both</td>
<td>Sept. to Nov. 1971</td>
<td>Orthogonal hot films</td>
<td>10</td>
<td>Unknown</td>
<td>Unknown</td>
</tr>
<tr>
<td>6. Haugen</td>
<td>S. W. Kansas</td>
<td>Land</td>
<td>Both</td>
<td>June to Sept. 1968</td>
<td>Sonic anemometer</td>
<td>5-66, 22-63</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>7. McBean</td>
<td>Ladner, B. C., Canada</td>
<td>Land</td>
<td>Both</td>
<td>Aug. 1969</td>
<td>Sonic anemometer</td>
<td>2</td>
<td>80</td>
<td>12-17</td>
</tr>
<tr>
<td>10. Mordukhovich and Tsvang</td>
<td>Rostov Region, USSR</td>
<td>Land</td>
<td>Both</td>
<td>July to Aug. 1964</td>
<td>Sonic anemometer</td>
<td>1, 4</td>
<td>Contin.</td>
<td>5-21</td>
</tr>
<tr>
<td>12. Smith</td>
<td>S. W. Lake Ontario</td>
<td>Water</td>
<td>Both</td>
<td>June 1972</td>
<td>Sonic anemometer</td>
<td>5-10</td>
<td>20</td>
<td>31-44</td>
</tr>
<tr>
<td>13. Tanner, et al.</td>
<td>Davis, California</td>
<td>Land</td>
<td>Both</td>
<td>May 1967</td>
<td>Anemoclinometer (3-d pressure sphere)</td>
<td>1, 4</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>14. Wieringa</td>
<td>Lake in Netherlands</td>
<td>Water</td>
<td>Both</td>
<td>Aug. to Sept. 1967</td>
<td>Aeolivane (trivane)</td>
<td>4-1, 8-4</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>15. Yokoyama</td>
<td>5km N. of Tokyo, Japan</td>
<td>Land</td>
<td>Both</td>
<td>Dec. 1968 to June 1969</td>
<td>Sonic anemometer</td>
<td>45, 180, 313</td>
<td>0-4</td>
<td>41§</td>
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Thus the equation must be regarded as a relatively simple compromise solution which is recommended at this time for practical use; it is subject to later modification on the basis of further measurements.

Whether there is an additional dependence on wave speed, as Davidson suggests, cannot be stated with certainty, since no other investigator so far has checked the relationships under the conditions of Davidson’s study. In the absence of such confirmation, we will then assume that (3) provides a satisfactory fit to the observations under all conditions.

In (3), the term containing $z/L$ represents energy produced by buoyancy, and that containing $\phi_m$ the energy produced mechanically. This can be seen from the fact that $(z/L)/\phi_m$ is the flux Richardson number. The factor 2.5 then implies that buoyancy is 2.5 times as effective as shear in producing vertical velocity fluctuations. This may be due to the fact that production of energy by buoyancy goes directly into vertical energy, whereas mechanical production goes first into longitudinal energy, and only later and, perhaps less efficiently, into vertical energy.

For the limit of large negative $z/L$, $\phi_\beta$ varies as $(z/[L])^4$, as required by the hypothesis of free convection and in good agreement with more complex theoretical models (see e.g., Deardorff 1972). The numerical factor in this relation comes out as 1.75, in good agreement with other estimates. The best estimate of $\sigma_\omega/\sigma_\beta$ under neutral conditions comes out as 1.3, although the uncertainty is still about 10%.

Given Eq. (1), a nomogram for $\sigma_\beta = \sigma_\omega/V$ has been constructed in Fig. 4. This nomogram clearly shows how $\sigma_\beta$ can be reduced by a factor of two when proceeding from land, with $z_0$ of order 10 cm, to water, with $z_0$ of order $10^{-3}$ cm. However, increasing stability can also reduce $\sigma_\beta$, going to zero for the ‘critical’ Richardson number where $\psi \to -\infty$.

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**DETERMINATION OF SURFACE STRESS FROM VERTICAL VELOCITY SPECTRA**

By D. Moravek, H. A. Panofsky and A. Weber*

**SUMMARY**

Many recent measurements of vertical-velocity spectra in the atmospheric boundary layer are combined in order to derive the dependence of the maximum of the logarithmic spectrum, normalized by surface stress, on stability. An empirical equation is suggested which fits the observations and has the proper limiting form for free convection. It is suggested that this equation can be used to derive surface stresses from relatively simple measurements of the most energetic part of the spectrum, and from a stability parameter.

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