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COMMENTS ON THE PAPER ‘ANGULAR MOMENTUM ADELECTION BY AXISYMMETRIC MOTIONS’ BY R. A. PLUMB (Q.J. 1977, 103, 479-485)

By R. D. ROSEN

Plumb’s contribution to the discussion of the constraints imposed on steady state, axisymmetric flows is welcome. His demonstration that such flows can exist provided that the surfaces of constant absolute angular momentum are “distorted sufficiently (usually in boundary layers)” is straightforward. His criticism of Starr’s (1974) theorem on axisymmetric flow is unwarranted, however, for in fact this theorem is not in conflict with Plumb’s result.

A conscientious reading of Starr’s paper reveals that one of the necessary conditions for the application of his theorem is that the rotational Rossby number, as defined by Starr, should be small (Starr, p. 517); in other words, that the surfaces of constant absolute angular momentum should not be much distorted. Starr required the small rotational Rossby number constraint to hold throughout the entire depth of the fluid, and he felt this was satisfied in the case of the terrestrial atmosphere’s general circulation. Starr recognized instances when this condition would not be met, as in certain dishpan experiments, in which case an axisymmetric flow could be observed. Plumb seems to acknowledge Starr’s discussion of this point in his footnote on p. 484, although he apparently misinterprets Starr’s definition of the rotational Rossby number (the rotational Rossby number is defined locally as a function of height and of distance from the axis of rotation, so that a large rotational Rossby number, in Starr’s sense, can imply strong velocity gradients in the boundary layers).

Robock (1975), using Starr’s approach, also tempers his discussion of the axisymmetric hurricane on the shape of the constant absolute angular momentum surfaces. Robock noted that in the boundary layer near the storm’s centre the rotational Rossby number for the flow would not be small. Therefore, these surfaces would be so distorted as to allow the hurricane circulation to be axisymmetric within at least a 3° radius from its centre for the very reasons Plumb states (Robock, p. 661; also compare Robock’s Fig. 2 with Plumb’s Fig. 3).

ACKNOWLEDGMENT

I am indebted to Dr Robert K. Crane for thoughtful discussions on the topic of this correspondence.

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REPLY

By R. A. Plum

It seems that Dr Rosen has misunderstood the main point of my paper (Plumb 1977, hereafter referred to as P), which showed that the arguments of Starr (1974a, hereafter referred to as S) were based on inconsistent reasoning and that the 'theorem' – that differential rotation cannot be maintained by axisymmetric motions alone – is invalid under all circumstances, e.g. for any value of viscosity or flow speed.

As Dr Rosen mentions, S sought to explain the contradiction between his theorem and the existence of differential rotation in axisymmetric annulus flows by arguing that in these flows the 'rotational Rossby number' \( R = \frac{\Omega}{\omega} \) (where \( \Omega \) is the angular velocity of the container and \( \omega \) the relative angular velocity of the fluid) is large. S proposed that in such cases the surfaces of constant absolute angular momentum \( \mu = r^2(\Omega + \omega) \) would then become severely distorted and that the viscous flux of \( \mu \) would become significant.

However, as noted in the footnote on page 484 of P, \( R \) is frequently small in laboratory annulus flows. I do not believe that I have misinterpreted the definition of \( R \), as Dr Rosen claims. For example, applying the definition in S to the numerical study of Williams (1967), shown in Fig. 4 of P, \( R \) nowhere exceeds 0.25. (In some other cases presented by Williams, it is still smaller.) The important point, discussed in P, is that the viscous flux can be (and, in a steady state, always is) significant, independent of the value of \( R \).

As an illustration of the invalidity of the theorem even at small \( R \), consider flow in an annulus bounded by inner and outer cylinders at \( r = a \) and \( r = b \) respectively. If \( \omega = 0 \) everywhere at time \( t = 0 \) then

\[ \Omega a^2 < \mu < \Omega b^2. \]

If the flow is inviscid, conservation of angular momentum

\[ \frac{D\mu}{Dt} = \left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) r^2(\Omega + \omega) = 0 \]

where \( (u, w) \) are the velocity components of the unspecified meridional circulation) demands that \( \Omega a^2 < \mu < \Omega b^2 \) for all \( t \), and at all \( r, z \). It follows that, throughout the evolution,

\[ -\Omega \frac{(b^2 - a^2)}{a^2} \leq \omega \leq \Omega \frac{(b^2 - a^2)}{a^2} \]

and hence that \( |R| < (b^2 - a^2)/a^2 \). Clearly, in an annulus of sufficiently narrow gap width, \( |R| \) can be constrained to be arbitrarily small. Now the results of Starr’s (1974b) time-dependent advection theorem imply that \( \omega = 0 \) everywhere for all \( t \). This is clearly in disagreement with (1) (except in the trivial case \( u = 0 \)).

Similarly, if viscosity \( \nu \) is non-zero, (1) becomes

\[ \left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) r^2(\Omega + \omega) = \frac{1}{r} \frac{\partial}{\partial r} \left( \nu r^3 \frac{\partial \omega}{\partial r} \right) + r^2 \frac{\partial}{\partial z} \left( \nu \frac{\partial \omega}{\partial z} \right). \]

According to S, the only possible steady state is \( \omega = 0 \) everywhere. Such a state is not a solution of (2) (if \( u \neq 0 \)). Therefore the theorem contradicts not only common experience in the laboratory and in numerical models of such phenomena as axisymmetric tropical cyclones, but also the fundamental equations of motion. The fallacy in the derivation of the theorem is discussed in detail in P.
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