NOTES AND CORRESPONDENCE

REFERENCES


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COMMENTS ON THE PAPER 'THE AERODYNAMICS OF OBLATE HAILSTONES'
BY S. THWAITES, J. N. CARRAS AND W. C. MACKLIN (Q.J., 103, 803–808)

By E. P. LOZOWSKI

Figure 3 in the paper by Thwaites, Carras and Macklin (1977) raises an important question, which the authors have apparently not addressed in their paper. This concerns whether the growth is controlled principally by the geometry of the spheroid orientation as a function of time (relative to a droplet stream which may be envisaged to be arriving from a fixed direction), or whether the complications of a turbulent, non-steady airflow, complex droplet trajectories, wake capture, lobe formation, surface water flow, splashing, and spin-off play a dominant role in the determination of the overall growth shape. This is a significant question for the physics of the growth mechanism, since the geometrical orientation effects are readily modelled, while the other phenomena (which are generally hidden under the disguise of a "collection efficiency") are too poorly understood at present to yield readily to a theoretical treatment. The purpose of this note is to try to address this question briefly, and to demonstrate that the relationship between the axis ratio and the gyration angle may be understood to be largely determined by simple geometrical factors. In so doing, the relative significance of the other effects may be put into perspective.

First, an apparent discrepancy between Figs. 2 and 3 must be pointed out. The authors state that the axis ratio, \( \alpha \), was determined from cross-section photographs such as those in Fig. 2 which were obtained in ‘dry growth’. Clearly, there is some measure of subjectivity involved in deciding precisely where to measure the axes in Fig. 2. Moreover, the printed photographs are probably not as suitable for the purpose as were the originals. Nevertheless, with an intrepid disregard for such reservations, the present author has measured the cross-sections of Fig. 2 to obtain the following values of \( \alpha \) as a function of \( \theta \) (in parenthesis): 0.10(5°), 0.37(15°), 0.59(25°), 0.73(35°), 1.45(80°). While it is not possible to claim second-digit accuracy, there appears to be a non-trivial disparity between these values at 5°, 15° and 80° and the corresponding dry growth points of Thwaites et al. in Fig. 3. Some resolution of this anomaly is desirable.

If all droplets arrive from the same direction, the mass of ice accreted in dry growth on a particular surface element will be proportional to the cross-sectional area of that element perpendicular to the flow. Further, when the ice mass is spread uniformly over the surface, the ice thickness will be proportional to the fractional cross-sectional area of the element. On this basis, it is possible to predict that for a gyrating particle, the ice thickness on any surface element will be proportional to the mean fractional windward-facing cross-section of the element during
the growth. Using this hypothesis, a relation between $\alpha$ and $\theta$ can be developed in a straightforward manner.

Relative to Fig. 1 of Thwaites et al., a space-fixed right-handed orthogonal coordinate system with unit vectors $i$, $j$, $k$, is defined with the origin at the centre of the spherical embryo, $i$ pointing to the left, $k$ pointing up, and $j$ pointing out of the page. Initially the spin axis lies in the ik plane, and a secondary axis, also in the ik plane, lies $90^\circ$ to its right. If the motors are started with the two axes in this orientation, their subsequent orientation at time $t$ is given by the vectors:

$$\text{spin axis: } \cos \theta i - \sin \theta \sin \omega_1 t j + \sin \theta \cos \omega_1 t k$$

$$\text{secondary axis: } - \cos \omega_2 t \sin \theta i - (\cos \omega_1 t \sin \omega_1 t \cos \theta + \sin \omega_1 t \cos \omega_1 t) j +$$

$$+ (\cos \omega_1 t \cos \omega_2 t \cos \theta - \sin \omega_1 t \sin \omega_2 t) k.$$  \hspace{1cm} (1)

Since these axes lie perpendicular to the surface (ignoring lobes), the fractional windward-facing cross-section at any time is the inner product of (1) and (2) with $-k$. According to the hypothesis, if growth is normal to the surface in the mean, the axis ratio $\alpha$ will equal the ratio of the average values of these two cross-sections, i.e.

$$\alpha = \langle \sin \theta \cos \omega_1 t \rangle / \langle \cos \omega_1 t \cos \omega_2 t \cos \theta - \sin \omega_1 t \sin \omega_2 t \rangle.$$  \hspace{1cm} (3)

where the symbol $\langle \rangle$ denotes a time average over the entire growth. When the surface has a leeward-facing component, no growth occurs (ignoring wake capture), and this effect must be taken into account in determining the average. The numerator can be easily averaged, as can the denominator under the assumption that $\omega_1 \gg \omega_2$. It is believed that this assumption will not seriously affect the results provided that the growth time $\gg 2\pi/|\omega_1 - \omega_2|$, and that $\omega_1$ and $\omega_2$ are not integral multiples. (For example, if $\omega_1 = -\omega_2$ and $\theta = 0^\circ$, the particle no longer rotates, and a symmetric ice accretion over the surface cannot occur.) After time averaging, the value of $\alpha$ is given by

$$\alpha = \sin \theta \sqrt{1 - \frac{1}{2} \sin^2 \theta - \frac{1^2 + 3^2}{2^2 + 3^2} \sin^4 \theta - \frac{1^2 + 3^2 + 5^2}{2^2 + 3^2 + 6^2} \sin^6 \theta - \ldots}.$$  \hspace{1cm} (4)

where the denominator is a series approximation to the complete elliptic integral of the second kind. Some values of $\alpha(\theta)$ computed from Eq. (4) are given in Table 1. For comparison, average values based on Fig. 3 of Thwaites et al. and values based on Fig. 2 (this author’s measurements) are also presented.

<table>
<thead>
<tr>
<th>$\theta$ (°)</th>
<th>Eq. (4)</th>
<th>Fig. 3</th>
<th>Fig. 2</th>
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<td>$0^\circ$</td>
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<tr>
<td>$90^\circ$</td>
<td>1.52</td>
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Table 1 suggests that the variation of $\alpha$ with $\theta$ is largely explainable on the basis of the simple geometrical considerations which are incorporated into the derivation of Eq. (4). The model values may be somewhat high near $90^\circ$, possibly due to centrifugal effects on the surface water. The most serious discrepancy occurs at low angles, where the growth of the minor axis is enhanced in practice because of the development of lobes, whose small dimensions and protrusion into the flow will enhance their collection efficiency. In view of the reasonable predictions of such a geometrically based accretion model, it is suggested that for the purpose of developing a heat
and mass transfer model for gyrating spheroids, the appropriate characteristic cross-sectional area to use would be the true average cross-section, rather than that containing the minor axis, as suggested by Thwaites et al.

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Reply

By S. Thwaites, J. N. Carras and W. C. Macklin

In his comments on our paper, Dr Lozowski raises three main points.

First, he questions whether the gyration angle is the main factor determining the shape of oblate hailstones or whether other effects (the turbulent airflow, complex droplet trajectories, wake capture, lobe formation, surface water flow, splashing, and spin-off of accreted water) also play an important role. We carried out experiments in both the dry and wet growth regimes and, as far as we could determine, there were no significant differences in the shapes of the gyrating hailstones in the two regimes (see Fig. 3 of our paper). The main effect of wet growth is to cause the lobe structure to be less pronounced, presumably due to the flow of liquid water over the surface. Consequently, our experiments show that at least some of the other possible effects mentioned by Dr Lozowski are of secondary importance.

Second, Dr Lozowski queries some of the measurements of the axis ratios displayed in Fig. 3 of our paper. As we point out, during the growth of the artificial hailstones a few layers of opaque ice were deposited in each artificial hailstone by temporarily reducing the liquid water concentration. These layers served to outline the shape and structure of successive stages in the growth and we used the eccentricity of these layers, rather than the overall shape, to determine the axis ratios. This explains the differences between our measurements and those made by Dr Lozowski on the photographs in Fig. 2. The differences are most marked at the smallest gyration angles because there was little actual growth in the minor axis direction due to the shielding effect of the more pronounced growth in the major axis direction.

Third, Dr Lozowski has calculated the expected axis ratios of hailstones for different gyration angles using a simple geometrical model. Except at low angles his results are in general agreement with ours and a possible reason for the discrepancy at low angles has already been given. However, we consider that his approach is over-simplified because it does not take into account the effects of the lobe structure. Whether his approach to the problem or whether the one outlined in our paper will be the more useful will have to be determined by further experiments designed to measure the collection efficiencies and the heat and drag coefficients of gyrating artificial hailstones under accretion conditions.

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