An analysis of Manley’s central England temperature data: I

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Summary

Manley’s (1974) temperature data for central England are subjected to analyses which suggest that they should be treated as a set of monthly time series. Secular variation is found for the months October to April inclusive. Extended methods dealing with spectral analysis are used to focus attention on the mainly longer-period oscillations. Oscillations of lengths 10–12 and 22–25 years are suggested for some months. Burg’s maximum entropy spectral method has been used to resolve signals in the very low frequencies. The combined variance associated with these oscillations is shown to increase for months near mid-year when the sun has its maximum north declination and to decrease for months on either side of June and July to a minimum when the sun has greatest south declination. Furthermore, the variance associated with months when the sun changes direction in declination and crosses the equator appears to receive boosts.

1. Introduction

Seldom do we have the opportunity to analyse a climatological time series of useful length. Such a series has been provided by Manley (1974) who presents monthly mean temperatures for central England for the period 1659 to 1973. That work is an updated version of Manley’s earlier papers (Manley 1953, 1959). Craddock (1965) carried out an extensive study of Manley’s earlier data using principal components analysis. Craddock’s analysis is also used by Kendall and Stuart (1966), and it is suggested that movement of a secular kind is present, together with an harmonic movement over the year and also over the winter season. The only work to date on Manley’s latest data set is by Shapiro (1975). He presents a brief analysis of the data taken as a string of 3780 months. The conclusion drawn is that significant relative variance is associated with oscillations having periods of 25-5, 12, and 6 months.

The present paper was intended to be short but going a little deeper than Shapiro. However, as is often the case with geophysical time series, the further we went the more interesting avenues of exploration arose. It has been decided to truncate the work having dealt with 13 series each of length 315 years. A second paper will deal with the temporal stability of possible oscillations, and investigate the plausibility of forecasting future temperature variation over central England.

2. Data arrangement

The temperature data as given by Manley can be analysed in two forms. Firstly, two time series of length $315 \times 12$, and $315$ by one, obtained by taking the terms as a string of consecutive monthly values, and a series for the annual values. Secondly, the data may be treated as 13 time series each of length 315 years; 12 for the monthly values, and one annual series. For the first course to be justified it is necessary to consider the $315 \times 12$ matrix of data points as being explained by two factors: one due to differences between the months (columns) and the other relating to differences between years (rows) and show that the latter factor is statistically insignificant. Should the factor relating to the temporal effect prove to be significant, then sampling can be assumed to have arisen from populations having
different parameters. In the event of the second alternative being adopted, the seasonal effect so strongly present in power spectra arising from adopting the first approach is, of course, removed. For the first case sampling is by the month, whereas in the latter it is by the year.

Which of the above approaches is most appropriate can be decided by carrying out a two-way analysis of variance on the data matrix (315 × 12) of temperature. Assume that each monthly temperature value, \( X_{ij} \), is composed of an overall or grand mean, \( \mu \), of the 315 × 12 temperature values; a factor relating to a temporal variation over years whose \( i \)th differential effect is \( \alpha_i \); a factor relating to months whose \( j \)th differential effect is \( \beta_j \); and an error component \( \varepsilon_{ij} \); then the following linear model can be postulated

\[
X_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}, \quad i = 1, 2, \ldots, 315, \quad j = 1, 2, \ldots, 12,
\]
and fitted as in Dyer (1976). The null-hypothesis of interest is that all the population row means are equal.

**TABLE 1. Sums of squares for the analysis of variance on the central England temperature data**

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Sum of squares</th>
<th>Degrees of freedom</th>
<th>Mean square</th>
<th>Variance ratio ( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row means</td>
<td>1450</td>
<td>314</td>
<td>4.62</td>
<td>2.75</td>
</tr>
<tr>
<td>Column means</td>
<td>79817</td>
<td>11</td>
<td>7256:10</td>
<td>4319</td>
</tr>
<tr>
<td>Error</td>
<td>5813</td>
<td>3454</td>
<td>1.68</td>
<td></td>
</tr>
</tbody>
</table>

Most of the variation in the temperature data arises from differences between column means (between months), as would be expected (Table 1). For the row means the critical \( F \) value at the 1\% level of significance with 314 and 3454 degrees of freedom is 1.15. Because the calculated \( F \) exceeds 1.15, we may reject our null-hypothesis and conclude that the population row means are not all equal. It follows that further analysis should be carried out on time series composed of individual monthly temperature values to remove the situation whereby different populations are mixed or combined.

3. Analytical techniques

A number of different techniques are used in this analysis, and to avoid confusion they may be tabulated as follows:

(i) test for homogeneity of monthly series (already carried out in section 2);
(ii) test for presence of trend;
(iii) remove trend using least-squares fitted straight line;
(iv) calculate serial correlations and hence obtain plots of correlograms;
(v) (a) filter temperature data using a difference filter when estimating spectral ordinates at the higher frequency end of the spectra; (b) filter temperature data using the coefficients in a five-term binomial expansion as weights when estimating spectral ordinates at the lower frequency end of the spectra;
(vi) obtain sample spectra for three different maximum lags namely 50, 75, and 100 years;
(vii) reclaim the spectra of the unfiltered data by dividing the ordinates of the spectra of the smoothed data by the square of the gain function belonging to the appropriate filter;
(viii) place 95\% confidence intervals on the spectra;
(ix) Apply Bürge's (1968, 1970, 1972) maximum entropy spectral technique to obtain better resolution of signals at low frequencies.
(a) Secular variation

Spearman’s coefficient of rank correlation, $r_s$, was used to test for trend in the temperature series. Unlike Pearson’s coefficient, usually used at lag one, it is sensitive to various types of trend. The values of $r_s$ for January to December and the annual means were: 0.176, 0.158, 0.184, 0.175, 0.085, 0.001, 0.029, −0.001, 0.076, 0.179, 0.147, 0.173 and 0.266 respectively. The underlined values are significant at the 1% level — except November, 5% — for a two-tailed test, the null hypothesis being that of no trend.

These results clearly suggest that blocks of months are behaving differently with respect to secular variation in temperature. Whereas the summer and part of the autumn seasons appear to be stationary in a temperature sense, the latter part of autumn, the whole winter

![Graphs of temperature data](image)

Figure 1. Temporal behaviour of the annual means of temperature for central England: Vertical bars represent the actual means; the smooth curve is the result of smoothing these means with a five term binomial filter to obtain better clarity. The upward trend is obviously slight and may be non-linear.
season, and that of spring are undergoing an upward trend in temperature. This general increase in monthly temperatures is reflected in the \( r_s \) value for annual means. Although many of the \( r_s \) values are significant, the degree of trend is slight. The time series for the annual means is shown in Fig. 1, where the smooth curve is obtained by using a five term binomial filter.

(b) The correlograms

Serial correlation coefficients for each of the thirteen series were calculated up to a maximum lag, \( m \), of 100 years by \( r(k) = c(k)/c(0) \), where \( c(k) \) is defined as

\[
c(k) = \frac{1}{n} \sum_{t=1}^{n-k} (x(t) - \bar{x})(x(t+k) - \bar{x}), \quad \text{and} \quad \bar{x} = \frac{1}{n} \sum_{t=1}^{n} x(t).
\]

\( x(t) \) is the temperature series, and the lag \( k \) taking values in the range \( 0 \leq k \leq m \). For the case where \( k = 0 \) we have \( c(0) \) giving the variance of the temperature series being analysed.

The correlogram for the time series of annual mean temperatures suggests that the series is somewhat oscillatory in nature (Fig. 2). The wave pattern is, however, complex and shows a number of superimposed oscillations of different lengths. Under these circumstances the correlogram is difficult to interpret. The primary period is probably rather long, with shorter periods in the range 20–25 years and less – e.g. of order 2 years. There is some evidence to suggest an oscillation of order 15 years. On the assumption that all the coefficients are zero, it is seen that sixteen have confidence intervals that do not include zero; the expected number for a random series would be 5% of the 100 coefficients calculated, i.e. 5. The confidence intervals were obtained by taking the standard error of the coefficients as 1/\(315–2\) and using the normality assumption

\[
r(k) - 1.96/\sqrt{313} \leq \rho(k) \leq r(k) + 1.96/\sqrt{313} \quad \ldots \quad (1)
\]

and hence

\[
r(k) - 0.11 \leq \rho(k) \leq r(k) + 0.11 \quad \ldots \quad (2)
\]

where \( \rho(k) \) represents the population correlation coefficient and \( r(k) \) the sample estimate. The correlograms for January to December are presented in Fig. 3 and are drawn to the same scale as in Fig. 2. The confidence limits given in Fig. 2 therefore apply to the correlograms of the individual month, and can be picked off with dividers.

Using the same criterion that was applied to the annual mean series, it is seen that all the monthly series have correlograms with more significant ordinates than would be expected by chance (Table 3). Without exception these correlograms are again complex in form. A
Figure 3. Showing the correlograms for the temperature time series of monthly mean values for central England.
TABLE 3. Showing for each month the number of serial correlation coefficients whose 95 per cent confidence interval does not contain zero. The expected number to arise by chance is 5

<table>
<thead>
<tr>
<th>Month</th>
<th>No. significant correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>8</td>
</tr>
<tr>
<td>February</td>
<td>6</td>
</tr>
<tr>
<td>March</td>
<td>6</td>
</tr>
<tr>
<td>April</td>
<td>9</td>
</tr>
<tr>
<td>May</td>
<td>12</td>
</tr>
<tr>
<td>June</td>
<td>12</td>
</tr>
<tr>
<td>July</td>
<td>9</td>
</tr>
<tr>
<td>August</td>
<td>11</td>
</tr>
<tr>
<td>September</td>
<td>13</td>
</tr>
<tr>
<td>October</td>
<td>7</td>
</tr>
<tr>
<td>November</td>
<td>10</td>
</tr>
<tr>
<td>December</td>
<td>8</td>
</tr>
</tbody>
</table>


discussion of their behaviour is best carried out in conjunction with that dealing with the spectra of the temperature series.

(c) Filtering the detrended data

Reliable spectral estimates can be obtained from a spectral analysis only if the spectrum under review is fairly flat. For an irregular spectrum, one with peaks, bias can be introduced into the sample spectral ordinates by the leakage which arises when the raw spectrum is smoothed by means of a lag window. One can either flatten the whole spectrum or remove the peaks in the frequency range where interest is not directed.

The correlograms shown earlier indicate the possibility of both long and short oscillations. For this reason it was decided to use two different filters: one suppressing the amplitude of long, the other of short, waves. The former can be removed by using a simple difference filter. That is, a new series is obtained by taking first differences on the series \( x(t) \), giving \( z(t) = x(t) - x(t-1) \), for \( 2 \leq t \leq n \).

The effect on the original series of applying this filter can be seen from a plot of the square of the filter's gain function \( |g(f)|^2 \), against frequency \( f \) (Fig. 4(a)), where \( g(f) = 2|\sin \pi f|, -\frac{1}{2} \leq f \leq \frac{1}{2} \), and the phase function is \( p(f) = \pi(f-\frac{1}{2}) \), \( 0 \leq f \leq \frac{1}{2} \), which gives the phase shift between the input and output series.

For the binomial filter the weights are obtained from the coefficients in the expansion of \((a+b)^4\). Each coefficient is divided by the sum of the weights so that their sum equals unity. Hence

\[
z(t) = 0.06x(t-2) + 0.25x(t-1) + 0.38x(t) + 0.25x(t+1) + 0.06x(t+2),
\]

for \( 3 \leq t \leq n - 2 \).

Also \( g(f) = |\cos 4\pi f| \) and no problems arise due to phase changes. The plot of \( |g(f)|^2 \) against \( f \) is given in Fig. 4(b).

(d) Computation of the spectra

A number of different equations are in common use to obtain the sample spectral ordinates over a given frequency range. We have used

\[
s(l) = \frac{1}{m} \left[ 1 + 2 \sum_{k=1}^{m-1} r(k) \lambda(k) \cos \frac{\pi lk}{m} \right], \quad l = 0, \ldots, m
\]

where \( r(k) \) has been defined in (2), \( m \) is the maximum lag and \( \lambda(k) \) is the lag window. Parzen's window was used, given by
\[
\lambda(k) = \begin{cases} 
1 - 6(k/m)^2 (1 - k/m) & \text{for } |k| \leq m/2 \\
2 (1 - k/m)^3 & \text{for } m/2 < k \leq m \\
0 & \text{elsewhere}
\end{cases}
\] (5)

The frequencies \( f \) can be obtained from \( k/2m \).

One of the problems encountered in spectral analysis is that of determining the maximum lag, \( m \). For \( m \) too large, instability, giving rise to spurious peaks, will arise. But one cannot play safe and keep \( m \) too small since this gives rise to a too smooth spectrum and

Figure 4. Plot of the square of the gain function for (a) difference filter and (b) for the five term binomial filter.

Figure 5. Showing the effect on the spectra of January temperature means for central England of varying the maximum lag, \( m \). For general purposes, a value \( m = 50 \) years is probably satisfactory.
hence poor resolution. The method adopted here consists of superimposing spectra calculated for different maximum lags and attempting to decide on the value of m at which instability commences. The result obtained for the January series is shown in Fig. 5. It would seem that increasing m beyond 50 years does not increase the number of peaks in the spectrum. So for general purposes this value of m was used, and increased when necessary. This was done when attention was directed at the low frequency end of the spectrum.

Before placing confidence intervals on the spectra, correction must be made for filtering. With \( s(f) \) the spectral ordinates of the input data (unfiltered) and \( s'(f) \) those of the output data (filtered) it is a straightforward matter to reclaim \( s(f) \) from \( s'(f) \). We use the relationship \( s'(f) = s(f).|g(f)|^2 \). This expression is the reason for plotting the square of the gain function against frequency, instead of the gain function itself. The former is the factor by which the filtered spectral ordinates must be divided in order to reclaim \( s(f) \).

![ANNUAL MEANS](image)

Figure 6. The spectrum of annual mean temperatures for central England. Even though trend has been removed, excessive variance is still associated with low frequencies. These frequencies are resolved into a peak at 100 to 200 years.

Confidence intervals for the spectral ordinates at any specific frequency \( f' \) can be obtained from

\[
d_s(f')/b \leq S(f') \leq d_s(f')/a, \text{ for } f' \neq 0, 0.5
\]

(6)

This is an equal-tail probability confidence interval for \( S(f') \) at a 100 \((1 - \alpha)\) per cent level. The equivalent degrees of freedom, \( d \), for the Parzen window is given by 3.7n/m, when \( n \) is the length of data record and \( m \) the maximum lag used. Chi-square tables are used to obtain values for \( a \) and \( b \) with the appropriate equivalent degrees of freedom and \( \alpha/2 \) value. If \( \log s(f) \) is plotted against \( f \) instead of \( s(f) \), then the width of the confidence interval is constant for all \( f \), since

\[
\log s(f') + \log (d/b) \leq \log S(f') \leq \log s(f') + \log (d/a)
\]

(7)

With formula (4) for the spectral ordinates, a random process would have a spectrum
Figure 7. In the spectra of monthly mean temperatures for central England few peaks are significant at the 95% level (see April). However oscillations of long wavelength are in evidence, these being supported by subsidiary peaks at some of their harmonics. Although peaks appear in most cases at the biennial oscillation, significance is lacking.

represented by a horizontal straight line at a height 1/m above the frequency axis. A confidence interval on any $s(f')$ that does not include 1/m can be taken as significantly different from a random process at the $\alpha$ level of significance.

(e) The spectra of monthly and annual mean temperatures

Significant peaks, at the 5% level, are not a feature of the temperature spectra (Figs. 6, 7, 8). The lower half of the 95% confidence interval is shown on the spectrum for April. Because the spectral ordinates are on a logarithmic scale, this interval is of constant width for all peaks and spectra. A random process would give rise to a flat spectrum, i.e. a horizontal line passing through 0.02(1/m). The peaks whose confidence interval does not include 0.02 can be assumed statistically significant.

It can be seen that January, February, April, May, August, September, October, November have each got one or two significant peaks. For a random process we could expect to get by chance 5% of the 51 ordinates showing significance, i.e. between 2 and 3. The spectra presented therefore fare rather poorly even for random series, which tends to
support the view that experience suggests to this author, namely that this test is conservative. Granger and Hatanaka (1966) give another criterion that may be used in an attempt to decide whether or not a spectral peak is real. It consists of looking for other peaks at frequencies that are harmonics of the basic one. The logic behind this is that no signal is likely to be pure.

The spectrum for the annual means has a major portion of its variance in a relatively short frequency interval at the low frequency end of the spectrum, together with a peak at a frequency of 0.47 cpy (2.13 years) (Fig. 6).

When the maximum lag is increased to 100 years (spectrum not shown) the resolution is improved and peaks at 100/200, 25 and 14.3 years appear, all of which are significant at the 5% level. The latter two are suggested by the two steps that occur for $m = 50$ years (Fig. 6). The correlogram and spectrum are mathematically equivalent; one gives a time domain the other gives a frequency domain description. A period of 100/200 years suggested by the spectrum is in agreement with the behaviour of the correlogram, which appears to have reached a minimum at something beyond a lag of 55 years. The same agreement applies to both the other peaks – 25 and 14 years. Also the behaviour of the ordinates at lags 2, 4, 6 are suggestive of a wave with period 2 years or thereabouts. It should be noted...
when interpreting the spectra that, unlike harmonic analysis, a spectral peak is taken as the ordinate at the centre of a frequency interval.

To deal with each pair of correlogram and spectrum for all the months would prove tedious. In general terms the correlograms of some months suggest deterministic components in the basic series. This view arises from the fact that in many cases the serial correlations do not damp out, even after making allowance for sampling disturbances. Furthermore to our minds, whereas some correlograms are suggestive of being oscillatory hence indicating a wave motion in the basic series, e.g. July, others have values which suddenly become large and this suggests that impulses are experienced by temperature at pseudo-regular intervals, e.g. January.

The spectrum for January is one of a number having a peak on or near zero frequency. When \( m \) is increased to 100 this peak is resolved and centred on a wavelength of 200 years. The next four peaks are at harmonics of 200 years. A significant peak is present at 3-13 years. The correlogram does little to clarify the situation, apart from suggesting a wave of the order 180 years.

For September, large serial correlations can be clearly seen at 10 years and some multiples thereof. They appear to be suppressed at 40, 50, 60, and 90 years by some other superimposed disturbance or disturbances. This is presumably because of the oscillation of long wavelength which is indicated by the general shape of the correlogram. The spectrum for this month shows a significant peak in the region of zero frequency and at a wavelength of 10 years. The peak at zero frequency becomes clear at a wavelength of 100 years, when the maximum lag is again increased to 100 years. A number of the peaks in this spectrum appear at harmonics of the 100 year wave.

If one considers the distribution of variance at the low frequency end of the spectrum over the months as a whole the result is interesting. It is convenient to commence with September, and it will be noted that a large proportion of the variance for this month, for October, and for November is present near zero frequency. This proportion decreases for December and remains low onto and including February. The two minor peaks in this month strengthen in March, and combine in April and May. June has very little variance at the low end of the spectrum, but this increases for July and shifts toward zero frequency in August. It would appear therefore that the temperature for September to January and to a lesser extent August has an oscillation of long wavelength (greater than 100 years). This suggests that, ignoring superimposed random fluctuations, any change in the temperatures for these months on a year-to-year basis should be more gradual than for the remaining months. That is: late summer, autumn and winter should vary less from year to year than the remainder of the year.

\( (f) \) Uncorrected spectra

Instead of smoothing the data and correcting the resulting spectra, \( s'(f) \), it is informative to investigate the uncorrected spectra, \( s(f) \), and the correlograms of the smoothed data. Because we cannot now place confidence intervals on the spectra, we used \( m = 100 \) because the loss in equivalent degrees of freedom was not important.

Smoothing data induces correlation between terms in the output series. The magnitude of the correlation and the terms over which it is present can be determined (Kendall and Stuart 1966). If smoothing is carried out over \( k \) terms using weights \( w(i) \), \( 1 \leq i \leq k \), it can be shown that the induced correlation \( c(s) \) is given by

\[
c(s) = \sum_{i=1}^{k-s} w(i)w(i + s) \Bigg/ \sum_{i=1}^{k} w^2(i), \quad |s| < k
\]  

\[8\]
Figure 9. Showing the correlograms corresponding to the spectra given in Fig. 8. The patterns are still confused in many cases, probably due to a number of waves being superimposed one on the other. For example a peak should appear at 40 years in the September graph, but interaction with another wave restricts the ordinates to the negative portion of the graph. These figures have been obtained using smoothed data.
For a five-term binomial filter our weights were 0.06, 0.25, 0.38, 0.25, 0.06 which gives \( c(s) \) values of 0.79, 0.39, 0.11, and 0.01 for lags one to four inclusive. From the foregoing, the first four serial correlations include induced correlation as a direct result of smoothing. In the correlograms that follow the first few ordinates are not displayed, since they convey little meaning, together with the fact that we have smoothed out the shorter oscillations on purpose.

A selection of correlograms is given (Fig. 9) which are somewhat easier to interpret than those for the unsmoothed data in Fig. 3. Distortion, however, still makes interpretation difficult. For example in September, the peaks that appear to be suggested at lags of 50, 70 etc. are attenuated because of the interaction of oscillations with longer wavelengths. However, it can be seen that signals with wavelengths in the neighbourhood of the two solar cycles are present in this figure.

Conflicting views have been put forward regarding a relationship, real or otherwise, between temperature and solar activity. King et al. (1974) and King (1975) described this relationship for central England July temperatures, whilst Shapiro (1975) found that no peak occurred in the spectrum for the temperature series taken as 315 x 12 months long.

The spectra for these smoothed series, not shown, indicate that the variance associated with signals comparable with those of the solar cycles vary in some organized way over months. For example, the variance associated with these frequencies increases over the months January to July. After July this variance decreases rapidly, increases again for September and then falls away for the remainder of the year. Toward the middle of the year the double solar cycle seems to be associated with most variance. On the other hand, for January and September it is the single cycle frequency that is most pronounced. This tends to be borne out by the correlograms (Fig. 9).

Since it is clear that any solar temperature relationship is month varying, use was made of the fact that the sun reaches its maximum north declination in June, maximum south declination in December, and crosses the equator in March and September. It was decided to investigate the relationship between the sun's declination and the combined variance associated with the two solar cycles for each month. Taking as a null hypothesis that the distribution of this variance over months is random, we would expect to find a rectangular distribution over months. That is, but for sampling fluctuations, the variances would present a horizontal line. That this is not the case is clearly shown in Fig. 10. It can be seen that most variance associated with these cycles is in June and July. The peak in variance is reached about three weeks after the sun has reached its position of maximum north declination. Furthermore, there is more variance associated with the solar cycles during the period when the sun is moving from south to north declination than the other way round. The variance also seems sensitive to the sun crossing the equator, and change in direction of movement. We see humps for March and September in Fig. 10, when the former occurs, and for December when the latter happens. It is likely, therefore, that the large amount of variance associated with June and July is accounted for partly by the maximum northern declination and partly because this is the point in the sun's apparent movement when it changes direction from moving south to north, to north to south.

(g) A reassessment of the longer waves

A recent and completely different approach to spectral analysis has been developed and introduced by Burg (1968, 1970, 1972). It is based on the concept of the entropy content of a time series and its maximization when subjected to certain restraints. The expression obtained by Burg for obtaining maximum entropy spectral estimates (m.e.m.) was
Figure 10. Showing the distribution of variance associated with the combined variance at the two solar cycles each month for monthly mean temperatures in central England (lower). The upper curve is the sun's declination over the months. Most variance is associated with June/July when the sun has greatest north declination, and least variance with those months when the sun has greatest south declination. When the sun changes direction, and when it crosses the equator, boosts in variance takes place. The variance associated with the months when the sun is moving from south to north is greater than for the opposite direction.

\[ P(f) = 2.\Delta t.Pm\left| 1 + \sum_{k=1}^{m} \gamma_k \exp(-2\pi jk\Delta t) \right|^2 \]  

(9)

where \( Pm \) is the mean output power of the series after whitening by a filter of length \( m \); \( \gamma_k \) are the prediction error filter coefficients; and \( \Delta t \) the sampling interval. The frequency, \( f \), is constrained to take values in the Nyquist interval, \(-1/(2\Delta t) \leq f \leq 1/(2\Delta t) \) (Wells and Chinnery 1973). The various theoretical aspects involved with the estimation of the prediction error filter coefficients are spread over the literature – concisely brought together by Ulyech and Bishop (1975). Resolution constraints are not present in (9) because the frequency response of a digital filter can be computed for arbitrary values of frequency; the window or filter adjusts itself to the data (Ulyech 1972a). This is a definite advantage over conventional methods when either the time series is short or the wavelength of the signal being investigated is of similar length to that of the data series (Currie 1973, 1974c). Because no fixed smoothing windows are applied m.e.m. provides unbiased estimates of spectral shape (Lacoss 1971; Currie 1974b).

Ulyech (1972b) showed that the m.e.m. correctly resolved the frequency of a sinusoid truncated to about one-half cycle. Furthermore he demonstrated that the frequency shifts associated with truncated sinusoids when spectral estimates are obtained by conventional methods are not present when the m.e.m. is applied. The technique is also appropriate for the determination of weak signals (Bolt and Currie 1975; Currie 1975a, b, 1974a).

A current disadvantage in using the m.e.m. is that its statistical distribution properties have not been resolved, and therefore confidence intervals cannot be placed on the spectral estimates (Currie 1974a). This provides the reason for not having used the m.e.m. throughout
the present paper. However, this aspect of the technique is being dealt with and it is hoped that a satisfactory outcome will be published later. It will be shown that the reality of spectral peaks can be approximately ascertained by considering signal-to-noise ratios (Banks 1969).

Using the m.e.m., peaks at the low frequency end of the spectra are, in most cases, clearly resolved. The length of the filter consisted of 60 terms, which was one of a number tried. This value gave satisfactory resolution of signals with stable spectra. The spectral ordinates were obtained at intervals of 0.0003 cpy (Fig. 11). These figures, when compared with Figs. 7 and 8, show the ability of the m.e.m. to resolve signals at the low frequency end of the spectra and to detect weak signals.

For the series of annual means the spectrum peaks at 94 years and the signal-to-noise ratio, s/n, is approximately 18. This value is more than adequate to indicate the physical reality of the peak. In the case of the spectra for monthly temperature series, they all exhibit satisfactory s/n values except May and July; in isolation February may be considered borderline (Fig. 11). There is an orderly frequency drift over some months, e.g. January to April, in an upward direction. The behaviour of the spectrum for May is difficult to explain, as is that for June. It is beyond the scope of this paper to delve deeply into this phenomenon. Suffice it to say that the early months appear to be grouped and have shorter long-term signals than those of the months toward the end of the year. The two sets of months are separated by the peculiar behaviour of May and June. It is possible that further analysis will permit combining the months within these groups to obtain mean periods for the indicated signals. The reason for the frequency shifts may become apparent when the series are analysed sequentially.

4. CONCLUDING REMARKS

It has been shown that the temperature data for central England should be analysed by the month. We feel that even this interval may be worth further reduction. A test for trend in the monthly and annual mean series indicated highly significant values for October round to April inclusive. The months May to September appear to be trend free. By the method employed, non-linear as well as linear trend could be detected. Linear trend is physically rather meaningless, unless we accept that the temperature of central England will go on increasing indefinitely. It is more plausible that an interval of a long oscillation was encountered. This agrees with the result of the conventional spectral analysis, and with that using Burg's m.e.m., which succeeded in resolving the peaks suggested by the former method. In general terms it would seem that the months are grouped: those at the beginning of the year behaving differently from those at the end and separated by a rather different pattern for May, June and July. The spectrum for the annual mean temperature series exhibits a clear peak at 94 years.

The serial correlations indicate that many of the temperature series are not wholly random. However significant peaks in the spectra were lacking in the conventional spectral analysis. This may be due to the presence of a number of oscillations which tend to blur both the correlograms and spectra and make interpretation difficult. Oscillations in the ranges 10–12 and 22–25 years were indicated and supported by the presence of their harmonics. The variance associated with them seems to vary in some organized way over the twelve months. The fact that the combined variance for these two oscillations are associated in pattern with the sun's declination suggests that solar activity and temperature relationships should be further investigated. This is another reason for using a smaller sampling interval, because if the relationship is weak there will be a tendency for it to be smoothed out if the time averages are too large. It may well be that sunspots provide too crude a measure of
Figure 11. The spectra for monthly and annual mean temperature series for central England computed using Burg’s m.e.m. Frequencies are at intervals of 0.0003 cpy. With the exception of June, wavelength coverage is 60 to infinity (years); for June it is 43 to 150 years. For all spectra, except May and July, the signal-to-noise ratios, s/n, indicate that the peaks are significant.
solar activity; perhaps other indicators should be used. Furthermore, different indicators may have more potent effects on meteorological parameters. The comparison so far has been subjective, but the outcome has logic in that the summer months have more variance associated with what could be the solar cycles, i.e. those months when the sun has high northern declination. The fact that the variance for months associated with change in direction of declination and the sun’s intersection with the equator receive boosts is particularly noteworthy.

As the presented analysis turned out, it posed more problems than it solved, and is we feel of value for this reason alone. In our opinion no work presented to date provides sufficient evidence to conclude with any useful degree of certainty that central England temperature variation is wholly random. The analysis of geophysical time series is seldom straightforward and those for the temperature of central England are promising to be no exception to this general rule.

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