Note on a preliminary scheme
for storm surge prediction using numerical models

By R. A. FLATHER and A. M. DAVIES

Institute of Oceanographic Sciences, Bidston Observatory, Birkenhead, England

(Received 14 November 1974; revised 16 June 1975. Communicated by Mr. G. R. R. Benwell)

SUMMARY

The initial stages of an investigation aimed at setting up a new system for the prediction of storm surges in the North Sea are described. The proposed scheme is based on the use of dynamical finite-difference models of the atmosphere and of the sea, the atmospheric model providing the necessary forecasts of meteorological data. These data are converted into the form required for input to the sea model, which then computes the associated storm surge.

Some preliminary results are presented and discussed and an indication given of the further development of the system.

1. INTRODUCTION

During recent years the use of primitive equation models for simulating atmospheric motion has become well established. Further, many of the problems associated with the application of these methods to the forecasting situation have been overcome, with the result that numerical models are now being employed in routine weather prediction.

Finite-difference techniques, employed in the above work, have also been widely used to solve the differential equations governing the motion of the sea under the action of tidal and meteorological forces, demonstrating the potentialities of sea models for storm surge prediction (Heaps 1969; Duun-Christensen 1971). However, in previous surge computations the necessary meteorological data have been either obtained from observations and therefore available only for hindcasts, or extracted by hand from weather charts—a cumbersome and to some extent subjective procedure, not well suited to operational forecasting. Further, the data derived have often taken the form of representative averages over a small number of quite large areas of the sea. In the present paper a description is given of a preliminary scheme, which seeks to overcome these difficulties by linking a numerical model of the atmosphere to one of the sea in a single dynamical surge prediction system. The essence of the proposed scheme is first to obtain forecast meteorological fields from the atmospheric model and then to process the data to give wind stress components and gradients of atmospheric pressure at individual grid points of the sea model, finally using this model—with the processed data as input—to predict the associated storm surge. The main advantage of the approach is that the procedures involved may be readily carried out by computer, since data exist throughout in digital form on regular meshes.

The long-term objective in applying the method to the North Sea is to establish an operational system for water level forecasting based on dynamical techniques. The requirements of such a system are to give warning of possible flooding of low-lying coastal areas around the Southern Bight; to provide, in advance, information needed for the successful operation of the future Thames Barrier; and to predict water levels offshore so as to facilitate navigation by deep-draught vessels in the shallow approaches to ports in the region. Although tidal variations are not as yet taken into account, the scheme in its present form is capable of producing a genuine forecast of meteorologically generated disturbances. The forecast could be available up to 18 hours before the start of the 12-hour period to which it
applies. Much work, however, remains to be done to improve the results, whose accuracy depends on the success of each stage in the prediction procedure.

Since the most accurate prediction of the tide at places where tide gauges are located is provided by methods based directly on analysis of time series of observations, it is envisaged that the contribution of the present system to the ultimate water level forecast would consist only of disturbances generated by meteorological forcing. For this reason, the forecasts obtained are evaluated by comparing calculated surge heights with residuals: values of (observed water level) – (predicted tide).

2. THE SEA MODEL

(a) Equations of motion and continuity

The dynamical equations for storm surges have been used in a number of different forms incorporating various assumptions (Proudman 1954; Reid and Bodine 1968; Duun-Christensen 1971). Following Duun-Christensen but taking east-longitude and latitude as space co-ordinates, the depth-averaged equations are

\[
\frac{1}{R \cos \phi} \left\{ \frac{\partial}{\partial \chi} (Du) + \frac{\partial}{\partial \phi} (Dv \cos \phi) \right\} + \frac{\partial \zeta}{\partial t} = 0, \quad (1)
\]

\[
\frac{\partial u}{\partial t} - 2 \omega \sin \phi \frac{v}{\cos \phi} = - \frac{g}{R \cos \phi} \frac{\partial \zeta}{\partial \chi} - \frac{1}{\rho R \cos \phi} \frac{\partial p_a}{\partial \phi} + \frac{1}{\rho D} (F^{(s)} - F^{(b)}), \quad (2)
\]

\[
\frac{\partial v}{\partial t} + 2 \omega \sin \phi \frac{u}{\cos \phi} = - \frac{g}{R \cos \phi} \frac{\partial \zeta}{\partial \phi} - \frac{1}{\rho R \cos \phi} \frac{\partial p_a}{\partial \phi} + \frac{1}{\rho D} (G^{(s)} - G^{(b)}), \quad (3)
\]

where the notation is:
- \( \chi, \phi \) east-longitude and latitude, respectively
- \( t \) time
- \( \zeta \) elevation of the sea surface
- \( u, v \) components of the depth mean current
- \( F^{(s)}, G^{(s)} \) components of wind stress on the sea surface
- \( F^{(b)}, G^{(b)} \) components of the bottom friction
- \( p_a \) atmospheric pressure on the sea
- \( D \) total depth of the water = \( h + \zeta \)
- \( h \) undisturbed depth of the water
- \( \rho \) density of the water, assumed uniform
- \( R \) mean radius of the earth
- \( g \) acceleration of the earth’s gravity
- \( \omega \) angular speed of rotation of the earth

The component directions are those of increasing \( \chi, \phi \) respectively, i.e. to the east and to the north. The depth mean currents are defined as

\[
u = \frac{1}{D} \int_{-h}^{h} u' \, dz, \quad \nu' = \frac{1}{D} \int_{-h}^{h} v' \, dz,
\]

where \( u', v' \) are components of horizontal current at depth \( z \) below the sea surface. Advective terms, which are generally considered to be small in the sea except in localized regions of strong currents and large velocity gradients (Bretschneider 1967), are omitted in Eqs. (2) and (3).

A quadratic law relating bottom stress to the depth mean current is adopted, giving

\[
F^{(b)} = k_b \rho u (u^2 + v^2)^k, \quad G^{(b)} = k_b \rho v (u^2 + v^2)^k, \quad (4)
\]
where $k_2$ is a constant for which an appropriate value is 0.0025 (Proudman 1953).

The problem is then to solve Eqs. (1) to (3) to find the variations of $\zeta, u, v$ over the sea area, given the changing distributions of wind stress and atmospheric pressure at the sea surface.

(b) Initial and boundary conditions

Because of the high rate of frictional dissipation in shallow seas, disturbances are damped quite rapidly, so that after one or two days the influence of the initial conditions becomes negligible in comparison with effects produced by the meteorological forcing functions. It is usual in hindcasting to take advantage of this fact by assuming that surges are generated from an initial state of rest, $\zeta = u = v = 0$ at $t = 0$, starting the calculation one or two days before the beginning of the period of real interest. When an operational prediction system has been established, initial conditions will always be available in the form of fields calculated and stored during the preceding forecast.

Conditions on $\zeta, u, v$ must also be satisfied along the lateral boundaries of the sea region. Thus, along a coastline, the normal component of current vanishes giving $q_n = 0$ for all $t \geq 0$ where $q_n = u \sin \beta + v \cos \beta$ and $\beta$ denotes the direction of the outward normal to the coast measured clockwise from north.

Several alternative conditions exist for open sea boundaries. First, elevation may be specified as a function of position and time

$$\zeta = \zeta(x, \phi, t),$$

where $\zeta$ represents the disturbance propagating across the boundary. A particular case of this condition for surges has $\zeta(x, \phi, t) \equiv 0$ (Veltkamp 1954; Groen and Groves 1962; Heaps 1969). However, as pointed out by Reid and Bodine (1968), such conditions do not take proper account of the damping due to loss of energy from the model to the neighbouring ocean since, for example, a plane wave incident on a boundary where $\zeta \equiv 0$ would be perfectly reflected with a phase change of 180°. To permit dissipation of energy across the boundary Reid and Bodine suggested that a radiation condition be used. We suppose that $\zeta$ represents the elevation associated with the externally generated surge entering the model. Then the internally generated disturbance reaching the boundary is $\zeta' = \zeta - \zeta$, since $\zeta$ is the total elevation. Applying a radiation condition to the internally generated part of the surge gives $hq_n = (gh)\zeta'$, where $q_n$ is the associated outward going current across the boundary. It is assumed here that the internally generated disturbance may be represented locally as a plane progressive wave travelling at right angles to the boundary. To close the problem, it is further assumed that the current $q_n$ associated with the input disturbance $\zeta$ is zero, so that the resulting boundary condition employed is

$$hq_n = (gh)\zeta'.$$

It should be pointed out that in order to use (5) or (6) in a surge prediction scheme, forecast values of $\zeta(x, \phi, t)$ must be provided. In the present system these are estimated from the meteorological forecast data as described in section 3.

(c) Method of solution

The solution of Eqs. (1) to (3) is obtained by means of finite difference techniques using a two-dimensional sea model developed from that described by Heaps (1969). The spatial mesh of the sea model covering the continental shelf surrounding the British Isles is shown in Fig. 1. Details of the new scheme are given elsewhere (Flather and Davies 1975).
The computer program written to perform the sea model calculations was designed to read fields of east and north components of atmospheric pressure and wind stress, and also input elevations $\xi$ at intervals from data sets stored on magnetic disc. Provided these fields contain forecast values, the solution obtained is a prediction of the surge.

3. The Meteorological Data

The Bushby–Timpson ten-level model on a fine mesh (Benwell, Gadd, Keers, Timpson and White 1971) currently employed in weather forecasting at the UK Meteorological Office, Bracknell, provides the essential atmospheric data. Ten-level model forecast runs of duration 36 hours are carried out twice a day. For the purpose of the present investigation, hourly values of the geopotential height, $H$, of the 1000mb pressure surface at meteorological model grid points covering the northwest European continental shelf (Fig. 1), were stored on magnetic tape. Hours 6 to 18 of each forecast were selected, giving a set of 13 spatial arrays of data spanning the 12-hour period covered. All forecast quantities required for input to the sea model have to be derived from the geopotential heights. The derivation involves the use of similar assumptions and empirical relationships to those introduced in earlier surge calculations (Heaps 1969, Duun-Christensen 1971). A brief description of the treatment of this data follows. For details of the derivation the reader is referred to Flather and Davies (1975).

The atmospheric pressure $p_a$ on the sea surface is obtained by applying the hydrostatic law so that $p_a = 1000mb + \rho_a g H$, where $\rho_a$ is the density of air assumed uniform and constant in the vertical between the sea surface and the 1000mb pressure surface. By taking appropriate differences of the pressure values obtained on the meteorological model grid, estimates can then be made of east and north gradients of atmospheric pressure and subsequently the east and north components of the geostrophic wind may be deduced.
The calculation of surface wind from geostrophic wind requires the use of an empirical law. However, although the optimum formulation of this law has been the objective of investigators for many years, no generally accepted formula has evolved. On the basis of measurements taken in the German Bight, Hasse and Wagner (1971) found that

$$w = 0.56 \hat{w} + b,$$

where $w$ and $\hat{w}$ are the magnitudes of surface and geostrophic winds respectively, and $b$ is a constant in the range 1.5 to 3.0 m s$^{-1}$ depending upon air–sea temperature differences. For the present work, (7) has been used with $b = 2.4$ m s$^{-1}$; the value appropriate to the case of a neutrally stable air column. Eq. (7) can be considered reasonably representative of the region of present interest, since it was obtained from measurements taken within the North Sea. The directions of surface and geostrophic winds are assumed to be the same.

The wind stress on the sea surface is computed using a quadratic law with the drag coefficient, depending on wind speed, employed by Heaps (1965).

The methods outlined above provide estimates of the necessary meteorological quantities at grid points of the atmospheric model, while the input data required for the surge calculation comprise values at appropriate grid points of the sea model mesh. These are

Figure 2. Weather charts for the storm surge of 26–30 March 1972.
obtained by interpolation using an extension of the method of differences to two dimensions (Buckingham 1957).

Finally, the surge input elevation \( \xi \) is estimated by applying the hydrostatic law to the sea and assuming some mean atmospheric pressure, \( \bar{p}_a \). Thus \( \xi = (\bar{p}_a - p_a)/\rho g \), with \( \bar{p}_a \) taking the value 1012mb. The computer program written to perform the calculations described produces output which is stored directly on magnetic disc for input to the sea model program.

4. Computation of Surges and Discussion of Results

A number of test computations have been carried out covering two surge cases (26–30 March 1972 and 28 March–6 April 1973) and employing the different open sea boundary conditions, (5) and (6), mentioned in section 2(b). Sample predictions for the period 26–30 March 1972 are now examined in relation to the meteorological situation and compared with observed residuals. The results presented were obtained using the radiation open boundary condition, (6). A summary of results obtained for the period 28 March–6 April 1973 is also given.

The meteorological conditions which produced the first surge are shown in Fig. 2. During the period 26–28 March 1972 a large though not very deep depression moved steadily south-eastwards from Iceland, crossing southern Scandinavia before turning north-east into the Baltic and filling slowly. A weak ridge of high pressure reached the North Sea in the early hours of 30 March. The surge residuals derived from observations, plotted in Fig. 3, and the computed co-disturbance lines shown in Fig. 4, indicate that the strong westerly and north-westerly winds acting over the North Sea raised water levels on the Dutch coast and in the German Bight by up to 120cm. The changes in elevation occurred slowly producing a very broad surge peak, see Fig. 3. Little disturbance was felt on the English coast.

Fig. 3 shows that the computations successfully reproduce the surge observed at almost all the coastal stations. Only at Esbjerg, where the computed surge is consistently 30 to 40cm lower than observed, can the results be considered less than satisfactory. Here the calculated surge values used in the comparison refer to an elevation point some 50km west of Esbjerg itself, and, since the greatest sea surface slope occurs in the east-west direction, see Fig. 4, it seems probable that the deficiency may be substantially accounted for by this fact alone. Surge disturbances introduced on the open boundary following the start of the calculation at 1800h on 26 March from an initial state of rest, produce the external surge which can be seen in Fig. 3 to propagate from Stornoway to Wick and thence down the east coast of Scotland and England. The erroneous surge peak occurring at Immingham at 0600h on 27 March can be attributed to this cause (see Fig. 4). It seems advisable, therefore, to allow a running-in period of at least 24 hours following the start of a computation from zero initial conditions. Thereafter, comparisons between calculated and observed surges can reasonably be made. With this in view, the discrepancies between calculated and observed surges at each port have been quantified by computing r.m.s. errors based on hourly values of the difference between the respective elevations for the last three days: 1800h 27 March to 1800h 30 March. For the present solution, the r.m.s. errors thus obtained are typically between 10 and 20cm; though larger errors appear at Cuxhaven, 23-2cm, and at Esbjerg, 33-3cm, and smaller errors at Lowestoft, 9-3cm, and at Bergen, 7-3cm.

The differences between the present solution using condition (6) and its equivalent using condition (5) are small within the North Sea. However, the solutions differ considerably in character at ports near the open boundary, the second containing a great deal more "noise" in the form of short period oscillations than is apparent when the radiation condition
Figure 3. Storm surge of 26–30 March 1972; ••••••••, residuals derived from observations; x-x-x-x-x, surge computed using the present scheme.
(6) is used. The source of this noise may perhaps be associated with the reflection of disturbances at the open boundary resulting from the use of (5). The relative absence of short-period variations from the first solution would then be due to the radiation of the associated energy outwards, as permitted in condition (6), and its subsequent dissipation in the neighbouring ocean. Since the accuracy of both solutions are similar (r.m.s. errors differ by at most 2 or 3 cm) the results obtained using the radiation condition would appear to be the more satisfactory.

A noticeable feature of the observed residuals at some ports is a semi-diurnal oscillation, which can be seen in Fig. 3 at Esbjerg, Cuxhaven and Southend, for example. It seems probable that the origin of these oscillations, which do not appear in the predictions, is tidal. It is well known that the influence of the tide on a storm surge is to produce an interaction which is associated with the non-linear terms in the hydrodynamical equations (Proudman 1955a, b). Tide-surge interaction has been shown to be of considerable importance in the Thames Estuary (Banks 1974), producing oscillations of tidal period in the surge residuals, and this mechanism is the most likely source of the fluctuations observed here. The amplitude of the oscillations suggests that the failure of the present solutions to take account of interaction may contribute significantly to the resulting errors. Some priority
should be given in the future to modification of the present preliminary system to allow tide and surge to be computed together within the sea model, thereby taking account of these effects.

The second surge event examined differed significantly from that described above in that the meteorological situation showed rapid changes. The principal surge activity was confined to the period 1–6 April 1973. A small depression developed west of Ireland in the early hours of 1 April and moved quickly east across northern England, deepening considerably and entering the North Sea near the Wash at about 1000h on 2 April. Strong northerly winds followed the depression; a wind speed of 65 knots being recorded at Kilnsea in the mouth of the Humber at midday. The depression continued to move east into the southern Baltic. The associated surges, including a small negative surge of about 12 hours duration in the Thames Estuary followed by a positive surge of 160cm and 24 hours duration in the Southern Bight, were predicted quite accurately. Later, a second depression from the Atlantic deepened and moved rapidly northeast reaching the Faroes at about 0600h on 4 April. The associated frontal system, bringing with it locally strong southerly and southwesterly winds, crossed the North Sea during the day, causing a large negative surge of up to −150cm at some places in the Southern Bight. This negative surge was not predicted by the model for reasons which are not clear. However, two factors which could have contributed to this failure and which should be examined in the future may be mentioned here. First, the discontinuity in pressure gradient at the front, perhaps poorly resolved in the original data, could have been further smoothed out by the subsequent treatment of the meteorological model output. The resulting winds may thus have been seriously underestimated in the neighbourhood of the front. A critical examination of the processing of the atmospheric data and revision of the system to make optimum use of the resolution available could bring some improvement in this aspect of the scheme. Second, since a given wind stress produces a gradient of the sea surface proportional to \(D^{-1}\) (see Eqs. (2) and (3)), substantial errors may arise locally because the sea model fails to represent adequately the areas of shallow water such as occur in the Thames Estuary, the Wash and the German Bight. Some improvement in this respect should be possible within the limitations of the present sea model grid size.

5. Concluding remarks

A first system employing numerical finite-difference models of the atmosphere and of the sea to forecast storm surges in the North Sea has been established. Some of the deficiencies of the preliminary scheme, pointed out in the text, are currently being examined with the hope that substantial reductions in the errors may result. If this proves to be the case, the potential of the system appears to be good. In particular, the ability of the scheme to predict the spatial distribution of a surge and its development could make it a useful supplement to existing techniques.

References


Flather, R. A. and Davies, A. M. 1975 The application of numerical models to storm surge prediction, *Institute of Oceanographic Sciences, Report* No. 16.


