An observed relation between the macroscale local eddy flux of heat and the mean horizontal temperature gradient

By G. B. TUCKER

CSIRO, Division of Atmospheric Physics, Aspendale, Victoria, Australia

(Received 13 April 1976; revised 8 June 1976)

SUMMARY

Frequent and detailed upper air observations at Laverton (38°S) for 30 days during September/October 1966 are analysed to determine the local horizontal eddy flux of heat and the mean horizontal potential temperature gradient. Components in the meridional and zonal directions are studied, but a more meaningful relation emerges when the vector eddy flux is considered in relation to the vector temperature gradient. The angle, \( \delta \), between these two vectors, and the down-gradient and cross-gradient exchange coefficients, \( K \) and \( R \), are derived. A 3-tiered structure between the surface and 30 km emerges, with down-gradient flux (\( K \) positive) in most of the troposphere and mid-stratosphere, and counter-gradient flux (\( K \) negative) in the lower stratosphere. The cross-gradient flux is in the direction of the thermal wind (\( R \) positive) at nearly all heights except in the lowest layers and between 20 and 24 km. \( \delta \) exhibits a fairly steady and systematic change throughout the height range analysed (2–28 km). Computations using standard upper air observations at selected Australian stations for 1970 show a mean annual pattern which is remarkably coherent with latitude (10–55°) and height (0–22 km), except in the tropical troposphere. Typical derived mid-latitude values for 1970 are:

<table>
<thead>
<tr>
<th>Layer</th>
<th>( K(10^8 \text{cm}^2 \text{s}^{-1}) )</th>
<th>( R(10^8 \text{cm}^2 \text{s}^{-1}) )</th>
<th>( \delta(°) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower stratosphere</td>
<td>-10</td>
<td>10</td>
<td>45</td>
</tr>
<tr>
<td>Upper troposphere</td>
<td>10</td>
<td>20</td>
<td>116</td>
</tr>
<tr>
<td>Lower troposphere</td>
<td>20</td>
<td>-10</td>
<td>206</td>
</tr>
</tbody>
</table>

Only part of the vertical variation in the lowest layers can be accounted for by surface frictional influences.

I. INTRODUCTION

The concept of a local eddy flux of heat, momentum, and moisture on a large scale and in a horizontal plane, was introduced by Priestley (1951) in an attempt to isolate and understand an aspect of the statistical dynamics of large-scale transient atmospheric systems. Since that time, although many observational studies of large-scale fluxes have been carried out, relatively little material has been published on the relation between these fluxes and the time-averaged state of the atmosphere. This is surprising because such a state must be defined in order to specify the fluxes. Further, it is generally accepted that parameterization using some such relation is necessary for future 3-dimensional climate models, and a zonal average relation, no matter how well based theoretically (Green 1970), cannot be applied with confidence at all geographic locations without some observational confirmation.

The neglect may be due in part to the lack of success of some earlier quantitative attempts to study the dynamics of time-averaged motion (e.g. Berson 1953) compared with the highly successful numerical simulation of ‘instantaneous’ synoptic scales manifested by the operational implementation of 24-hour numerical weather prediction. Much attention, however, has been focussed on zonally averaged eddy fluxes. These have been shown to be particularly useful as an indicator of energy transformations within the atmosphere (Lorenz 1955) and hence as a means of monitoring this aspect of the comparative behaviour between general circulation models and the real atmosphere.

One result of the zonally-averaged flux studies has been the synthesis of a generalized scheme (Newell 1964) in which the atmosphere from surface to about 80 km, is shown to comprise four tiers. The lowest tier (0 to 10–15 km), corresponding to the troposphere,
exhibits in the meridional direction a poleward, down-gradient heat flux: it achieves a net conversion from available potential energy to kinetic energy. The next tier (10-15 to 25 km), corresponding to the lower stratosphere, also exhibits a poleward heat flux, but this is counter-gradient; it achieves a conversion from kinetic energy to available potential energy. The third and fourth tiers (25 to 50, and 50 to 80 km) have some features in common with the first and second tiers, respectively, but the regimes are not confined to one hemisphere; they extend from winter to summer poles. Reed and German (1965) have deduced a mechanism which explains this zonally-averaged scheme, but the tiered structure has not been examined in local regions.

Clapp (1970) has made an observational study of the relation between local eddy heat flux and the horizontal temperature gradient, integrated from the surface to 25 mb. This analysis was carried out for 5-year average winter and summer seasons in the northern hemisphere using data for the period May 1958—April 1963 from the M.I.T. general circulation library. Fluxes parallel and perpendicular to the isotherms were expressed as \( (VT)_n = R k \times VT \) for the northern hemisphere, and \( (VT)_n = -KV \bar{T} \), where \( s \) and \( n \) represent components parallel to the isotherms (in the thermal wind direction) and perpendicular to the isotherms (along the temperature gradient toward lower mean temperatures). \( k \) and \( V \) represent a unit vector pointing upward and the horizontal gradient operator. \( V \) is the horizontal component of the wind vector, \( T \) is the temperature. \( R \) and \( K \) are exchange coefficients. An overbar represents a time average.

Clapp was seeking some form of systematic variation in \( R \) and \( K \) in order to parameterize the local horizontal eddy heat flux or, more particularly, the divergence of this quantity. He found that, while values of \( K \) were roughly proportional to temperature gradients (with some major exceptions, particularly over the Himalayan and Rocky Mountains), the field of \( R \) varied sharply along the isotherms. Clapp concluded that there is a need to seek new empirical parameters to relate statistically the observed fluxes and the mean fields.

One of the features of this analysis which may have obscured the sought relation is the vertical integration of data from the surface to 25 mb. An observational experiment in Australia has enabled a detailed examination to be made of the vertical variation of the local eddy heat flux and the mean temperature gradient, but only for one month. Using standard upper air soundings the study was subsequently extended to a selection of observing stations chosen to sample a wide range in latitude and so determine the wider relevance of an emerging relation between transient eddies and the local mean temperature gradient.

Figure 1. Vertical profiles of \( \vartheta \partial \vartheta \) and the meridional gradient of mean potential temperature, \( \partial \theta / \partial y \), at Laverton (38°S), from LSSE data.
Eddy Flux of Heat

2. Data

The vertical structure of the flux:gradient relation was examined using detailed and frequent (three-hourly) observations of wind and temperature at Laverton (38°S 145°E). Data covered 31 consecutive days during September/October 1966, and extended from the surface into the mid-stratosphere (about 30 km). Details are given in The Laverton serial sounding experiment (Bureau of Meteorology, Australia, 1968). Future reference will be made to these LSSE data.

For the extension to other stations, standard upper air data for the year 1970 were used:

<table>
<thead>
<tr>
<th>Location</th>
<th>Latitude (°S)</th>
<th>Longitude (°E)</th>
<th>Normal number of daily soundings</th>
<th>Temperature</th>
<th>Wind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Darwin</td>
<td>12</td>
<td>131</td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Townsville</td>
<td>19</td>
<td>147</td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Port Hedland</td>
<td>20</td>
<td>118</td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Alice Springs</td>
<td>24</td>
<td>134</td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Charleville</td>
<td>26</td>
<td>146</td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Eagle Farm (Brisbane)</td>
<td>27</td>
<td>153</td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Guildford (Perth)</td>
<td>32</td>
<td>116</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Williamtown (Sydney)</td>
<td>33</td>
<td>152</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Adelaide</td>
<td>35</td>
<td>139</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Laverton (Melbourne)</td>
<td>38</td>
<td>145</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Hobart</td>
<td>43</td>
<td>147</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Macquarie Island</td>
<td>55</td>
<td>159</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

(additional wind soundings were generally confined to the troposphere)

Covariances between winds and temperatures (calculated using simultaneous observations) were adjusted by a correction factor. This factor was the ratio between the standard deviation of the wind component soundings used in calculating the covariance, and the standard deviation using all wind soundings. In general this factor was between 0.9 and 1.1, necessitating only minor adjustments.

Horizontal temperature gradients were calculated from the observed winds using the thermal wind assumption. This procedure was adopted because of the widely spaced network of upper air observing stations: the assumption is consistent with the vertical filtering techniques used in processing the Laverton serial sounding data (Tucker 1973) and with the smoothing implicit in standard upper air soundings. Obviously, as very low latitudes are approached the temperature gradient determined in this way becomes increasingly inaccurate.

In the computations the potential temperature, \( \theta \), was used, rather than temperature.

3. Detailed Vertical Profiles

(a) Meridional and zonal components

The northward component of local eddy heat flux, as represented by the covariance \( \overline{v'\theta'} \), and the northward temperature gradient, \( \partial \theta / \partial y \), are given as functions of height in Fig. 1. The tiered structure of the atmosphere is evident, with counter-gradient flux between 13 and 24 km and down-gradient flux above and below. A pronounced minimum in \( \overline{v'\theta'} \) appears at 10.5 km, a little above the level at which the meridional temperature gradient is a maximum.
At Laverton during this period the vertical mean (virtual) temperature structure is marked by a major discontinuity at 11.1 km, the mean level of the lower tropopause, whereas the lowest (virtual) temperature occurs at 17.4 km and represents the southward extension of the tropical tropopause.

The eastward components of flux and temperature gradient, given in Fig. 2, show no such simple structure. Indeed the association appears artificial and unlikely to be useful. The large values of $\overline{\theta'}$ at the higher levels are a notable feature. They occur at the same levels as the high values of $v'w'$ previously reported (Tucker 1973), and suggest a layer of enhanced activity in the middle stratosphere.

**(b) Vector relations**

The vector relation between $\overline{\nabla \theta'}$ and $\nabla \theta$ is illustrated in Fig. 3, (inset), the angle, $\delta$, between the two being positive for the flux lying clockwise of the gradient. In Fig. 4 the flux is resolved along the temperature gradient in the direction of lower temperatures, and divided by the temperature gradient magnitude to determine $K = -|\overline{\nabla \theta'}| \cos \delta / |\nabla \theta|$; similarly, along the isotherms in the direction of the thermal wind, $R = |\overline{\nabla \theta'}| \sin \delta / |\nabla \theta|$.
Levels at which the probable error of the estimates are large (taken as \(|\nabla \theta| < 0.15 \times 10^{-7} \text{ K cm}^{-1}\)) are indicated on the diagram. These quantities are analogous to the horizontal exchange coefficients in Clapp’s vertically-integrated analysis.

In all three profiles the three-tiered structure of the atmosphere up to about 28 km emerges clearly.

Throughout most of the troposphere (2–9 km) 90° < \(\delta\) < 180°, indicating that the eddy heat flux has components toward lower temperatures and in the direction of the thermal wind. This is reflected in both \(K\) and \(R\) being positive. The method of data analysis, involving vertical filtering of winds and temperatures (Tucker 1973) precluded determinations below about 2 km.

In the lower stratosphere (10–20 km) 0° < \(\delta\) < 90°, indicating that, while the flux is now toward higher temperatures, there is still a component in the thermal wind direction. Thus although \(K\) has become negative, \(R\) remains positive. Singularities occur in the vicinity of 13 km where \(\nabla \theta \to 0\).

Above 25 km the regime has characteristics similar to those in the troposphere, with both \(K\) and \(R\) positive.

Thus it would appear that by integrating vertically from the surface to 25 mb (25 km) Clapp was combining two distinct layers which, in terms of the eddy heat flux resolved along the temperature gradient, have opposite characteristics; although, of course, because of the low densities involved at the upper levels, their contribution to the net energy flux is not large.

Typical values of \(K\), \(R\) and \(\delta\) for the three tiers are given below, although it should be noted that a fairly systematic change in \(\delta\) occurs throughout the height range studied (Fig. 3).

<table>
<thead>
<tr>
<th></th>
<th>Upper and mid-troposphere</th>
<th>Lower stratosphere</th>
<th>Mid-stratosphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K) (10^{10}\text{ cm}^2\text{ s}^{-1})</td>
<td>+5</td>
<td>-2</td>
<td>+10</td>
</tr>
<tr>
<td>(R) (10^{10}\text{ cm}^2\text{ s}^{-1})</td>
<td>+5</td>
<td>+2</td>
<td>+10</td>
</tr>
<tr>
<td>(\delta) (degrees)</td>
<td>135</td>
<td>45</td>
<td>135</td>
</tr>
</tbody>
</table>

No direct comparison with Clapp’s result is possible but the values of both \(K\) and \(R\) for the troposphere are somewhat larger from this single sample than might be expected from his northern hemisphere integrated values.
4. Variation with Latitude

Studies of the local eddy momentum fluxes by Brooke (1971) show that contributions to the flux from scales of between two and four cycles per day are not large. This suggests that the normal frequency of conventional upper air soundings effects a reasonable sampling of the macroscale eddy flux. Therefore, in order to explore the wider significance of the tiered structure revealed by the detailed LSSE soundings, analyses of conventional upper air data were undertaken at a number of Australian stations selected to cover a wide latitude range. The sampling problem is exacerbated at high levels, so all but the gross features of the vertical profiles are poorly represented in the stratosphere. Where sampling falls below about 50% of the 12-hourly releases, monthly flux computations become virtually meaningless. Even with the correction possible using the additional wind soundings, results from the once-daily soundings at northern Australian stations must be suspect at all levels.

<table>
<thead>
<tr>
<th></th>
<th>Guildford</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K</td>
<td>R</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan.</td>
<td>-16</td>
<td>-11</td>
<td>-2</td>
<td>-17</td>
<td>-9</td>
<td>-5</td>
<td>+26</td>
<td>+4</td>
</tr>
<tr>
<td></td>
<td>+41</td>
<td>-30</td>
<td>(+14)</td>
<td>(+5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1971</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feb.</td>
<td>+13</td>
<td>-11</td>
<td>+9</td>
<td>-5</td>
<td>+13</td>
<td>-11</td>
<td>+10</td>
<td>-20</td>
</tr>
<tr>
<td></td>
<td>-1</td>
<td>-16</td>
<td>-</td>
<td>-</td>
<td>+2</td>
<td>+2</td>
<td>+23</td>
<td>-33</td>
</tr>
<tr>
<td></td>
<td>-27</td>
<td>-9</td>
<td>-16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mar.</td>
<td>-4</td>
<td>+1</td>
<td>+3</td>
<td>-32</td>
<td>-8</td>
<td>-10</td>
<td>-15</td>
<td>-49</td>
</tr>
<tr>
<td></td>
<td>+2</td>
<td>+23</td>
<td>-33</td>
<td>-27</td>
<td>-9</td>
<td>-16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apr.</td>
<td>+8</td>
<td>+2</td>
<td>+10</td>
<td>+2</td>
<td>+12</td>
<td>+8</td>
<td>+12</td>
<td>-17</td>
</tr>
<tr>
<td></td>
<td>+26</td>
<td>+5</td>
<td>+15</td>
<td>-5</td>
<td>+14</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>May</td>
<td>-14</td>
<td>-23</td>
<td>+5</td>
<td>-11</td>
<td>-20</td>
<td>-25</td>
<td>-5</td>
<td>-29</td>
</tr>
<tr>
<td></td>
<td>+8</td>
<td>-11</td>
<td>+31</td>
<td>+17</td>
<td>+1</td>
<td>-14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>June</td>
<td>+17</td>
<td>-1</td>
<td>+9</td>
<td>+20</td>
<td>-6</td>
<td>+7</td>
<td>+6</td>
<td>+24</td>
</tr>
<tr>
<td></td>
<td>+17</td>
<td>+18</td>
<td>+33</td>
<td>+7</td>
<td>+3</td>
<td>+7</td>
<td>+11</td>
<td>+18</td>
</tr>
<tr>
<td>July</td>
<td>+18</td>
<td>+14</td>
<td>+35</td>
<td>+29</td>
<td>+8</td>
<td>-2</td>
<td>+35</td>
<td>+21</td>
</tr>
<tr>
<td></td>
<td>+33</td>
<td>+35</td>
<td>+26</td>
<td>+1</td>
<td>+26</td>
<td>+1</td>
<td>+26</td>
<td>+16</td>
</tr>
<tr>
<td>Aug.</td>
<td>+19</td>
<td>+38</td>
<td>+16</td>
<td>+13</td>
<td>-2</td>
<td>+17</td>
<td>+22</td>
<td>+28</td>
</tr>
<tr>
<td></td>
<td>+32</td>
<td>+4</td>
<td>+2</td>
<td>-7</td>
<td>+15</td>
<td>+15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sept.</td>
<td>-5</td>
<td>+7</td>
<td>0</td>
<td>+5</td>
<td>+19</td>
<td>+20</td>
<td>+11</td>
<td>+2</td>
</tr>
<tr>
<td></td>
<td>+12</td>
<td>-6</td>
<td>-8</td>
<td>+4</td>
<td>+5</td>
<td>+5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oct.</td>
<td>-1</td>
<td>-10</td>
<td>+16</td>
<td>+5</td>
<td>+15</td>
<td>-9</td>
<td>+33</td>
<td>+19</td>
</tr>
<tr>
<td></td>
<td>+41</td>
<td>+21</td>
<td>+51</td>
<td>0</td>
<td>+26</td>
<td>+4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nov.</td>
<td>-5</td>
<td>-13</td>
<td>+21</td>
<td>+9</td>
<td>+9</td>
<td>+5</td>
<td>+7</td>
<td>+5</td>
</tr>
<tr>
<td></td>
<td>+32</td>
<td>+20</td>
<td>+27</td>
<td>+13</td>
<td>+15</td>
<td>+7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dec.</td>
<td>+4</td>
<td>+1</td>
<td>+3</td>
<td>-10</td>
<td>-10</td>
<td>-6</td>
<td>-4</td>
<td>+5</td>
</tr>
<tr>
<td></td>
<td>+4</td>
<td>+8</td>
<td>+12</td>
<td>+56</td>
<td>+6</td>
<td>+7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+1971</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The main feature of monthly calculations was a great scatter in derived values (cf. Table 1). However, when the entire year's data were grouped to obtain annual values of the fluxes and the temperature gradients, and hence of $K$ and $R$, a more coherent pattern emerged.

Annual profiles of $K$ and $R$ are shown in Figs. 5(a) and (b). Values are not plotted at the maximum wind level (about 12.5 km) because the horizontal temperature gradient generally attains very small values there, giving spurious results for $K$ and $R$.

A notable consistency is evident among stations south of Alice Springs (24°S), with signs of a distinct latitude variation. Notwithstanding the sampling deficiency, the tiered structure of the atmosphere emerges; values in the upper levels of the $K$ profile for Macquarie Island are consistent with the third, mid-stratosphere, tier having been sampled. Also, all stations between 30 and 40°S show a minimum value for $K$ in the mid-troposphere; this is also a feature of the detailed results in Fig. 5. However, there are two marked discrepancies between these mean annual results for 1970 and those using the more detailed LSSE soundings. First, the annual values of $K$ and $R$ in mid-latitudes are generally three or four times smaller; second, the $R$ profiles in the lowest 5 km of the middle and high latitude stations show consistent negative values, i.e. a flux component in the reverse direction to the thermal
Figure 5. (a) Vertical profiles of $K$ for the year 1970 at Australian upper air stations selected to sample a wide latitude range, from standard upper air soundings. Profiles are discontinuous at levels where very small values of $\nabla \theta$ lead to unrealistically large values of $K$. (b) Vertical profiles of $R$ for the year 1970, at Australian upper air stations selected to sample a wide latitude range from standard upper air soundings. Profiles are discontinuous at levels where very small values of $\nabla \theta$ lead to unrealistically large values of $R$. 
wind (this is discussed below). In addition, the low-latitude stations exhibit no consistent pattern except perhaps in the 15–20 km region for which the sampling is worst.

For the six stations south of 30°S the average value of $K$ at 10 km is about $15.10^9 \text{ cm}^2 \text{s}^{-1}$, and $R$ is about $20.10^9 \text{ cm}^2 \text{s}^{-1}$. Thus $|\nabla \theta|/|\nabla \theta| \approx 25.10^9 \text{ cm}^2 \text{s}^{-1}$, which is in reasonable conformity with an estimate by Morel and Larcheveque (1974) of $16.10^9 \text{ cm}^2 \text{s}^{-1}$ for the large-scale exchange coefficient, based on the dispersion of numbers of EOGE balloons at about the 200 mb level in the 30–50°S latitude zone.

$\delta$ as a function of latitude and height is given in Fig. 6; it shows quite a smooth transition across the tropopause, at least for latitudes south of 30°S.

![Figure 6](image)

The variation of $\delta$ with latitude and height for the year 1970 at Australian upper air stations selected to sample a wide latitude range, from standard upper air soundings.

The persistence of $225^\circ > \delta > 180^\circ$ in the lowest few kilometres is a notable feature of the results. It also occurs in the very lowest layers of the LSSE data. Mechanistically this is consistent with the observed synoptic structure. Transient surface pressure waves in a zonally-aligned temperature field have a zero or only a very slight SE–NW tilt (in the southern hemisphere), reflected by the streamlines at the 'geostrophic level' of about 2 km; but the surface friction effect gives the streamlines at the surface an asymmetric shape in which southerlies have a stronger westerly component than northerlies, and consequently both $\nabla \theta$ and $\nabla \theta$ are negative (i.e. $270^\circ < \delta < 180^\circ$). At somewhat higher levels, during the stage of active baroclinic development, the tilt becomes strongly SE–NW, hence $\overrightarrow{\nabla \theta}$ is negative and $\overrightarrow{\nabla \theta}$ positive ($180^\circ < \delta < 90^\circ$). This effect is also consistent with $\overrightarrow{\nabla u}$, which is generally negative in the mid-troposphere of southern middle latitudes, becoming positive in the lowest layers. Thus surface friction influences induce a tilt in the opposite sense to that in the upper troposphere where most of the poleward eddy flux of momentum occurs. (These arguments are elaborated in the following section.)

The lower stratosphere feature of $90^\circ > \delta > 0^\circ$ appears to be the result of transient systems maintaining the same dynamical shape as in the upper troposphere but the meridional temperature gradient being reversed. A reversal does not generally occur in the zonal temperature gradient.

Much less coherence exists between low-latitude stations where local eddy fluxes are generally much smaller but so are the temperature gradients, resulting in variable and often spuriously high computed values for $K$ and $R$. 
5. Surface frictional influences

Observational and theoretical studies show that the transient eddy flux of momentum occurs at high levels in the troposphere and that at lower levels the horizontal tilt of wave structures in the flow patterns, which primarily accomplishes this flux, is less marked. Indeed at the surface in middle latitudes both low pressure and high pressure systems show little asymmetry about a N–S central axis.

| TABLE 2. Average yearly values of the covariance $\overline{w' u'}$ (m²s⁻²) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | Whenupai        | Lord Howe       | William-        | Laverton        | Adelaide        | Forest          |
|                 | (Auckland)      | Island          | town (Sydney)   | (Melbourne)     |                 |                 |
| Latitude (°S)   | 37              | 32              | 33              | 38              | 35              | 31              |
| Longitude (°E)  | 174             | 159             | 152             | 145             | 139             | 131             |
| Level (mb)      |                 |                 |                 |                 |                 |                 |
| 200             | -35.7           | -64.0           | -27.5           | -42.1           | -65.8           | -58.5           |
| 310             | -25.2           | -41.1           | -33.7           | -36.4           | -48.1           | -64.9           |
| 410             | -17.3           |                 |                 |                 |                 |                 |
| 510             | -9.0            | -15.1           | -13.5           | -19.6           | -17.2           | -30.6           |
| 600             |                 | -4.1            |                 |                 |                 |                 |
| 700             | +1.3            | -9.1            | -7.9            | -14.7           | -13.1           | -11.9           |
| 800             | +6.7            |                 |                 |                 |                 |                 |
| 900             | +14.0           | +0.5            | -2.6            | -5.0            | -2.7            | +3.4            |
| Surface         | +4.5            | +2.1            | +0.6            | +2.3            | +4.1            | +1.5            |
| Level (ft x 10⁻²)|                 |                 |                 |                 |                 |                 |
| 200             | 40              |                 |                 |                 |                 |                 |
| 310             | 30              |                 |                 |                 |                 |                 |
| 410             |                 |                 |                 |                 |                 |                 |
| 510             |                 | 20              |                 |                 |                 |                 |
| 600             |                 |                 |                 |                 |                 |                 |
| 700             |                 |                 |                 |                 |                 |                 |
| 800             |                 |                 |                 |                 |                 |                 |
| 900             |                 |                 |                 |                 |                 |                 |
| Surface         |                 |                 |                 |                 |                 |                 |

Source: De Lisle (1969)
June 1957–May 1961
Source: Maher and McRae (1964)

A notable feature, however, of the $\overline{w' u'}$ covariance along the latitude of maximum poleward momentum flux (about 35°) is that in the very lowest layers the sign of the covariance reverses. This is shown by annual values for stations in the vicinity of 35°S in the Australia–New Zealand region, given in Table 2, derived from published upper wind statistics (Maher and McRae 1964; De Lisle 1969). Similar sign reversals in the lowest levels appear in seasonal and in most mean monthly values.

If we assume that at the top of the friction layer the wave structures have no tilt, then the effect of surface friction in the lower layers can be assessed in the following way.

Assuming the simplest structure of surface geostrophic wind components given by $u_g(x) = 0$ and $v_g(x) = V_g \cos(2\pi x/D)$, where $x$, $y$ are the W–E and S–N axes, $X$ is the eastward distance from the $v$(max) longitude, $D$ is the wavelength in the $x$ direction, and $V_g = v_g$ (max); whence

$$\overline{w' u'} = \frac{1}{D} \int_0^D u_g v_g \, dx = 0$$

At some lower level, $l$, frictional effects can be represented by an angular deviation of the wind towards lower pressure, $\alpha_l$, and a wind-speed reduction factor $d_l$. So that (for the southern hemisphere)

$$u_l(x) = d_l V_g \cos(2\pi x/D) \sin \alpha_l,$$

$$v_l(x) = d_l V_g \cos(2\pi x/D) \cos \alpha_l.$$
Thus
\[\int_0^d u_i v_i \, dx = \int_0^d d_i^2 V_i^2 \cos^2(2\pi x/D) \sin \alpha_i \cos \alpha_i \, dx\]
and, assuming \( d_i \) and \( \alpha_i \) are independent of \( x \),
\[\overline{v_i u_i} = \frac{1}{2} d_i^2 V_i^2 \sin \alpha_i \cos \alpha_i\]
Near the surface, reasonable values for \( d \) and \( \alpha \) (Pasquill 1971; Clarke and Hess 1974) are: over the sea, 0.6 and 16°; over bushy vegetation, 0.5 and 24°.
Therefore, assuming a typical value of \( V_i \) to be 10 m s\(^{-1}\), in the southern hemisphere
\[\overline{v' u'} \text{ (over the sea surface)} \approx +4 \text{ m}^2 \text{s}^{-2}\]
\[\overline{v' u'} \text{ (over bushy vegetation)} \approx +2.3 \text{ m}^2 \text{s}^{-2}\]
It appears therefore that part of the sign reversal of covariance \( v' u' \) in the lower levels, shown in Table 2, might be due to the influence of surface friction on surface pressure patterns. However, the covariance values in Table 2 change sign at too high a level at some stations for this to be a complete explanation.
This analysis can be extended to consider the effect on the local eddy flux of sensible heat near the surface. Let us assume that the temperature associated with the wind pattern is represented by \( T_i = t_c + t_i(x) \), where \( t_c = \text{constant} \) and \( t_i(x) = -k v_i \), and \( k \) is also constant. Then
\[\int_0^d u_i T_i \, dx = -k \int_0^d u_i v_i \, dx\]
and
\[\frac{u_i T_i}{v_i} = -\frac{1}{2} k d_i^2 V_i^2 \sin \alpha_i \cos \alpha_i = -\frac{1}{2} k u_i v_i\]
Similarly
\[\int_0^d v_i T_i \, dx = -k \int_0^d v_i^2 \, dx\]
and
\[\frac{v_i T_i}{v_i} = -\frac{1}{2} k d_i^2 V_i^2 \cos \alpha_i = -k v_i v_i \cot \alpha_i\]
Under these circumstances the mean temperature gradient is in the northward direction and the vector eddy heat flux is veered on this by the angle
\[\delta = \arctan\left(\frac{u_i T_i}{v_i T_i}\right)\]
\[= \begin{cases} 196^\circ, & \text{over sea surface} \\ 204^\circ, & \text{over bushy vegetation} \end{cases}\]
These figures are broadly consistent with observations of both \( \delta \) and \( v' u' \) in the lowest levels of middle-latitude Australian stations, given in Fig. 6 and in Table 2.

6. CONCLUSION

It is apparent from the detailed LSSE results, and confirmed by upper air observations at standard levels, that the atmosphere exhibits a distinct vertical pattern in its local eddy-flux:gradient relation, together with a latitude coherence – at least in middle latitudes. There is, however, a considerable variation from month to month in the relation derived from monthly upper air observations, as shown in Table 1 for \( K \) and \( R \) at about 5 1/4 km. The reason why the device of taking the average of 12 monthly values should give a more mutually consistent pattern of variation in the latitude-height plane, and why this should be very similar to the detailed LSSE results for one month at one latitude, is not at all obvious.
On a month-to-month time-scale the derived \( K \) and \( R \) values suggest that atmospheric processes do not exhibit a consistent flux:gradient relation, whereas when averaged over a year more order is revealed. In this case the general agreement of annual values and the LSSE results must be seen as probably largely fortuitous. It is notable that there is a tendency
for large positive values of $R$ and, to a lesser extent, $K$, to occur in winter, the most turbid
time of year, and for negative values of $R$ to occur in summer (Table 1). If this is true, it does
not augur well for the use of statistical dynamical models (using this form of flux parameter-
ization) to describe atmospheric changes on monthly or seasonal time scales, until a
reliable seasonal variation can be established or a completely non-geographic parameteriza-
tion devised. It does imply, however, some sort of serial monthly or seasonal compensation
in order that the annual pattern should show such a consistent flux: gradient relation.

An important feature of the observational study discussed above is the vertical variation
which appears to occur in a systematic way from the surface through the troposphere into
the mid-stratosphere. Analytical baroclinic theory is not so far sufficiently advanced to give
any information on the detailed vertical structure of developing waves in the real atmosphere
because of the mathematical complexity of the problem. Current investigations using two-
level models (e.g. Hollingsworth 1975) do not suggest any notable change in trough-
longitude tilt between wave development in the two levels.

An obvious next step in the study of local eddy-flux:gradient relations on this scale
is an examination of the association that emerges from explicit general circulation models.
Data exist both for case studies of individual wave development at numerous levels and also
for a determination of the vertical and geographical distribution of time-averaged
statistical relations. The pattern of divergence of horizontal eddy flux can also be computed,
and this is perhaps a more directly relevant quantity to be represented in statistical–dynamical
models of climate.

Of course, the extent to which standard upper-air soundings can adequately sample the
total eddy flux has still not been firmly established. Further detailed observational exper-
iments in different geographic locations and at different times of year are required to deter-
mine the most appropriate parameterization for use in climate modelling.

REFERENCES

Berson, F. A. 1953 A quantitative analysis of large-scale patterns with special

Brooke, R. A. 1971 Fluxes of momentum and mass from 2 km to 28 km at
Laverton, Victoria, during the serial sounding experi-
ment, Spring 1966, Ibid., 97, 110–117.

Bureau of Meteorology 1968 The Laverton serial sounding experiment, Met. Summary,
Commonwealth of Australia, Melbourne.

Clapp, P. F. 1970 Parameterization of macroscale transient heat transport for
use in a mean-motion model of the general circulation,

Clarke, R. H. and Hess D. 1974 Geostrophic departure and the functions A and B of
Rossby-Number similarity theory, Boundary Layer Met.,
7, 267–287.


Green, J. S. A. 1970 Transfer properties of the large-scale eddies and the general
circulation of the atmosphere, Quart. J. R. Met. Soc.,
96, 157–185.

Hollingsworth, A. 1975 Baroclinic instability of a simple flow on the sphere, Ibid.,
101, 495–528.

Lorenz, E. W. 1955 Available potential energy and the maintenance of the

Maher, J. V. and McRae, J. N. 1964 Upper wind statistics, Australia, Met. Summary, Common-
wealth of Australia, Melbourne.

Morel, P. and Larcheveque, M. 1974 Relative dispersion of constant-level balloons in the 200 mb
general circulation, J. Atmos. Sci., 31, 2189–2196

Newell, R. E. 1964 Circulation of the upper atmosphere, Scientific American,
210, 62–74.
Pasquill, F.


Priestley, C. H. B.


Reed, R. J. and German, K. E.


Tucker, G. B.