


Yokoyama, O. 1971 An experimental study on the structure of turbulence in the lowest 500 meters of the atmosphere and diffusion in it, *Reports of the National Institute for Pollution and Resources*, Japan, No. 2, pp. 115.

Department of Meteorology, College of Earth and Mineral Sciences, Pennsylvania State University, 503 Deike Building, University Park, Pennsylvania 16802. 16 June 1975

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Determination of Surface Stress from Vertical-Velocity Spectra

By D. Moravek, H. A. Panofsky and A. Weber*

Summary

Many recent measurements of vertical-velocity spectra in the atmospheric boundary layer are combined in order to derive the dependence of the maximum of the logarithmic spectrum, normalized by surface stress, on stability. An empirical equation is suggested which fits the observations and has the proper limiting form for free convection. It is suggested that this equation can be used to derive surface stresses from relatively simple measurements of the most energetic part of the spectrum, and from a stability parameter.

* North Carolina State University.
1. Theory

The idea to determine surface stresses from spectra is not new; but usually such determinations are based on observations in the inertial subrange. Measurements in this region, however, require instruments capable of responding to high-frequency fluctuations, particularly close to the ground. In fact, frequently the minus five-thirds law appears to be satisfied, but the spectral region analysed is not isotropic, and stress estimates are biased.

It is suggested here to use the most energetic part of spectra of velocity components for the determination of surface stress, where measurements are less demanding. The technique requires that the spectra satisfy Monin–Obukhov similarity theory, a condition probably best satisfied for the vertical-velocity spectra. Therefore, this note will be confined to this velocity component.

It turns out that the function $nS_{\omega}(n)$ (where $n$ is frequency and $S_{\omega}(n)$ the spectral density of vertical velocity) is quite flat near its maximum; hence, the magnitude of the maximum can be established with little uncertainty, even if the frequency at this maximum is not known accurately.

We shall denote the maximum value of $nS_{\omega}(n)$ by $Q$. Then, according to similarity theory, $Q/\omega_{*}^{2} = F(z/L)$, where $\omega_{*}$ is the friction velocity, $F$ a universal function, $z$ the height and $L$ the Monin–Obukhov length. Once $F(z/L)$ has been determined, this equation can be used to determine the surface stress, $\tau = \rho \omega_{*}^{2}$ from measurements of $Q$ and a stability parameter only ($\rho$ is the air density).

It is the purpose of this note to use numerous recent measurements of $Q$ to assess the characteristics of $F(z/L)$.

2. The observations

Observations were obtained from 14 different sources, 7 over water or ice, and 7 over land. All the data over the ocean, and most over land were collected in the surface layer; two of the sets of land observations refer to much higher levels.

The sources for the observations over water or ice were: Donelan and Miyake (1973); Weiler and Burling (1967); Miyake, Stewart and Burling (1970); Pond et al. (1971); Denman and Miyake (1973); S. D. Smith (1972); Miyake et al. (1971).

Over-land observations were obtained from: Busch (1973); Busch and Panofsky (1968); Kaimal et al. (1972); Kaimal and Haugen (1967); Shaw, Silversides and Thurtell (1974), Yokoyama (1969), and Allen Weber (unpublished).

When the stability parameter was given in the form of a Richardson number, the same procedures were used to convert this into $z/L$ as recommended in the companion paper (Merry and Panofsky 1975). Where the spectra had been normalized by the variance of vertical velocity rather than by $u_{*}^{2}$, only near neutral situations were analysed and the ratio of variance to $u_{*}^{2}$ was taken as 1·69.

![Figure 1](image)

Figure 1. Eq. (2) fitted to average observations from various sources.

At first, two separate graphical analyses were performed:

First, all over-sea observations of $Q/\omega_{*}^{2}$ were plotted against $z/L$. The scatter was enormous, and only a few average points were evaluated for unstable near-neutral conditions.

Second, all over-land data were plotted in the same fashion, except for Weber’s observations. The scatter was much less, and estimates of average less uncertain. Such averages were estimated for all $z/L$ values from $-0·4$ to $0·3$ in intervals of $0·1$.

If all vertical-velocity spectra were similar, $F$ should vary with $z/L$ in the same way as $(\sigma_{\omega}/\omega_{*})^{2}$. In that case, we would have, from the companion paper,
\[ F(z/L) = A [\phi_m - 2.5z/L]^2, \quad \ldots \quad (1) \]

where \( A \) is a constant.

However, spectra are more peaked in stable than unstable air. Therefore the ratio \( Q/\sigma_w^2 \) should be largest in stable stratifications. This condition can be satisfied by lowering the factor of \( z/L \) in (1). In particular, quite a good fit to the observations can be obtained by replacing the factor 2.5 in Eq. (1) by 2. We then take

\[ Q/\mu^2 = F(z/L) = 0.426[\phi_m - 2z/L]^2 \quad \ldots \quad (2) \]

Fig. 2 tests Eq. (2) on the basis of independent data, obtained by A. Weber at 18m and 91m on the meteorological tower of the Savannah River Laboratory in South Carolina. The instruments used were Climet cup anemometers and bivanes.

![Figure 2. Test of Eq. (2) on observations at Savannah River Laboratory.](image-url)

Apparently Eq. (2) fits these observations reasonably well, and we conclude that it is as good an estimate for \( F(z/L) \) as can be obtained at present. Among other things, it satisfies the free-convection condition that \( Q \) must become independent of stress at large negative \( z/L \).

However, as was mentioned in the companion paper, an equation derived from ratios of scattered data is likely to be biased toward larger values. Fig. 2 suggests that the increase of \( Q/\mu^2 \) with \( z/L \) for \( z/L > 0.1 \) is actually slower than suggested by Eq. (2).

In summary, it is suggested that stress can be estimated by first determining the spectrum density \( Q \) of vertical velocity in the region in which \( nS_w(n) \) has a maximum, and then computing the stress from

\[ \tau = \rho \mu^2 = 2.35Q[\phi_m - 2z/L]^{-3} \quad \ldots \quad (3) \]

Suggestions for estimation of \( \phi_m \) have been given in many places, e.g., in the companion paper, and by Businger et al. (1971). The same papers also suggest relations between \( z/L \) and Richardson number.

It is likely that such stress estimates are more accurate in unstable and neutral air than the considerable scatter in Fig. 2 would indicate. Probably much of the scatter in that figure is due to the known large random variability of stresses measured by the eddy-correlation technique. In stable air such estimates are unreliable and biased toward low values. When more measurements are available Eqs. (2) and (3) should be modified for use in stable air.

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Yokoyama, O.

Department of Meteorology,
College of Earth and Mineral Services,
Pennsylvania State University,
503 Deike Building,
University Park,
Pennsylvania 16802.
16 June 1975


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551.557.4 : 551.558.1

Comments on the paper by R. S. Pastushkov 'The effects of vertical wind shear on the evolution of convective clouds'

By J. H. Fenner

Taking great interest in the subject of effects of wind shear on convective clouds, I read the paper by Professor Pastushkov (Quart. J. R. Met. Soc., 101, pp. 281-291), hoping to be able to apply it to thunderstorm forecasting. Unfortunately, his $E_0$, the initial energy of atmospheric instability and $\Delta H$, the height of the unstable layer' take on unrealistic values for the earth's atmosphere. From his Table 1, p. 284 (using SI units), $E_0$ takes on values between 1700 and 2800m$^2$/s$^2$ for 'strong' convection, and between 1500 and 1900m$^2$/s$^2$ for 'weak'. $\Delta H$ is either 2500 or 3500m. Using values $E_0 = 2800$ and $\Delta H = 3500$, and solving

$$E_0 = g \int_0^{\Delta H} \frac{T(z) - T_0(z)}{T_0(z)} \, dz = 2800 \text{m}^2/\text{s}^2$$

This is not a realistic value for $E_0$. It seems that $E_0$ should be a function of the length scale of the atmosphere, and $\Delta H$ should be a function of the height of the atmosphere.