The application of complex demodulation to meteorological satellite data

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SUMMARY

A method of performing zonal harmonic analyses on near-polar orbiting satellite data is presented. This method, known as complex demodulation, yields the time variation of the spectral components of global scale meteorological parameters. Experimental data from the selective chopper radiometers on board the Nimbus 4 and 5 weather satellites are examined in order to gain an insight into the temporal behaviour of the stationary and travelling components of stratospheric zonal temperature fields.

1. INTRODUCTION

Global scale atmospheric phenomena are important in the redistribution of energy throughout the atmosphere and planetary waves in both temperature and pressure fields are a dominant feature of meteorological charts. These waves can be conveniently examined using zonal harmonic analysis, although the exact method used will depend on the data source and the particular phenomena being investigated.

Data collected synoptically from conventional weather stations and ships can be translated onto a regular latitude/longitude grid and Fourier analysis of the grid point data round a circle of latitude will yield amplitude and phase information on the spectral components for the latitude at that particular time. In this case, data from the purely spatial domain are transformed into a pure frequency domain. Applying this analysis to successive days gives the day-to-day variation of the spectral components. These conventional data have provided useful information for many years, particularly over the northern hemisphere (e.g. Hare 1965; Muench 1965; Benwell 1966; Kao 1970). However, over the vast oceanic regions of the southern hemisphere fewer meteorological data are available, but with the advent of meteorological satellites in the early 1960s it became possible for the first time to obtain a more uniform global coverage.

In general, meteorological satellites are launched in near-polar orbits and the fixed orbital period results in a particular latitude being traversed at regular intervals in space and time. Consequently there is an important difference between conventionally obtained and satellite-derived data. Any harmonic analysis of such satellite data should therefore take account of the spatial and temporal nature of the observations. Fourier transform techniques can be used to carry out the transformation from a mixed temporal and spatial domain into a frequency domain. These methods have been used to obtain both short and long period spectral estimates of the derived temperature field (see Chapman et al. 1974 and Chapman 1974) and yield information on the mean amplitude and phase of the spectral components during the period of analysis.

In addition to examining the wave structure it is also important to determine the time variations of the individual components which constitute the power spectrum. We have obtained 3–4-day running means of these, using complex demodulation. The purpose of this paper is to introduce the theory of this technique and to indicate its relevance to planetary scale wave analysis in satellite meteorology. Section 2 provides the necessary

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mathematical background and in section 3 we discuss specific examples of the application of this technique to radiance data obtained from the selective chopper radiometers (SCR) flown on the Nimbus 4 and 5 satellites.

2. MATHEMATICAL BACKGROUND

The temperature \( T \) at any point in the atmosphere is a function of latitude \( \theta \), longitude \( \lambda \), height \( z \) and time \( t \), and may be expanded in a Fourier integral:

\[
T(\theta, z, \lambda, t) = \int_{-\infty}^{\infty} \sum_{n=0}^{\infty} A_{on}(\theta, z) \exp\{i(\omega t - n\lambda)\} d\omega
\]

where \( A_{on}(\theta, z) \) is a complex amplitude, \( \omega \) is the frequency and \( n \) the zonal wavenumber of the planetary waves. \( (\omega < 0, \omega > 0 \) and \( \omega = 0 \) correspond to westward travelling, eastward travelling and static wave components respectively.\)

In practice, measurements of \( T \) are available only over a finite time interval, \( m\tau \), where \( \tau \) is the orbital period of the satellite and \( m \) a number of orbits. In this case the Fourier integral may be replaced by a summation

\[
T(\theta, z, \lambda, t) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{\infty} A_{ln}(\theta, z) \exp\{i(2\pi l/m\tau - n\lambda)\}
\]

\( T \) is not known as a continuous function of \( \lambda \) and \( t \), but only as a series of values corresponding to the sub-satellite points for a particular latitude. Successive measurements of \( T \) occur at time \( t = j\tau \) and longitude \( \lambda = -j\Omega \), \( j = 0, 1, 2 \ldots \) \((m - 1)\), where \( \Omega \) is the angular velocity of the earth. From now on we drop the explicit reference to \( \theta \) and \( z \) and concentrate on a fixed latitude and height. Eq. (2) now becomes

\[
T_j = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{\infty} A_{ln} \exp\{ij(2\pi l/m + n\Omega\tau)\}
\]

A discrete Fourier transform of the series of measurements \( T_j \) would give amplitudes \( A_k \), \( k = 0, 1 \ldots m/2 \) (we assume \( m \) is divisible by two), where

\[
T_j = \sum_{k=0}^{m/2} A_k \exp(i2\pi jk/m)
\]

The coefficients \( A_k \) are obtained from the inverse of Eq. (4), i.e.

\[
A_k = \sum_{j=0}^{m-1} T_j \exp(-i2\pi jk/m)
\]

Comparing Eqs. (3) and (4) we note that the coefficients \( A_{ln} \) cannot be deduced unambiguously from the satellite measurements, but only the coefficients \( A_k \), where (since \( k \) is integral) \( k \propto l + nm\Omega\tau/2\tau \), so that a value for \( n \) must be assumed to determine \( l \). An examination of a typical spectrum obtained from SCR observations (Fig. 1) will help to clarify the situation. For example a component at 1.7 c. day\(^{-1} \) could be produced by a wavenumber-1 eastward travelling wave of frequency 0.7 c. day\(^{-1} \) or a wavenumber-2 westward travelling wave of frequency 0.3 c. day\(^{-1} \). In practice this ambiguity is not a serious problem since the dominating planetary scale waves generally have 'periods'\(^* \) of several days and hence frequencies less than 0.5 c. day\(^{-1} \).

This method yields reliable estimates of the spectral components of the temperature field provided the statistical properties of the generating processes remain constant over the interval \((m\tau)\) of analysis (i.e. stationary time series). However, difficulties arise when

\* Here 'period' refers to the time taken for a travelling planetary wave to complete one circuit of the earth, as distinct from the lifetime of a wave component.
these properties change with time, since then the resulting spectra cannot be easily related to the generating processes. One solution is to divide the series of observations into a number of shorter sections and then transform each section as described above. Although better estimates of the amplitudes of the spectral components are obtained the frequency resolution is degraded.

An alternative method is to transform the time series into a mixed frequency and time domain instead of into a pure frequency domain. The resulting complex amplitudes, which vary with time, will reflect changes in the statistical properties of the series, and a close study of these variations can provide information on the basic generating processes.

Complex demodulation, which is described below, is one of several techniques which may be used to perform such transformations. A more detailed description is given in Bingham et al. (1967) and an example of its use in the analysis of economic time series can be found in Godfrey (1965).

Consider a series of measurements of a parameter $x(t)$, over a time interval $T$, where $t$ is time, $t = 0, 1 \ldots (T-1)$. The series is at first centred about a chosen frequency $\omega_0$ by multiplying $x(t)$ by $\exp(-i\omega_0 t)$. The two resulting series (real and imaginary) are separately smoothed using a filter of bandwidth $\Delta \omega$, say. These series provide an estimate of the time variation of the complex coefficients in the frequency band $\omega_0 \pm \Delta \omega/2$ and can be examined either in their complex form or more usefully in terms of their amplitude and phase.

Turning now to the specific application of complex demodulation to satellite data we consider the real part of Eq. (3), i.e.

$$T_j = \sum_t \sum_n a_{in} \exp[i(j(2\pi t/m + n\Omega t) + \Phi_{in} - n\Lambda_{in})]$$

where $a_{in}$ are real amplitudes, $\phi_{in}$ are the phases, $\Lambda_{in}$ are the longitudes of the maxima at $t = 0$, and these variables are related to the complex amplitudes $A_{in}$ in Eq. (3) by $A_{in} = a_{in} \exp[i(\phi_{in} - n\Lambda_{in})]$. The complex demodulates at zonal wavenumber $p$ are given by

$$T_{pj} = \sum_{t=\pm \frac{1}{2} \Delta t} \frac{1}{2} a_{tp} B_{tp} \exp[i(j2\pi t/m + \phi_{tp} - p\Lambda_{tp})]$$

$$= C_{pj} \exp(i\psi_{pj})$$

(7)
where $B_t$ is the transfer function of the low pass filter at the frequency $2\pi f / m \tau$, $\Delta l$ is the bandwidth of the filter such that $2\pi \Delta l / m \tau = 0.5 \text{c.day}^{-1}$, and $C_{pj}$ and $\psi_{pj}$ are the amplitude and phase respectively of the $p$th wavenumber. (Details of suitable filters are given in the appendix.)

If only a single component with zonal wavenumber $p$ is present in the time series then the behaviour of both $C_p$ and $\psi_p$ with time is simple. However, in general more than one component will be present and therefore the time variation of the amplitude and phase will be more complicated. In such cases interpretation of these variations can be aided by constructing models containing different numbers of components with various frequencies, amplitudes and phases. Models containing two and three components have been constructed by the authors and the results utilized in the following section, although details are not presented here.

3. **Analysis of satellite data**

(a) _Source and reduction of data_

In this section we describe the specific application of complex demodulation to the analysis of stratospheric temperature data derived from both Nimbus 4 and 5 SCR soundings. (Details of the instruments can be found in Abel et al. (1970) and Ellis et al. (1973).)

The data used are obtained from the day sections of the satellites’ orbits (ascending mode) at each orbital crossing of the selected latitude (in this analysis 60°S). Orbital smoothing is carried out by fitting a quadratic expression to the data values; the quadratic extends to 6° of latitude either side of the latitude of interest. Due to various operational

![Amplitude and Phase Variation](image)

**Figure 2.** *Zonal wavenumber 1: (a) amplitude and (b) phase variation with time for channel 1 (~45 km), 5 September–14 November 1970, latitude 60°S (Nimbus 4 SCR data).*
considerations certain orbits of data are lost and therefore some of the points in the resulting time series are missing. In the data analysed, at most two or three consecutive orbits are missing and where these occur linear interpolation along the latitude circle is used. Blackbody equivalent temperatures are then calculated from the radiances. These temperatures represent a weighted mean of atmospheric temperature over a particular slab of atmosphere. (Further information on the height resolution (given by the weighting function) of the SCR channels can be found in Abel et al. (1970) and Ellis et al. (1973).)

The first data set used, discussed in (b) below, is taken from channel 1 of the SCR on Nimbus 4. This channel is designed such that the maximum sensitivity to infrared radiation (weighting function peak) occurs at \( \sim 45 \) km. The Nimbus 5 data, discussed in (c) below, are taken from the SCR channel B23, which has a weighting function peaking at around 40 km.

(b) 5 September–14 November 1970

The variations of amplitude and phase with time for zonal wavenumber 1 at 60°S are shown in Fig. 2. The marked amplitude oscillations which are a noticeable feature of these diagrams are accompanied, for the most part, by only small phase variations. The nearly constant phase is a characteristic of both static and standing waves but an examination of the amplitude allows the two possibilities to be distinguished since the amplitude of a static wave remains constant with time. The observed variations can be explained by a simple three-component model consisting of one static wave and two travelling waves with frequencies of \( \pm 0.08 \) c.day\(^{-1}\). These conclusions are confirmed by an examination of the Fourier spectrum of the satellite data for this period (Fig. 1). After 6 November the phase changes linearly with time whilst the amplitude at first falls and then rises slightly.

![Figure 3. As for Figure 2 except for zonal wavenumber 2.](image-url)
These variations suggest that only two wave components are now present: a static wave and an eastward travelling wave of frequency $\sim 0.1 \text{c.day}^{-1}$. Although a simple linear superposition of a wave component can explain the observed variations for the majority of the period, there are instances when such a simple-minded approach is not valid; for example the amplitude and phase changes from 25 October to 6 November. In such instances we conclude that the driving forces are undergoing some change and are therefore introducing nonlinear terms which are not considered in the simple models.

The variations of amplitude and phase with time for zonal wavenumber 2 are presented in Fig. 3. We will concentrate our discussion on the first part of this period, to 15 October, as it is in this section that the amplitudes are greatest and the waves therefore most important. During this period significant changes in amplitude occur, accompanied by a continuous almost linear change of phase with time. This type of amplitude and phase variation can be produced by a simple two-wave model consisting of a static wave and an eastward travelling wave (frequency $\sim 0.08 \text{c.day}^{-1}$). Once again these conclusions are confirmed by reference to Fig. 1. However, this simple model does not completely explain the observed variation unless the amplitudes of the constituent components vary with time – indicating changes in the driving forces. The amplitude and phase variations indicate that the travelling wave is dominant for most of the time except for approximately 10 days, from 15 to 25 September, when the static wave dominates.

(c) 5 July–4 August 1974

This period is particularly interesting as it includes the important southern hemisphere sudden stratospheric warming first observed by Barnett (1975). In Fig. 4 we present plots of the amplitude and phase variations of zonal wavenumber 1 at 60$^\circ$S. The most noticeable

![Figure 4](image-url)  

Figure 4. Zonal wavenumber 1: (a) amplitude and (b) phase variation with time for channel B23 ($\sim 40$km), 5 July–4 August 1974, latitude 60$^\circ$S (Nimbus 5 SCR data).
features of these plots are the increase in wave activity, resulting in amplitudes in excess of 19 K around 26 July, and the almost constant phase over the whole period. Referring to the Fourier spectrum of the satellite data given in Fig. 5 we find that a broad peak is centred on zonal wavenumber one which is significantly different from that shown in Fig. 1 (where three well defined components can be seen). In contrast, therefore, to the previous study, (b), no simple model can be used to explain the observed variations in amplitude and phase. The most likely explanation for the large amplitude fluctuation from 22 to 31 July is that changes occur in the external driving forces. Evidence for this is
given by Barnett (1975); associated with the sudden warming are significant coolings in the tropics which suggest a meridional redistribution of heat.

The variations of amplitude and phase with time for zonal wavenumber 2 at 60°S are presented in Fig. 6. Large amplitudes are in general associated with steady eastward phase changes whilst minima in amplitude are associated with oscillations of the phase. The superposition of a static component and a travelling component (frequency ~0.1 c. day⁻¹) broadly explains the observed amplitude and phase variations and agrees with the spectrum shown in Fig. 5. Although averaged over the whole period, the amplitude of the travelling component is greater than that of the static component (see Fig. 5), the oscillation in phase suggests that during 5–6 days around 20 July, the static wave dominates. Consequently, although the main features of the observed variation can be explained by a simple linear model, a complete description requires that the amplitudes of these two waves vary with time.

(d) Cine film

Finally, we briefly discuss a further application of this technique. It is often useful in meteorology to observe as directly as possible the time variation of meteorological parameters on a global scale. One method of recording these variations is by means of a cine film and since the Nimbus SCR data give almost global coverage of the stratospheric temperature fields over extended periods these are an obvious choice as the basis for such a film. Complex demodulation provides the zonal harmonic components from which hemispheric temperature fields at a particular time can be reconstituted. The nature of this method results in smoothing-out of the high-frequency components and by choosing a suitable time interval, the field at successive intervals of time can be recorded as successive frames of the cine film. Essentially this is an interpolation technique based on the physically reasonable assumption that the maximum frequency of the travelling waves is <0.5 c. day⁻¹.

An outline of the computational steps involved in this process is given below.

1. The complex demodulates for a particular channel are computed for each latitude from 80°N to 80°S in 4° latitude steps.
2. Every eighth data value, for each wavenumber and latitude, is selected. This introduces negligible error because the filter function has the effect of producing 3–4-day running means of the zonal components.
3. The data are reconstituted to form temperature fields for successive values of time (~14½ hours). These fields are interpolated onto a latitude–longitude grid (41 × 37; 4° latitude and 10° longitude intervals) and further time interpolation is carried out with an interval ~30 min.
4. The required hemisphere (21 latitudes) is selected and the stereographic contour maps are prepared.
5. The contour maps are recorded on magnetic tape for later processing by the SC4020 computer at the Atlas Computer Laboratory.

The above process condenses data from a period of some 76 days onto a film lasting 5 or 6 minutes. At present one colour film of Nimbus 4 data has been generated and a number of films of Nimbus 5 data are in preparation.

4. Conclusions

A technique has been presented which enables the time variation of zonal harmonic components of satellite-derived data to be determined. Although complex demodulation can stand on its own as an analytical tool, maximum use of the method is gained when it
is used in conjunction with Fourier transform techniques. These techniques can be used to establish which spectral components are present in a set of data, but they do not yield information on the time variation of these components. Complex demodulation enables these variations to be monitored, providing useful information on the development of planetary scale waves. This is of particular importance during periods of sudden stratospheric warmings when large amplitude changes can occur over a period of a few days.

On many occasions the time development of a particular wavenumber can be adequately described by the linear addition of two or three spectral components; however, instances do occur when such simple models are not adequate. Complex demodulation provides information which can help in identifying when nonlinear effects are important.

Complex demodulation has also been shown to have an important application in the smoothing of data in order to present the longer term changes in the hemispheric temperature fields on a cine film.

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Some suitable spectral windows used in complex demodulation are defined by

\[ B_l = \left[ \sin \left( \frac{(S+1)\pi l}{m} \right) \right] \left[ \sin \left( \frac{\pi l}{m} \right) \right]^K \]

where \( l \) is the number of cycles in a period \( T = m\tau \), \( m \) is the total number of orbits in the period of analysis, \( \tau \) is the orbital period of the satellite, \( S \) and \( K \) are integers which define the bandwidth of the filter.

In the time domain the windows defined above can be obtained by taking \( K \) moving averages (of length \( S+1 \)) of the demodulated data; i.e. each pass is represented by

\[ y(t) = \frac{1}{S+1} \sum_{i=-S}^{S} x(t+i) \]

where \( x(t+i) \) is the original complex series defined for \( 0 < t+i < T-1 \) and \( y(t) \) is the new smoothed series defined for \( \frac{1}{2}S < t < T - \frac{1}{2}S - 1 \). After four passes of this function (producing the Parzen window) a total of \( 4S \) datum values will be lost, so that for the Parzen window the final series will only be defined for \( t = 2m \ldots T-2m-1 \).

Figure 7. Comparison of normalized spectral windows; (a) cosine bell; (b) Bartlett window; (c) Parzen window.
Bartlett ($K = 2$) and Parzen ($K = 4$) windows, together with the cosine bell, are shown in Fig. 7 (for further details see Priestly 1965). The spectral advantages of the Parzen window over the other two filters is apparent from this diagram. There are no negative peaks (which are present with the cosine bell) and the subsidiary peaks are a smaller fraction of the main peak than for the other two functions. The first subsidiary peak is approximately 4.5% of the main peak for the Bartlett window whilst for the Parzen window the ratio is approximately 0.2%. 