A limited area nested numerical weather prediction model: Formulation and preliminary results

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(Received 10 February 1977; revised 23 May 1977)

SUMMARY

A limited area, six-level, primitive equations numerical weather prediction model has been developed to provide improved 24- to 36-hour forecasts for the Australian region. Important features of the model include semi-implicit time differencing, and a nesting procedure which enables boundary values to be updated from a hemispheric spectral model. Preliminary results from the model indicate considerable operational potential.

1. INTRODUCTION

One of the major concerns of the Australian Numerical Meteorology Research Centre (ANMRC) is the development of a limited area numerical weather prediction model capable

![Graph showing skill scores at 300 mb, 500 mb, and MSL from 1970 to 1975]
of providing improved operational 24- to 36-hour forecasts over the Australian region. At present the operational numerical weather prediction models employed by the Australian Bureau of Meteorology include a hemispheric spectral model developed by Bourke (1974), and an Australian region filtered baroclinic model developed by Maine (1972). Although the latter model has replaced manual prognosis methods in the mid troposphere, its forecasting skill has remained stationary (and at m.s.l. marginally less than manual) for a number of years (Fig. 1). Moreover, some of the scaling assumptions inherent in the model exclude general applicability to the important and extensive Australian tropical regions; it is a dry model; and there is virtually no representation of boundary layer transfer processes.

By relaxing some of the above restrictions, it is hoped that the model described in this paper will ultimately lead to improved operational prognosis accuracy over the Australian region. As a primitive equations model it is not subject to the same scale limitation and, furthermore, it includes many more physical processes such as moist and dry convection, vertical diffusion, and surface exchanges of heat, momentum and moisture. The time extent of prognosis accuracy should also be improved by nesting the new model within the hemispheric spectral system. Despite these potential advantages no deterioration in relative computational efficiency is anticipated due to the use of an economical semi-implicit time differencing technique.

In a broader application context, the developments reported herein represent part of a continuing international trend towards the application of limited area primitive equation models for routine forecasting purposes, e.g. Okamura (1975), Howcroft (1971), Bushby and Timpson (1967). The use of more efficient semi-implicit time-differencing algorithms in an operational limited area context is not as common, although Burridge (1975) and Asselin and Robertson (1976) have reported successful applications.

2. DESCRIPTION OF THE MODEL

The limited area model described here is an adaptation, to a Lambert conformal projection of the Australian region, of the southern hemispheric model developed by Gauntlett et al. (1976) (hereinafter referred to as G). Briefly, the model is a six-level, primitive equations model using a semi-implicit time-differencing scheme. The equations are written in flux form and include representation of processes such as vertical and horizontal diffusion, moist and dry convection, and topography.

The primitive equations, expressed in flux form, are:

the momentum equations

\[
\frac{\partial \mathbf{u}}{\partial t} = -m \left\{ \frac{\partial}{\partial x} (\mathbf{u} u) + \frac{\partial}{\partial y} (\mathbf{u} v) \right\} - \frac{\partial}{\partial \sigma} (\hat{\mathbf{u}} \mathbf{u}) + f \mathbf{u} - p_s \frac{\partial \phi}{\partial x} - RT \frac{\partial p_s}{\partial x} +
\]

\[
m^2 K_m \left\{ \frac{\partial}{\partial x} \left( p_s \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( p_s \frac{\partial u}{\partial y} \right) \right\} + F_x
\]

\[
\left( 1 \right)
\]

\[
\frac{\partial \phi}{\partial t} = -m \left\{ \frac{\partial}{\partial x} (\mathbf{u} \phi) + \frac{\partial}{\partial y} (\mathbf{v} \phi) \right\} - \frac{\partial}{\partial \sigma} (\hat{\mathbf{u}} \phi) - f \mathbf{u} - p_s \frac{\partial \phi}{\partial y} - RT \frac{\partial p_s}{\partial y} +
\]

\[
m^2 K_m \left\{ \frac{\partial}{\partial x} \left( p_s \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( p_s \frac{\partial v}{\partial y} \right) \right\} + F_y
\]

\[
\left( 2 \right)
\]
the thermodynamic equation

$$\frac{\partial}{\partial t}(p_\sigma T) = -m^2\left\{\frac{\partial}{\partial x}(\bar{u} T) + \frac{\partial}{\partial y}(\bar{v} T)\right\} - p_\sigma^2 \frac{\partial}{\partial \sigma}(\bar{\sigma} T) + \frac{RT\omega}{c_p} +$$

$$m^2 K_T \left\{\frac{\partial}{\partial x} \left( p_\sigma \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( p_\sigma \frac{\partial T}{\partial y} \right) \right\} + \frac{p_{\bar{\theta}}}{c_p}$$

(3)

the water vapour equation

$$\frac{\partial}{\partial t}(p_r) = -m^2\left\{\frac{\partial}{\partial x}(\bar{u} r) + \frac{\partial}{\partial y}(\bar{v} r)\right\} - p_\sigma^2 \frac{\partial}{\partial \sigma}(\bar{\sigma} r) +$$

$$m^2 K_r \left\{\frac{\partial}{\partial x} \left( p_\sigma \frac{\partial r}{\partial x} \right) + \frac{\partial}{\partial y} \left( p_\sigma \frac{\partial r}{\partial y} \right) \right\} + F_r - p_r C$$

(4)

the continuity equation

$$\frac{\partial p_\sigma}{\partial t} = -m^2\left\{\frac{\partial}{\partial x}(\bar{u}) + \frac{\partial}{\partial y}(\bar{v})\right\} - p_\sigma \frac{\partial \bar{\sigma}}{\partial \sigma}$$

(5)

and the hydrostatic equation

$$\frac{\partial \bar{\sigma}}{\partial \sigma} = -\frac{RT}{\bar{\sigma}}$$

(6)

The symbols used above are fairly standard and well known. A complete definition is given in the appendix.

Following Robert et al. (1972) two new dependent variables are defined:

$$P = p_\sigma(\phi + RT_0 \ln p_\sigma), \quad W = \bar{\sigma} - (\sigma m^2 / p_\sigma) \left\{\frac{\partial}{\partial x} \left( \int_0^1 \bar{u} d\sigma \right) + \frac{\partial}{\partial y} \left( \int_0^1 \bar{v} d\sigma \right) \right\}$$

(7)

where $p_\sigma$ is a space- and time-independent constant and $T_0$ varies only in the vertical. Eqs. (1)–(5) may then be transformed into finite difference equations for $\bar{u}^{2t}, \bar{v}^{2t}, \bar{W}^{2t}$, and $\bar{P}^{2t}$, in the manner described in G, whilst for Eq. (6) the finite difference representation is analogous to that used by Smagorinsky et al. (1965). The equation defining $P$ at time $t + \Delta t$ is a Helmhotlz equation and takes the form

$$m^2 \nabla^2 P_k^{2t} + M_k F_k^{2t} = R_k$$

(8)

where, for the six-level model under consideration and for the vertical distribution of levels shown in Fig. 2, $M_k$ is a 6 × 6 tri-diagonal symmetric matrix dependent only on basic model parameters, $R_k$ is a column vector of dimension 6, dependent on information available at timesteps $t$ and $t - \Delta t$, and the bar operator signifies time averaging according to

$$\bar{P}^{2t} = \frac{1}{2}(P(t + \Delta t) + P(t - \Delta t))$$

(9)

Following the solution of Eq. (8) and the inversion of Eq. (9) to obtain $P(t + \Delta t)$, the values of all remaining dependent variables are then directly obtained in accordance with procedures outlined in G.

Significantly, it has been found that the space-dependent ‘linearization’ of the pressure gradient terms used in the hemispheric version (p. 205 of G) was not necessary over the Australian region because of the much lower topography. A considerable saving of time was therefore made since it was not required to resort to the cycling techniques used for the solution of the equation, equivalent to Eq. (8), in the hemispheric model.
Figure 2. The vertical distribution of primary variables in the semi-implicit model. Full levels are indicated by dotted lines; vertical boundary conditions are also shown.

The grid spacing of the Australian region grid is about 200 km and the semi-implicit scheme allowed a timestep of 30 minutes to be taken. Larger timesteps have been tested, but no systematic attempt has yet been made to find the limit for the scheme.

3. LATERAL BOUNDARY CONDITIONS

It was decided at the outset to provide a nesting facility as well as simple static inflow/outflow boundary conditions. Boundary tendencies are readily available from the multilevel hemispheric spectral model developed by Bourke (1974) which is now used operationally by the Australian Bureau of Meteorology. The nesting procedure enables new information to enter the forecast domain during the prognostic period.

Experiments have been conducted with two alternative boundary procedures, each being of the so-called 'one way interactive type'. The essentials of the first approach (hereinafter referred to as method A) are summarized in Table 1. At inflow points, all variables are updated by tendencies provided by the hemispheric spectral model. At outflow points, the tendencies of surface pressure and tangential velocity are extrapolated from the interior, mixing ratio and normal velocity are held constant, and temperature is updated according to a backward Lagrangian advection technique analogous to that described by Shapiro and O'Brien (1970). Finally, at the penultimate rows a (1,2,1) filter is applied to the normal velocity component following the suggestions of Chen and Miyakoda (1974).

The second boundary approach (hereinafter referred to as method B) is based on a more pragmatic philosophy and follows closely the suggestions made by Davies (1976) involving the 'dynamic relaxation' of the variables of the limited area model in the vicinity of the lateral boundaries to the externally prescribed flow. Thus, on the r.h.s. of each of the primary forecast equations (1) to (5), an additional term of the general form
\(-K(\tau - \bar{\tau})\) is included where \(\tau\) represents the limited area model values of \(\bar{u}, \bar{v}, \bar{T}, \bar{r}\) and \(p_{\bar{a}}\). \(\bar{\tau}\) represents the corresponding spectral model values of the same primary variables; \(K\) is a continuous function which is nonzero only within four grid units of the boundary and is defined by \(K = \{1/(2\Delta t)\} \cdot d^2\), where \(d\) is the distance in grid units from the boundary and \(a = 0.15\).

### Table 1. Lateral Boundary Conditions (Method A) for the Australian Region Semi-implicit Model

<table>
<thead>
<tr>
<th>Boundary variable</th>
<th>Inflow</th>
<th>Boundary condition</th>
<th>Outflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_T)</td>
<td>All variables</td>
<td>(V_T^{b+1} = V_T^{b-1} + (V_T^{b+1}<em>{(b-2)} - V_T^{b-1}</em>{(b-2)}))</td>
<td></td>
</tr>
<tr>
<td>(V_N)</td>
<td>updated from</td>
<td>(V_N^{b+1} = V_N^{b-1})</td>
<td></td>
</tr>
<tr>
<td>(P_a)</td>
<td>hemispheric</td>
<td>(P_{ab}^{b+1} = P_{ab}^{b-1} + (P_{ab}^{b+1}<em>{(b-2)} - P</em>{ab}^{b-1}_{(b-2)}))</td>
<td></td>
</tr>
<tr>
<td>(T)</td>
<td>spectral</td>
<td>(T_{b+1}^{b+1} = T_{b-\Delta x, b-\Delta y}) where (\Delta x = x^b_{b+1}\Delta t) and (\Delta y = y^b_{b+1}\Delta t)</td>
<td></td>
</tr>
<tr>
<td>(r)</td>
<td>model tendencies</td>
<td>(r_{b+1}^{b+1} = r_{b}^{b-1})</td>
<td></td>
</tr>
</tbody>
</table>

\(V_{TN}, V_{Nb}, P_{ab}, T_b, r_b = \) boundary values of tangential and normal velocities, surface pressure, temperature and mixing ratio

\(T_{b-\Delta x, b-\Delta y} = \) temperature at point distant \(\Delta x, \Delta y\) from boundary

### 4. Initialization

A number of initialization procedures have been tested. These included mixed gradient/geostrophic winds (Maine and Seaman 1967), linear balance winds, and nonlinear balance winds. All these methods were tried and were followed, and also not followed, by a dynamic initialization procedure before the commencement of the prognosis. It was found that the combination of a nonlinear balance step followed by 30 cycles of dynamic initialization in the manner described by Gauntlett (1975) provided the most satisfactory initial fields. However, the magnitude of the improvements afforded by dynamic initialization following a nonlinear balance were not sufficient to justify the additional computing time, at least in the current computing environment. These findings accord with the recent results of Temperton (1976).

It should also be noted that although approximately one-third of the forecast region lies in the tropics, no attempt has hitherto been made to provide initialization procedures compatible with low latitude requirements. This is a serious shortcoming due largely to the paucity of available wind data in the region. The first moves have been made towards a tropical analysis and initialization scheme in anticipation of greatly improved wind data coverage following the launching of the Japanese geostationary satellite in 1977. These include the development of an improved analysis scheme, in which observations of the wind and mass fields are analysed separately, and then subsequently blended using variational principles (Seaman et al. 1977). This results in geopotential fields which reflect wind data more adequately than the previous Cressman 'successive correction' method. Also, four-dimensional assimilation schemes are being tested by Bourke in connection with the operational hemispheric spectral model mentioned above.

### 5. Results

To date, statistics have been prepared for fifteen synoptic situations and have been compared with the performance of the operational filtered model for the same situations. About half of these are 'winter' situations and the other half 'summer' situations. A plot of the S1 skill scores (Tewele and Wobus 1954) for each model (non-nested) is shown in
Fig. 3. Skill-score comparisons between non-nested semi-implicit and filtered models for 15 situations.

In assessing the practical value of these results it is important to point out that although the filtered model has a slight vertical resolution advantage (seven levels compared with six in the primitive equation model), the horizontal resolutions in the models are identical (approx. 200 km) and there is virtually no difference in relative computational efficiency. A significant improvement in forecasting skill has been obtained at all levels except 200 mb, and the gain is most pronounced between 700 and 300 mb where the average improvement is about 8 points. The decrease in skill at 200 mb is due to the choice of $\sigma$-surfaces in the semi-implicit model. The top two levels in the model were at $\sigma = 0.087$ and 0.250, which resulted in a substantial vertical extrapolation error in the vicinity of the tropopause. This constraint will be relaxed in the immediate future with the inclusion of an extra level and a better distribution of existing levels.

It is of interest to compare in some detail the non-nested* semi-implicit forecast with that produced by the operational baroclinic filtered model. Such a comparison is made in Fig. 4. It can be seen that the forecast for the operational model is characterized by the unrealistic intensity of both the west and east coast anticyclones, the slow movement of the trough over southeastern Australia, and a general lack of finer detail. The primitive equation model, on the other hand, is more realistic in detail and the predicted intensity and movement of nearly all significant synoptic features are improved. This satisfactory performance of the primitive equation model is further underlined by the forecast results (non-nested) for 500 mb shown in Fig. 5. For further comparisons, Fig. 6 includes two nested primitive equation m.s.l. forecasts for the same situation using methods A and B, together with the corresponding forecast from the multilevel spectral model. The latter are interpolated from the hemisphere to the Australian region to provide the required boundary tendencies. The spectral model has predicted well the phase speeds of the higher latitude systems but it is of insufficient resolution (wavenumber 15) to maintain the finer detail present, for example in the complex low pressure system to the south of the continent.

As far as the relative merits of the two nesting procedures evaluated are concerned, it would appear that method B (Fig. 6(b)) is superior to method A (Fig. 6(a)) for the very important reason that more consistency is maintained between the ensuing limited area forecast and the external forecasting imposed by the spectral model. This is especially evident in the southwest and northwest sections of the forecast where method B has had relatively more success in representing respectively the intensity of the southern Indian Ocean depression and the falling pressures in the vicinity of a developing low latitude cyclone. Method A by contrast is inferior in both respects although it does enjoy a superior position verification.

* Non-nested in this context refers to time-invariant boundary values on inflow points and time-dependent outflow boundary values as outlined in Table 3.
Figure 4.  (a) Initial analysis for 23 z 6 August 1973; (b) 24 h manual verification analysis valid 23 z 7 August 1973; (c) 24 h operational filtered baroclinic prognosis valid 23 z 7 August 1973; (d) 24 h semi-implicit prognosis (non-nested) valid 23 z 7 August 1973.
for the high pressure system over Western Australia. This may be fortuitous however and could be associated with upstream errors, notably the higher pressures near the western boundary at 35°S and the consequent sharpening of the frontal zone to the immediate southwest of Western Australia.

Note that the method B nested forecast and the spectral forecast agree closely in respect of their poor prediction of the Western Australian anticyclone. All the limited area forecasts, and especially the method B nested forecast, show superiority over the spectral model in the definition and evolution of the complex low pressure area operating over southeastern Australia. It is fairly obvious, however, that more extensive evaluations are required before any positive conclusions are drawn concerning the relative merits of the two nesting procedures evaluated. Such evaluations are proceeding.
Figure 6. (a) 24 h semi-implicit m.s.l. nested (method A) prognosis valid 23 z 7 August 1973; (b) 24 h semi-implicit m.s.l. nested (method B) prognosis valid 23 z 7 August 1973; (c) 24 h manual verification analysis valid 23 z 7 August 1973; (d) 24 h spectral m.s.l. prognosis valid 23 z 7 August 1973.
A number of other (non-nested) situations were studied in detail, and some of the results are worthy of close attention. One of these (23 z 25 August 1973) is notable for the development of a cut-off low over southeastern Australia with associated heavy rainfall. This situation was of particular interest because the development took place over the relatively data-rich land mass and it was hoped that considerable success would be obtained. The semi-implicit model, as shown in Figs. 7(a), (b), (c), predicted the synoptic development and the rainfall pattern with pleasing accuracy. A comparison of observed and
predicted rainfall was carried out over southeast Australia; they were found to accord very well both in intensity and distribution (see Fig. 8).

Another occasion examined was a summer situation (23 z 5 February 1976) in which a tropical cyclone had crossed the coast, weakened, and then re-intensified owing to a strong influx of moisture. Flood rains were experienced over much of inland Australia leading, in some cases, to the highest river levels in recorded history. It was not expected that the model would perform particularly well because, as mentioned above, the whole prognosis system from analysis, through initialization, to the prognosis model physics, is directed towards the mid- and high-latitudes. Little allowance had been made for tropical systems. This was indeed the case, as can be seen in Figs. 9(a), (b), and (c). The tropical low was significantly weakened and a secondary low pressure centre off the West Australian coast was over-developed. In marked contrast with the mid-latitude cut-off low described in the preceding paragraph, the rainfall pattern predicted by the model was disappointing. Neither the great intensity nor the widespread nature of the actual rainfall was captured by the model. The model was re-run replacing the GFDL convective adjustment scheme (Smagorinsky et al. 1965) by a recently developed version of the convective scheme described by Krishnamurti and Moncrief (1971), but only a small change in the distribution of rainfall was obtained.

No problems have been encountered when the model has been run out to 36 hours and even beyond, to 48 hours. A plot of various domain integrals is given in Fig. 10 for two typical situations. The domain integral of m.s.l. pressure behaves well and shows no signs of systematic oscillation. The mean kinetic energy is also conserved very well apart from an initial decrease caused mainly by the sudden introduction of surface drag at the commencement of the integration.

A few brief comments on computer requirements may be of interest. The computing facilities available are 160 kilobytes on an IBM 360/65. The semi-implicit primitive equations package consists of three parts: initialization, prognosis, and display, requiring 3, 16, and 2 minutes of CPU time respectively. The total elapsed time for the three parts is about 30 minutes, for a 24-hour prognosis.

6. CONCLUSIONS

The first phase of development and preliminary testing of a limited area semi-implicit numerical weather prediction model for routine forecasting over the Australian region has been completed. The next step, apart from some minor tuning such as choice of optimal \( \sigma \)-surface distribution, is to run the model in an exhaustive operational trial over several
months of summer and winter situations. These trials have commenced and the results will be described in a future report on the operational performance of the model.

The model has performed in a very encouraging manner up to this point. Over a reasonable number of situations (fifteen), chosen in different seasons, the semi-implicit primitive equations model has shown marked superiority over the current operational filtered model, particularly in the mid troposphere. There appears to be no reason why this superiority should not be confirmed by further comparative testing. Positive aspects of the model include its ability to retain the fine detail of significant synoptic features such

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**Figure 9.** (a) Initial analysis for 23z 5 February 1976; (b), (c) Semi-implicit forecast (non-nested) and verifying analysis for 23z 6 February 1976.
as the complex low of Fig. 4 and the cut-off depression of Fig. 7, and to maintain the sharpness and speed of frontal systems compared with the filtered model. Finally, the nesting facility allows the forecast to be extended realistically out to at least 36 hours, compared with the 24-hour forecasts provided by the operational filtered model.

Additional developments of the model which should be mentioned include an efficient staggered grid formulation, and the incorporation of a version of the Arakawa–Schubert (1974) cumulus convection scheme. Some results from these modifications have been published separately (McGregor 1977) and need not be discussed here.

There remain a number of deficiencies of the model which must be attended to in the near future. The most severe deficiency is in the tropical regions where the model does not perform well during periods of active development owing to the heavy bias of the existing analysis and initialization scheme towards mid-latitudes. As described in section 4, the launching of the Japanese geostationary satellite in 1977 will greatly enhance the density of tropical wind data. More advanced analysis and initialization schemes are being developed in anticipation of this improvement. The quality of the m.s.l. prognosis can also be considerably improved even in mid-latitudes. As yet, little attempt has been made to represent adequately the surface boundary layer, and only simple mixing length concepts with no diurnal variation have so far been used. Considerable effort is now being directed toward relaxing these restrictions.

ACKNOWLEDGMENTS

We would like to express our appreciation of the technical help given by H. Scott; and to W. Bourke and R. Seaman for many hours of critical discussion.

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APPENDIX

Symbols included in the text

$c_p$ specific heat of air at constant pressure

$C$ rate of change of mixing ratio due to condensation

$f$ Coriolis parameter

$F_x$ rate of change of momentum in the $x$ direction due to vertical diffusion

$F_y$ rate of change of momentum in the $y$ direction due to vertical diffusion

$k$ the number of a vertical level

$K_h$ the horizontal coefficient of eddy viscosity for heat

$K_m$ the horizontal coefficient of eddy viscosity for momentum

$K_r$ the horizontal coefficient of eddy viscosity for mixing ratio
\textit{m} \quad \text{map scale factor for Lambert conformal projection}

\textit{p} \quad \text{pressure}

\textit{p_*} \quad \text{surface pressure}

\textit{\dot{q}_c} \quad \text{rate of heating due to condensation and convection}

\textit{u} \quad \text{earth velocity component in the } x \text{ direction}

\textit{v} \quad \text{earth velocity component in the } y \text{ direction}

\textit{r} \quad \text{mixing ratio of water vapour to dry air}

\textit{R} \quad \text{gas constant of air}

\textit{t} \quad \text{time}

\textit{T} \quad \text{temperature}

\omega \quad \frac{dp}{dt}

\phi \quad \text{geopotential height of a } \sigma\text{-surface}

\sigma \quad \frac{p}{p_*}, \text{ the vertical model coordinate}

\dot{\sigma} \quad \frac{d\sigma}{dt}

\dot{u} \quad \frac{p_* u}{m}

\dot{\theta} \quad \frac{p_* v}{m}