A parameterization scheme for non-convective condensation including prediction of cloud water content

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(Received 18 April 1977; revised 22 November 1977. Communicated by Dr K. A. Browning)

SUMMARY

A model for non-convective condensation processes has been developed. The model allows condensation to begin before relative humidity reaches 100%. The liquid water content of clouds is a prognostic variable of the model. The rate of condensation is a function of relative humidity and moisture flux convergence. The micro-physical processes involved in the formation of clouds and precipitation are parameterized by assuming that the rate of precipitation formation is a function of the amount of cloud water. Evaporation from falling rain is taken into account.

Quantitative tests with the model indicated that it yields reasonable evolution times and water content of clouds, and gives reasonable precipitation amounts.

1. INTRODUCTION

Most, if not all, numerical models of synoptic-scale motion or of the general circulation, handle non-convective condensation in a fairly simplified way. This is probably partly because a straightforward and simple formulation of this type of condensation gives fairly realistic precipitation without substantially adding to the model complexity. Clouds are usually not considered at all. In some GCMs, clouds are taken into account using climatological data, with no dynamic interplay between the cloudiness of the model atmosphere and its motion. However, in the NCAR model (Washington 1974), a simple dynamic coupling and feedback mechanism is provided by making cloudiness a function of relative humidity. The complexity of precipitation and latent heat release calculations varies considerably from model to model. In some, excess humidity with respect to the saturation value is simply converted to precipitation and associated heat release. In others, several parameters are introduced to facilitate tuning for various specific forecasting aspects (e.g. the model by Davies and Olson 1973).

Increasing emphasis has been placed on the need to improve the handling of condensation – especially the associated cloudiness – in dynamic models during the past few years. One of the major reasons is the incipient modelling work of climate within the frame of GARP, objective II. Here, of course, it is very important that condensation and clouds are treated as correctly as possible in all respects. A review of these questions is made by Arakawa (1975).

Although some effects of condensation and cloud might be disregarded from a dynamical point of view in short-range weather forecasting, the inclusion of those effects may still be relevant: for example, with respect to the calculation of the energy budget at the earth's surface and to operational forecasting of cloudiness and precipitation.

The purpose of the present paper is to design a fairly simple condensation model yielding not only release of latent heat and precipitation but also cloud mass, for possible use in a large-scale dynamical model, which provides the input parameters: wind, temperature and moisture. The fine details of the condensation process – formation of cloud droplets and their growth to rain drops, evaporation from drops, etc. – are para-
meterized. A threshold relative humidity at which condensation starts is considered, hence allowing for sub-grid-scale cloud cover.

Derivation of the model equations is given in section 2. Section 3 contains an extended discussion of the parameters of the model. Some numerical integrations with a one-dimensional version of the model have been carried out. The results, presented in section 4, provide a guide to characteristic features of the model cloud with respect to different external forcings and to the sensitivity of the cloud to changes in the model parameters.

The important aspect of the representation of cloud on a vertical sub-grid scale is not, however, considered in the present study.

2. Model Equations

In formulating a set of equations to describe condensation on the scale of a typical grid size and greater, we must consider two aspects which are more or less independent. First, we shall allow condensation to start before the relative humidity given by the numerical model has reached 100%. This implies that condensation occurs in only a portion of the grid square; this sub-grid-scale feature has therefore to be parameterized. As we allow for fractional cloud cover, we shall consider the possibility of evaporation of droplets or drops in clear parts of the grid square as a consequence of sub-grid-scale eddy motion. This evaporation, and large-scale convergence of moisture, will raise the humidity in cloud-free parts and consequently cause a gradual shrinking of those parts as condensation develops. Second, we shall consider the existence of cloud mass. Consequently, we have to formulate how the condensate is partitioned between precipitation and cloud building.

(a) Condensation in a fraction of the grid square

Returning to the first-mentioned aspect: consider a grid square which we take as unit area, denoting the cloud-free fraction by \( a \). Then the net heating rate resulting from condensation and evaporation in the unit area is given by

\[
Q = (1 - a)Q_c - a(E_c + E_r)
\]

(1)

where \( Q_c \) is the rate of release of latent heat in the cloudy portion and \( E_c \) and \( E_r \) are the cooling rates due to evaporation of cloud droplets and rain drops, respectively, regarded as taking place in the cloud-free portion of the grid square. Hence \( Q_c, E_c \) and \( E_r \) are sub-grid-scale quantities.

The thermodynamic equation and the moisture equation that are representative for the grid square may then be written

\[
\frac{\partial \theta}{\partial t} = A(\theta) + (\theta/T)(Q/c_p)
\]

(2)

\[
\frac{\partial q}{\partial t} = A(q) - Q/L
\]

(3)

where the operator

\[
A(\theta) = -\nabla \cdot [\nabla (\psi(\theta)) - \partial (\omega(\theta))/\partial p - \text{div eddy flux}(\theta),
\]

and \( q \) is the specific humidity (\( \approx \) mixing ratio of water vapour). Variables not explained have their conventional meaning. Introducing relative humidity \( U = q/q_s, q_s(\theta) \) being the saturation value of \( q \) at temperature \( \theta \) (or \( T \)) we shall utilize the following relations:

\[
\frac{1}{q} \frac{\partial q}{\partial t} - \frac{1}{U} \frac{\partial U}{\partial t} = \frac{1}{q_s} \frac{\partial q_s}{\partial t} = \frac{\varepsilon L}{RT} \frac{\partial \theta}{\partial t} + \left( \frac{\varepsilon L}{c_p T} - 1 \right) \frac{\partial p}{\partial t}.
\]

(4)
The last equality expresses the Clausius-Clapeyron relation \((\varepsilon = 0.622)\). Considering partial derivatives with respect to \(t\), \(x\) and \(y\), the last term is zero since we are using the \(p\) system. Combining Eqs. (2), (3) and (4) we now eliminate the time derivatives for \(\theta\) and \(q\) and obtain

\[
Q = (M - Lq_x \partial U/\partial t)(1 + US_q)^{-1}.
\]

\[(5)\]

where \(S_q = (\varepsilon L^2/Rc_p)(q_s/T^2)\) and \(M = LA(q) - US_q c_p(T/\theta)A(\theta)\). Relation (5) shows that since we allow condensation to appear before the grid square is saturated \((U = 1)\), we have to decide how the converged vapour should be shared between condensation and a general moistening \((\partial U/\partial t \neq 0)\). Before discussing this problem let us briefly note a couple of details. The quantity \(M\) is the amount of vapour available (per unit time) for condensation and moistening; the second term of \(M\) accounts for possible expansion or compression of the air. The denominator in Eq. (5) arises because the rise of temperature and saturation humidity should keep pace with each other as the release of latent heat takes place. Note also that the \(S_q\) term cannot be neglected until \(q_s \approx 0.5 \times 10^{-3}\) (at \(\sim 350mb\) in the standard atmosphere).

The rate of heating caused by condensation is thus obtained from Eq. (5), provided we know the rate of change of \(U\). The most usual and straightforward approach employed is to assume that all the convergence, expressed by \(M\), results in condensation. That is, \(U\) is at steady-state, possibly at a value \(< 1\). Furthermore, it is usually assumed that all condensed water precipitates immediately and hence no clouds are formed. In that case, Eqs. (2), (3) and (5) form a closed system from which we obtain diagnostically the heating by condensation and the rate of precipitation.

Returning to the full equation (5), we will derive a formulation for the rate of change of relative humidity. Let us consider \(a\), the clear fraction of the unit area where the specific humidity is \(q_0 = U_0q_e\) and the change of humidity is contributed to by large-scale convergence (or divergence), evaporation and expansion or compression. The integrated rate of change over this area is given by

\[
\partial q/\partial t = (a/L)(M_o + E_c + E_r)
\]

\[(6)\]

where \(M_o\) is a quantity \(M\) with \(U = U_0\).

We now make the assumption that this increase (or decrease) is used to enlarge (or reduce) cloud cover over a portion of \(a\) of such precise size that \(q\) changes from \(q_0\) to \(q_e\) in that portion. Furthermore, we assume that the relative humidity, \(U_o\), of the cloud-free part of the grid area remains constant.

As we are also going to consider the presence of cloud water (mixing ratio \(m\) representative for the grid square) we shall include this for completeness. Leaving out second-order effects, which would come from changes in \(q_s, q_0\) and \(m\), the humidity rise per unit time in that portion of \(a\) thus becomes

\[
(q_s - q_0 + m)\partial a/\partial t = q_e(U_s - U_0 + m)(\partial a/\partial t)
\]

\[(6a)\]

As this will be provided for by the change expressed in Eq. (6a), we obtain

\[
\partial a/\partial t = \frac{a}{L[q_s(U_s - U_0) + m]}(M_o + E_c + E_r)
\]

\[(7)\]

where \(U_s = 1\).

Since Eq. (7) is valid from the moment condensation begins in the area, we also find that \(U_0\) is the value at which condensation is allowed to begin. By analogy with relation (1) we have

\[
U = (1 - a)U_s + aU_0
\]

\[(8)\]
Differentiating and utilizing Eq. (7) we obtain
\[ \frac{\partial U}{\partial t} = \frac{U_s - U}{L(q_s(U_s - U_0) + m)} (M_0 + E_c + E_r) \] . \quad (9)

For a model, in which we do not take cloud mass into account, the system is now closed with the aid of relation (9). Inserting this relation into Eq. (5) it is easily seen that at the moment when \( U = U_0 \) there is no heat liberated, but the convergence, \( M \), merely increases \( U \). As soon as \( U > U_0 \), relation (9) shows that the \( \partial U/\partial t \) term in Eq. (5) only utilizes part of \( M \) while the rest is brought into condensation, and this latter part becomes larger as \( U \) increases.

(b) Amount of water in clouds and precipitation

In this approach to describing the quantity and rate of change of cloud water, we shall not consider the distribution of droplet sizes, but only the mass mixing ratio \( m \). The vertical air motions that we are concerned with in this context have a typical magnitude of a few cm s\(^{-1}\), up to a maximum of some tens of cm s\(^{-1}\) in highly limited areas. By assuming a log-normal distribution of droplet sizes (see, e.g., Matveev 1967) we find an average terminal fall speed of cloud droplets of 1.5–2 cm s\(^{-1}\). Thus, we may expect that the vertical air motion and the terminal fall speed of droplets roughly cancel in the greater part of an area containing non-convective condensation. Therefore, vertical motion terms are omitted in the derivation of the prognostic equation for cloud water content and only lateral motion of cloud droplets is considered.

The fundamental source of cloud water is naturally condensation, the sinks being precipitation and the evaporation of cloud droplets transported into the clear part of the area. Thus we get an equation for the rate of change of cloud water:
\[ \frac{\partial m}{\partial t} = -\nabla.(V m) + (1 - a)(Q_c/L) - a(E_c/L) - P \] . \quad (10a)

or if we use relation (1):
\[ \frac{\partial m}{\partial t} = -\nabla.(V m) + Q/L - P + a(E_r/L) \] . \quad (10b)

where \( P \) is the rate of precipitation formation (mass of water per unit mass of air per unit time) at level \( p \).

For the purposes of this study we now have a closed system of equations consisting of (2), (3), (5), (9) and (10b) and the diagnostic relation (8). However, we have not yet formulated expressions for the rates of evaporation and of precipitation formation \( E_c \), \( E_r \) and \( P \) respectively. As mentioned above, it is assumed that cloud droplets may enter the clear parts of the area as a result of lateral transports. The mechanism for these transports is either sub-grid-scale eddy diffusion or large-scale advection from neighbouring grid points. This matter is not elaborated in the present study. In the quantitative experiments we have set \( E_c = 0 \) since its effects cannot properly be regarded in a one-dimension model.

Evaporation from raindrops is considered to occur as follows. Those drops that form in one volume (grid box) are not subject to evaporation within that same volume. Thus it is merely those drops that enter from above that may evaporate in the clear fraction of the box in question. The rate of evaporation is assumed to be proportional to the sub-saturation, \( U_s - U_0 \). In a layer \( \Delta p \) centred around level \( p \) we find that the rate of cooling per unit mass due to evaporation is
\[ E_r(p) = (g/\Delta p)(U_s - U_0)\bar{P}(p - \frac{1}{2}\Delta p) \] . \quad (12)

where \( \bar{P}(p - \frac{1}{2}\Delta p) \) is the integrated precipitation rate at level \( p - \frac{1}{2}\Delta p \) with possible evaporation in layers above considered. Note that the cooling rate, representative for the grid square,
is $aE$, and hence proportional to $U_s - U$. The expression is thus also applicable to cloud-
free layers through which the rain may pass.

Our remaining concern now is how to describe the fraction of the condensed water
which falls out as rain at each moment. Since we are not considering micro-physical
processes of condensation we have to resort to parameterization. We do so by relating the
rate of precipitation formation to the amount of water in the cloud so that the higher the
rain intensity the denser the cloud.

The relation employed is

$$P = C_0 m [1 - \exp\{- (m/m_r)^2\}] = C_m m$$  \hspace{1cm} (13)

where $C_0$ and $m_r$ are parameters that will be discussed below. The dimension of $C_m$
(and $C_0$) is time$^{-1}$, implying that $C_m^{-1}$ yields a typical time of conversion of droplets
to raindrops. Ogura and Takahashi (1971) used a similar formulation in a paper on
convective cloud modelling; however, they assumed a linear relation (i.e. $C_m = \text{const}$). In
our approach the functional form of $C_m$ is meant to account qualitatively for the
likelihood that the drop size spectrum broadens as cloud density increases. This
widening of the spectrum makes the processes of coalescence and accretion by rain more
efficient, whereby the conversion time $C_m^{-1}$ becomes gradually shorter and may eventually
reach the limiting value $C_0^{-1}$. Small values of the ratio $m/m_r$ yield a conversion time that
is comparatively long, which in turn results in a cloud that is practically non-precipitating.
As $m/m_r$ approaches unity, a marked reduction of conversion time takes place and the
cloud becomes an efficient precipitator. We note that the parameters $C_0$ and $m_r$ and the
form given to $P$ (Eq. (13)) qualitatively function in the same way as the threshold and rate

The parameter $m_r$ should consequently be a representative value of the water content
at which a cloud typically comes into a well-developed precipitating state. Instead of the
mixing ratios $m$ and $m_r$ in the exponent in relation (13) it might seem equally natural to
use concentrations. This aspect will be discussed in section 3.

It appears that the range of typical values of the concentrations of liquid water in
precipitating clouds is about 0.35–0.65 g m$^{-3}$ (Mason 1971). Therefore the value
$m_r = 0.5 \times 10^{-3}$ (corresponding to 0.5 g m$^{-3}$ at 800–850 mb) has been used in the reference
experiment.

Regarding the value of $C_0$, we find from Mason that a few hours are required for
layer cloud droplets to grow before an appreciable rate of precipitation may be observed.
In the reference experiment the value $C_0 = 10^{-4}$ s$^{-1}$ has been adopted. This corresponds
to a conversion time of 2.8 hours. Additional values of $C_m^{-1}$ as a function of $m$ are given
in Table 1.

<table>
<thead>
<tr>
<th>TABLE 1. THE CONVERSION TIME, $C_m^{-1}$, IN HOURS FOR A FEW $m$ VALUES. $m_r = 0.5 \times 10^{-3}$; $C_0 = 10^{-4}$ s$^{-1}$</th>
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<tbody>
<tr>
<td></td>
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<tr>
<td>$m \times 10^3$</td>
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<tr>
<td>$C_m^{-1}$</td>
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The $m$ value of column ‘a’ is representative of a relatively dense fog; the other columns
show conversion times for $m$ values discussed in previous paragraphs.

We shall summarize the model system discussed in this section by reviewing the
sequence of steps to be considered at each timestep and each grid point when the model
is incorporated in a prediction model. First we investigate whether the stratification is
convectively unstable or stable. (If convectively unstable, then further steps should be taken in accordance with a model that treats the parameterization of convection.) If stable, then compare $U$ with the threshold value $U_0$. If $U \geq U_0$ we continue to the checking of $M$ and $m$. If now either $M > 0$ or $m > 0$ (there may be subsidence but the clouds not yet completely dissolved, $m > 0$) then calculations according to the condensation model should be carried out. These are:

(i) from values of the previous timestep, evaluate $M$ and Eq. (9) and then $Q$ from Eq. (5); the predicted value of $U$ is also obtained from Eq. (9);

(ii) predict $\theta$ and $q$ (Eqs. 2 and 3); predict $m$ from Eq. (10b) using $P$, $a$, and $E_r$ values of the previous step;

(iii) then evaluation of Eqs. (8), (13) and (12) completes the calculation of one timestep.

3. SOME GENERAL VIEWPOINTS REGARDING THE PARAMETERS OF THE MODEL

In this section we shall discuss qualitatively plausible relations of the parameters to the synoptic-scale dynamical model, and their impact on the result of the condensation model.

Before proceeding to the discussion, it seems appropriate to remark that in the present design the gross energy budget is independent of how the various parameters are chosen. These merely determine the partitioning of converged moist energy between change of relative humidity and release of latent heat and the distribution of the condensation product between cloud mass and precipitation. It is also important to remember that the spatial resolution of condensation processes is determined by the resolution employed in the dynamical model; the condensation model only gives average effects over the grid box. If the condensation that might occur on a sub-grid scale is required in a deterministic sense, then one has either to increase adequately the resolution in the model system, or to use a separate (detached) model that - from the information about the average condensation, given by the large-scale model - yields the distribution of clouds and precipitation within the grid box.

One of the most obvious factors that would influence the sub-grid-scale distribution of condensation is probably the shape of the underlying terrain. It seems that this aspect must be of importance when assigning values to $U_0$. If, for instance, the mesh size is such that it contains a mountain ridge, it is quite likely that condensation would appear essentially on the windward side, whilst the air would be relatively dry on the leeside. So to allow for this sub-grid-scale condensation to occur in a model with that grid size, it is necessary to set $U_0 < 1$. This is assuming that the wind direction is more or less perpendicular to the ridge line. Clearly the situation would be different if the wind blew parallel to the ridge; then the value of $U_0$ would presumably be closer to unity.

Furthermore, we may state that $U_0$ should generally tend to decrease as the grid size increases. The larger the grid square, the more likely it is that merely a fraction of it contains condensation.

In the above case it may well be that, even in a near-steady state, only part of the whole grid square would have condensation in the real case. We may therefore speculate whether $a$ should then have a minimum value $a > 0$. Allowing for possibilities of this type would, however, require certain modifications of the governing equations for energy conservation. Quantity $a$ is a variable that enables us to let the condensation appear while $U$ still is less than unity and still changing in time, by and large in pace with the synoptic changes. The partial cloud cover will hence be reflected by the value of $a$ at any time, but experiments with a three-dimensional model have to be performed in order to assess
the confidence with which that value can be utilized as a precise prognostic quantity.

The parameters $C_0$ and $m_*$ and hence the conversion time $C_m^{-1}$ (Eq. (13)) could conceivably be given a more elaborate form based on the following reasoning. At low temperatures, say at high altitude or in high latitudes in winter, ice particles are more likely to be present in the condensate. These would tend to shorten the conversion time, and consequently speed up the transition from a non-precipitating to a precipitating stage. Furthermore, this transition would occur at a relatively low density of the cloud. Hence a plausible relation to temperature would be that in an interval – 265 to 255 K, say – both $C_0^{-1}$ and $m_*$ change from one value, valid on the warm side, to another and smaller value, valid on the cold side of the interval.

The rate of rain formation in a cloud layer may be enhanced considerably as a result of accretion of cloud water by rain falling through this layer from higher cloud. This effect could conceivably also be accounted for within the present frame of parameterization by letting $C_m(p)$ be an increasing function of $P(p)$.

Still another aspect, which is related to the one in the preceding paragraph, is mentioned for completeness. The thickness of a cloud layer also influences the rate of precipitation formation. Empirical studies show that clouds hardly precipitate if they are thinner than 500 m and that they have to be 2–3 km thick to yield continuous and intense rain (Mason 1971). The way of including these effects will be dependent on a model's vertical resolution and/or possible parameterization of vertical sub-grid-scale condensation.

It was remarked in section 2(b) that the mixing ratios, $m$ and $m_*$, are used in the exponent in relation (13). Instead of $m_*$ we could consider a fixed concentration as the parameter. This alternative exponent would decrease with height as the square of air density. By putting $P = Q/L$ in Eq. (10b), we find that the alternative form would yield a relatively higher steady-state cloud water concentration at high altitudes. A decisive comparison has to await until data on high-altitude concentrations become available and/or until the present model is tested in a three-dimension dynamical model.

4. Numerical results

The model equations derived in section 2 have been integrated numerically for a few illustrative cases, with various values of some of the parameters. Only temporal and vertical variations are considered. The vertical domain between 100 and 1000 mb contains 10 levels ($\Delta p = 100$ mb). A straightforward upstream differencing (first-order approximation in time and space) is adopted.

What is considered to be the reference experiment has the following characteristics:
The prescribed $p$ velocity has an approximately parabolic shape with zero at the top and the bottom and maximum upward velocity of about 5 cm s$^{-1}$, $W_{\text{max}} = 5$ (see Fig. 1).
The prescribed temperature profile follows a moist adiabat, with a vertical mean that is about the same as for the standard atmosphere.

From the prescribed $p$ velocity and temperature, the term for steady-state release of latent heat (Eq. (5) with $U = U_0$) was calculated and is shown in Fig. 1. The parameter values are $U_0 = 0.85$, $C_0 = 10^{-4}$ s$^{-1}$, $m_* = 0.5 \times 10^{-3}$.

At initial time $U = U_0$, and $m = 0$ at all levels; $m$ is set $= 0$ at the top and the bottom levels at all times. The timestep is 30 minutes.

The two prognostic variables of this specific model are thus $U$ and $m$ (Eqs. (9) and (10b)). The other quantities $a$, $E$, and $P$ are diagnostically calculated from Eqs. (8), (12) and (13). In addition to these model variables, the rate of precipitation at the ground, obtained from Eq. (5) with $\partial U/\partial t = 0$ and no cloud water storage permitted, is computed and denoted by $\bar{P}_d$. Therefore $\bar{P}_d$ gives the maximum possible rate of precipitation, with
Figure 1. Prescribed vertical velocity in cm s$^{-1}$ (---) and prescribed maximum release of latent heat in K day$^{-1}$ (-----).

which it is interesting to compare the corresponding quantity $\bar{P}$ obtained in the cloud model integration.

The integration is carried out to 48 hours by which time a steady state is nearly reached. The accumulated precipitation was then 26·2 mm, while accumulation of $\bar{P}_d$ has yielded 34·8 mm. The evolutions of $\bar{P}$ and $\bar{P}_d$ during the first 18 hours are displayed in Fig. 2. We notice that $\bar{P}_d$ slowly increases because the relative humidity increases. For the case with cloud mass included it is seen that $\bar{P}$ is small during a build-up period of about 5 hours. Then $\bar{P}$ increases fairly rapidly and after another 5 hours it is about 70% of the maximum possible rate as depicted by the ratio $\bar{P}/\bar{P}_d$.

Fig. 3 shows the vertical distribution of cloud mass at 6, 12 and 48 hours. This quantity also exhibits the relatively rapid change that takes place between 6 and 12 hours. We furthermore notice that there is a tendency for a lowering of the altitude of maximum concentration.

Figure 2. Reference case: predicted and diagnostically calculated rates of precipitation (mm day$^{-1}$) and their ratio (%) as functions of time. $A = \bar{P}_d$, $B = \bar{P}$, $C = \bar{P}/\bar{P}_d$. 
Figure 3. Reference case: cloud water (g m\(^{-3}\)) as a function of height at 6 hours (A), 12 hours (B), 48 hours (C).

If vertical advection of \(m\) had been included in Eq. (10b) we could expect a somewhat smoother distribution and the maximum concentration at a higher elevation than exhibited in Fig. 3. A rough numerical estimate indicates, however, that no substantial change of the shape of the curves in Fig. 3 would result.

5. Model response

(a) Response to various external forcings

In order to get an indication of how the evolution and the steady-state concentration of cloud water respond to other intensities of large-scale moisture convergence, the prescribed vertical velocity has been changed in two cases. The vertical profiles are the same as in the reference case. In experiment EW1 the velocity is doubled (\(W_{\text{max}} = 10\) cm s\(^{-1}\)) and in EW2 the velocity is reduced to 1/10 (\(W_{\text{max}} = 0.5\) cm s\(^{-1}\)). In EW2, the integration had to be carried out to 96 hours before a reasonably steady state was reached.

Fig. 4 shows the steady-state concentrations of EW1 and EW2 together with the reference case. We notice that there is no one-to-one correspondence between changes in vertical velocity and cloud water content. In absolute terms, the differences are most conspicuous at and below the level of maximum forcing. If the mixing ratio, \(m\), were plotted instead, we would find that the stronger the forcing, the closer is the resemblance between the vertical shapes of the forcing and the mixing ratio, at least for large \(m\). This is a consequence of the assumed relation between \(m\) and \(P\) (Eq. (13)). For a sufficiently large \(m\) (or rather \(m/m_p\)), the steady-state value of \(m\) is practically directly proportional to the forcing.

There is no drastic qualitative difference between the results of the reference case and EW1. Naturally, in the steady state, \(\bar{P}\) is doubled and so, approximately, is the accumulated precipitation. The ratio \(\bar{P}/\bar{P}_d\) increases somewhat faster in EW1 than in the reference case, but the difference is not remarkable. Hence in EW1 the ratio reaches 70%.
in about 7 hours. There is, of course, a clear difference in absolute terms. The rate of precipitation at 5 hours in EW1 is about the same as the steady-state value of $\bar{P}$ in the reference case.

In EW2 the temporal variation of the ratio $\bar{P}/\bar{P}_e$ is as depicted in Fig. 5 (note that the time scale is different from that in Fig. 3). The ratio has just reached the 10% level at about 24 hours, and it is not until 66 hours that the ratio reaches 70%. The same feature also appears when other results of EW2 and the reference case are examined. Namely, the cloud water concentration at 48 hours is about 1/3 of that in the reference case, while the ratio between vertical velocities is 1/10. The ratio of accumulated precipitation at 48 hours is 1/30. Thus with a weak vertical velocity, the model essentially produces only cloud, amounts of precipitation being almost negligible.

To obtain an idea of how the present model describes layers of clouds, one integration has been performed where the initial relative humidity is 50 and 65% at some of the pressure levels, shown in column $t = 0$ in Fig. 6. All other conditions are the same as in the reference case.

Figure 4. Steady-state distribution of cloud water (g m$^{-3}$) in the reference case (B), in the case $W_{\max} = 0.5$ cm s$^{-1}$ (A), in the case $W_{\max} = 10$ cm s$^{-1}$ (C).

Figure 5. Temporal variation of $\bar{P}/\bar{P}_e$ in the case $W_{\max} = 0.5$ cm s$^{-1}$.
The vertical distribution of cloud mass at 48 hours is displayed in Fig. 6. We notice that the densities of the two cloud layers are not significantly different from the densities at corresponding levels in the reference case (Fig. 3, curve C). In comparison with the reference case, the accumulated precipitation is reduced more than might be supposed from the cloud mass. This is because part of the precipitation evaporates in the cloud-free dry layers, and raises the humidity there. This effect may be seen by comparing the relative humidities at 0 hour and 48 hours in Fig. 6. The accumulated amount of rain at 48 hours is 7-2 mm, which is 27% of the reference case value. The accumulated value obtained from $P_d$ is 12-3 mm which is 35% of the corresponding value in the reference run. To some extent the result of this case resembles that of the case with weak vertical motion (EW2): well-developed clouds exist, but relatively little precipitation reaches the ground.

Figure 6. Case with layers: vertical distribution of cloud water (g m$^{-3}$) at 48 hours. Columns under $U$ show variation of relative humidity with height at times indicated.

One experiment has been performed in which the appearance of subsidence is simulated once cloud and precipitation have developed. Starting at 15 hours of the reference case, the vertical velocity is prescribed to change linearly with time such that the velocity profile acquires a reversed sign and the maximum downward motion is 0.5 cm s$^{-1}$ at 24 hours. From then on, subsidence is fixed. The resulting rate of precipitation shows decaying intensity, the characteristic time being about the same as that of the build-up stage, i.e. of order 8-10 hours. Comparing the strength of the forcing of the two stages (5 cm s$^{-1}$ upwards versus 0.5 cm s$^{-1}$ downwards) the decay time must be regarded as relatively short. That this is indeed reasonable, we may realize by looking at the terms $Q/L$ and $P$ of Eq. (10b).

During the build-up stage those two terms have opposite signs and thus counteract, whilst during the subsiding stage they are both negative and hence cooperate in reducing the amount of cloud water. The decay time might still seem to be unrealistically long. However, it must be kept in mind that typical effects of subsidence, such as a temperature rise and subsequent relative humidity reduction, were not imposed on the large-scale parameters. A more elaborate test has to wait until the condensation model is included in a dynamical prediction model.
In the preceding experiments we have looked at the response of the condensation model to various external forcings. The results produced by the model under different simulated synoptic situations appear to be rather realistic with respect to the existence of clouds and rate of precipitation.

(b) Response to chosen parameters

It is naturally important to get an idea of how sensitive the resulting distributions of cloud mass and precipitation are to changes in the parameters $U_0$, $C_0$ and $m_r$. A few experiments for this purpose have therefore been performed.

To test perfectly the influence of $U_0$, this condensation model has to be implemented in a large-scale dynamical model. Nevertheless, one experiment has been run which differs from the reference case in that $U_0 = 0.75$. As may be expected, it now takes longer to reach the maximum possible rate of precipitation. The ratio $P/P_a$ is about 70% after 12 hours; the corresponding time in the reference case was 10 hours. The steady-state distribution of cloud mass is, of course, the same as in the reference case.

Two experiments were run in which $C_0$ had different values ($0.5 \times 10^{-4}$ and $2 \times 10^{-4} \text{s}^{-1}$) from that employed in the reference case ($10^{-4} \text{s}^{-1}$). The resulting vertical distributions of cloud mass at 48 hours are depicted, together with the reference case, in Fig. 7. We find that changes in the value of $C_0$ have a noticeable effect on the amount of cloud water. However, the percentage change in $m$ is not as large as the percentage change in $C_0$. This is a consequence of the functional form of $C_m$. The characteristic response to changes in $C_0$ appears to be that, as $C_0$ increases, the vertical variation of $m$ becomes less pronounced and the level of maximum concentration lowers somewhat.

In two other integrations, the parameter $m_r$ was changed by a factor of two from the reference value. The resulting cloud water content for the three values of $m_r$ are shown in Fig. 8. We notice that a change in $m_r$ produces about the same change in cloud mass as does a similar change in $C_0$.

The following common features of the results of the four experiments are noted.

![Figure 7. Steady-state distribution of cloud water (g m$^{-3}$) for various values of $C_0$.](image)
As seen from Figs. 7 and 8, it seems to be easier to cause an overestimation of cloud water by choosing $C_0$ too small or $m_r$ too large than to cause an underestimation by setting $C_0$ too large or $m_r$ too small. Furthermore, there are naturally clear differences between the experiments during the first 6 to 12 hours regarding the evolution of clouds and precipitation. The latter is very small during that period, however. The considerably higher and approximately equal rates that appear in the subsequent development in all cases therefore rapidly make the difference in the accumulated amounts small. Hence at 20 hours, deviations from the reference value have already been reduced to 10–20% (in absolute terms less than about 1.5 mm). The precise values of $C_0$ and $m_r$ are thus not too critical for the accumulated precipitation, while the resulting concentration of cloud water shows a significant dependence on these parameters.

6. **Concluding Remarks**

The condensation model developed here appears to yield realistic quantitative results. It seems to be worth the effort to include this model in a large-scale dynamical model for further investigation. This is necessary to obtain a more comprehensive insight into the merits and weaknesses of the present parameterization. For example, we concluded in section 5(b) that a full dynamical system is required for an adequate test of the parameter $U_0$ and of the effect of subsidence on existing clouds. The parameterized relation between cloud mass and precipitation has to be tested further to find optimum values for $C_0$ and $m_r$ or to see if other relations have to be employed, e.g. the ratio of concentrations instead of $m/m_r$ in Eq. (13). It is worth while pointing out that a 30-minute timestep has been used in the integrations. This may be defended both with regard to numerical stability and to truncation. As may be realized from the value of $C_0$ and from the results shown above, the typical timescale that is involved is of the order of several hours. The same is true for the convergence of water vapour (i.e. the quantity $M$ in section 2) since it is governed by synoptic-scale motion. The inclusion of this parameterization of condensation processes in a dynamical model would of course require some additional storage space in a computer, but the increase in computation time would be quite moderate.
While this paper was being reviewed the author had an opportunity to make one 36 h prediction on real data with a complete prediction model including the present condensation model. The results of this preliminary test are indeed encouraging.

ACKNOWLEDGMENT

The present study was begun during the author's stay at the Dynamic Prediction Research Division of the Atmospheric Environment Service, Canada, under contract No. OSV3-0262.

REFERENCES

Arakawa, A. 1975 Modelling clouds and cloud processes for use in climate models, GARP Publications Series, No. 16.

Davies, David and Olson, M. P. 1973 Precipitation forecasts at the Canadian Meteorological Centre, Tellus, 25, 43–57.


Mason, B. J. 1971 The physics of clouds, Oxford University Press (chiefly chapters 3 and 6).

