Three-dimensional lee-wave pattern

By B. GJEVIK and T. MARTHINSEN

Institute of Mathematics, University of Oslo, Norway

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SUMMARY

Satellite photographs are analysed to study the lee-wave patterns generated by isolated islands in the Norwegian Sea and the Barents Sea.

In situations where the wave motion is confined to the lower atmosphere (trapped waves) the waves are located within a wedge-shaped wake behind the islands. Both the diverging wave type, where the crests are orientated outwards from the centre of the wake, and the transverse wave type, where the crests are nearly perpendicular to the wind direction, are observed. The former wave type is, however, the more common.

In certain situations a long single-crested wave is observed at Jan Mayen. The wave appears on photographs as a straight line in the cloud layer. In one case it extends sideways from the island to a distance of about 350 km. Wave kinematics is used to obtain the phase lines for a steady wave pattern for different atmospheric models. The theory is found to explain some important features of observed wave forms.

1. INTRODUCTION

A vast literature is available on atmospheric lee waves. Most is concerned with waves generated by mountain ridges where the motion of an individual air particle is essentially confined to a vertical plane perpendicular to the ridge. The three-dimensional wave motion which occurs when waves are generated by isolated mountain peaks has received considerably less attention. A general mathematical treatment of this problem is very involved: the few works we know of are Kochin (1938), Scorer and Wilkinson (1956), Wurtele (1957), Palm (1958), Crapper (1962), and Blumen and McGregor (1976). These are mainly theoretical studies of the wave motion for rather special atmospheric situations, and the results are not compared with observations. The works by Kochin, Scorer and Wilkinson, and Crapper show that trapped waves are confined to a wedge-shaped region behind the wave source and that the horizontal structure of the wave pattern is similar to the well-known ship-wave pattern. The waves are of two types: transverse waves with the wave crest nearly perpendicular to the wind direction; and diverging waves which radiate out from the wake. Scorer and Wilkinson remark that the latter wave type is unlikely to be observed in the atmosphere.

We have studied satellite photographs of lee waves generated by isolated islands in the Norwegian Sea and the Barents Sea and the observational results are presented in section 2. Contrary to Scorer's statement, the diverging wave type is found to be rather common on these photographs. Since the existing theoretical results refer to atmospheric models which are not applicable in our case, we have used wave kinematics to obtain the phase lines of the waves for more realistic atmospheric models (section 3). We have found that theoretical results which are based on linearized wave theory explain many features of the observed wave pattern. There are, however, discrepancies between theory and observation and the theory is unable to explain the single-crested wave which is observed at Jan Mayen in certain situations. This wave form has to our knowledge not previously been described in the literature.

From a reviewer of this paper we learned that a recent publication by Scorer (1978) contains some satellite pictures of three-dimensional lee-wave patterns together with a general discussion of the topic. A rectified version of Fig. 2(a) is included in Professor Scorer's book, which also shows two pictures of vortex streets at Jan Mayen.
2. OBSERVATIONAL RESULTS

Studies of satellite photographs show that three isolated islands in the Norwegian Sea and the Barents Sea often generate atmospheric lee waves. These islands are Jan Mayen (71.5°N 8.5°W), Bear Island (74.4°N 19.0°E) and Hopen (76.6°N 25.3°E). Waves are particularly frequent in the wake of Jan Mayen. This island is also known to generate vortex streets, and vortex shedding at Jan Mayen has been studied by Wehner (1949), Mohr (1971) and Fjellheim (1973). Other references to works on atmospheric vortex streets are given in the review article by Berger and Wille (1972).

Before proceeding, we give a short description of the topography of the three islands.

In the eastern part of Jan Mayen a volcanic mountain, Beerenberg, rises to a height of 2277 m. The mountain is nearly conical with a diameter at the foot of about 15 km. The western part of the island also is mountainous with the highest top at 769 m. The terrain between the mountains in the east and in the west is low.

There is a low plateau at about 30-40 m in the northwestern part of Bear Island. The southeastern part of the island is mountainous with three main peaks of 536, 440 and 360 m.

Hopen island consists of a long narrow ridge with tops between 250 and 370 m.

![Map of Jan Mayen, Bear Island and Hopen showing the main topographical features.](image)

Figure 1. Maps of Jan Mayen, Bear Island and Hopen showing the main topographical features.

In Fig. 1 the main topographical features of the three islands are shown. On all three islands the Norwegian Meteorological Institute operates meteorological stations. From Hopen only surface observations are available, but from Bear Island and Jan Mayen there are also regular upper air soundings.

We have examined VHRR photographs in the visible and infrared band, taken by the satellites NOAA 4 and NOAA 5 in the period August 1976 to September 1977. Unfortunately, the photographs available to us are geometrically distorted, and data read from the photographs are corrected for scale distortion; the degree of distortion can be assessed by comparison between Fig. 2(a) and Fig. 5.15.v in Scorer (1978). We found lee waves at Jan Mayen on 7 days, at Bear Island on 2 days, and at Hopen on 1 day. Vortex shedding at Jan Mayen occurred on 11 days.

Here we choose to present four cases with particularly well-developed lee-wave patterns. We believe that these cases demonstrate typical and general features of the wave motion and we give an analysis of these cases.

(a) Case 1: Jan Mayen, 1 Sept. 1976, about 1117 GMT

A photograph of the Jan Mayen region with a remarkable lee-wave pattern is shown in Fig. 2(a). The waves are revealed by a cloud layer, and the waves are clearly generated
Figure 2. Section of a VHRR photograph taken by NOAA 5. The scale indicates the flight direction of the satellite. The scale perpendicular to the flight direction is diminished by a factor 0.72. (a) Jan Mayen 1 Sept. 1976, 1117 GMT (visible band). On the left, Scoresby Sound, east Greenland. (b) Jan Mayen 29 Dec. 1976, 1122 GMT (infrared band). (c) Jan Mayen 8 Oct. 1976, 1314 GMT (infrared band). (d) Spitzbergen 19 Sept. 1976, 1137 GMT (visible band).

by the island's influence on the air flow. The snow-covered top of Beerenberg appears on the photograph as a white spot at the upstream front of the wave train.

Figure 3(a) shows a display of upper air data recorded by a radiosonde at nearly the same time as the photograph. These observations show that a temperature inversion extends from about 1400 to about 2600 m and that the wind direction in the lower part of the atmosphere is nearly constant. The surface report indicates cumulus and cumulonimbus clouds with base at 300–600 m. The radiosonde shows that the air above 1800 m is very
Figure 3. Upper air data: temperature (solid lines), dew point (thick dashed lines), wind (direction and speed, in knots), and stability (N). In the temperature diagram (left) wet adiabats are dash-dot lines and dry adiabats thin dashed lines. (a) Jan Mayen 1 Sept. 1976, 1115–1220 GMT. (b) Jan Mayen 29 Dec. 1976, 1115–1153 GMT. (c) Jan Mayen 8 Oct. 1976, 1115–1227 GMT. (d) Bear Island 19 Sept. 1976, 1115–1234 GMT.

dry, and that the cloud layer appearing on the photograph is formed by horizontal spreading at the inversion.

A striking feature of the wave pattern is its lack of symmetry; this indicates that the terrain on the island has an important effect on the wave pattern. The line formed by the converging wave crests at the centre of the wake corresponds closely to the wind direction above the cloud layer. We will therefore consider this line, which extends southwards from the top of Beerenberg, as the reference axis of the wake.

Close inspection of the photograph reveals that an abrupt change in the wave phase takes place along a straight line west of the central axis. This indicates that the wave pattern west of the reference axis consists of two wave trains of different origin. The western wave train is probably generated by the mountains on the western part of Jan
Mayen while the wave train diverging from the reference axis is generated by Beerenberg. The latter wave train is confined within a sector of about $17^\circ$. We measured the wavelength at the front of the western wave train to be about 9 km and found the wave crests to make an angle of about $30^\circ$ with the central axis. Assuming stationary wave crests, this leads to a phase velocity of about $18 \text{ m s}^{-1}$.

Within a sector of about $24^\circ$ east of the reference axis and close to the island, there is a train of diverging waves. These waves are clearly generated by Beerenberg and they seem to be similar to the waves generated on the western side of the mountain.

A straight dark lane in the cloud layer radiates from the eastern slope of Beerenberg out to about 350 km from the island. The clearance in the cloud layer is probably caused by downward motion of the air. The width of the lane is about 14 km and it makes an angle of about $52^\circ$ with the wind direction. If the lane is stationary with respect to the ground, the propagation velocity relative to the air flow is about $27 \text{ m s}^{-1}$. Linear wave theory (see section 4) is unable to explain this high propagation velocity or the fact that the lane appears to propagate with nearly unchanged form.

(b) Case II: Jan Mayen, 29 Dec. 1976, 1122 GMT

Also on this photograph (Fig. 2(b)) a straight clear lane in the cloud layer radiates out from the eastern side of Jan Mayen where the snow-covered top of Beerenberg can be seen as a weak white spot. The width of the lane is about 14 km and it makes an angle of about $38^\circ$ with the wind direction. The propagation velocity (with respect to the air) is found, under the assumption that the lane remains stationary with respect to the island, to be about $11 \text{ m s}^{-1}$. We note that, although the width of the lane is the same in cases I and II, the propagation velocity is considerably lower in the latter case.

Upper air data recorded at nearly the same time as the photograph are displayed in Fig. 3(b). The wind direction varies little with height, and the wind is from the same direction as in case I. As in case I there is a low-level temperature inversion, but in this case it extends only from 840 to 1330 m above sea level. The clouds over Jan Mayen are reported to be cumulus and cumulonimbus, with base at 200-300 m. The air above 840 m is very dry and it is unlikely that the tops of the clouds penetrate the inversion layer. Cloud bands and cloud cells associated with convection currents in the air, caused by heating of the air over the sea, are seen on the photograph.

A high cloud (white on the photograph) is formed over the southwestern part of the island. It is an indication of strong vertical motion over the mountains in this part of the island. The wavy cloud band downwind from Beerenberg indicates that the wake is unstable and that atmospheric conditions are close to those which lead to vortex shedding. In other situations where a shallow stable layer intersects Beerenberg in a similar way, vortex shedding is frequent (Fjellheim 1973). The series of satellite photographs which we have examined confirm this. In none of these cases are lee waves found to be excited.

(c) Case III: Jan Mayen, 8 Oct. 1976, 1314 GMT

In this case the wind is from the east and there is an isothermal layer between 1720 and 2600 m above sea level (Fig. 3(c)). The cloud base (patches of stratus) is reported to be at 100-200 m, with a layer of altocumulus above. Above 1720 m the air is dry and we therefore assume that this level marks the top of the cloud layer. The photograph (Fig. 2(c)) shows that these clouds are formed at the crests of the waves, and that a wake is formed by higher clouds (white on the photograph). The wave crests on the southern side of the wake extend further outwards from the wake than the wave crests on the northern side. This
may be an effect of the mountains on the southwestern part of Jan Mayen. Close inspection of the photograph also reveals a slight distortion of the wave crests. The waves on the northern side of the wake are confined within a sector of about 12°.

(d) Case IV: Bear Island and Hopen, 19 Sept. 1976, 1137 GMT

The satellite photograph for the Spitzbergen region (Fig. 2(d)) shows two lee-wave patterns southeast of Spitzbergen. The more northerly of these patterns is generated by Hopen and the more southerly by Bear Island. Upper air data from Bear Island recorded at about the same time as the photograph are presented in Fig. 3(d). A strong temperature inversion extends from about 700 to about 1500 m. Fog is observed at the ground and the air is saturated up to about 800 m, above which the air is dry. The waves are therefore probably made visible by a cloud layer at about 800 m. The wind direction is from WNW and varies little with height.

The wave pattern at Bear Island consists of straight-crested waves, the crests making an angle of about 65° with the wind direction. The wave crest on the northern side of the centre of the wake extends farther out than that on the southern side, and the former waves are confined to a sector of about 21°. This asymmetry in the wave pattern may be due to the shape and location of the three mountain tops on Bear Island. The average wave length measured along the centre of the wake is about 8·5 km.

The waves generated at Hopen are of the transverse type with wave crests almost perpendicular to the wind direction. The waves are confined within a sector of about 49° and the average wavelength along the central axis of the wave train is about 9·5 km. There are no upper air data from Hopen. The temperature variation with height may be nearly the same as at Bear Island. The weather maps indicate, however, that the wind speed at Hopen is probably lower than at Bear Island.

The four cases examined show typical, although remarkably well-developed, lee-wave patterns. We note that the waves generated at Jan Mayen are of the diverging type and that waves of the transverse type are observed only at Hopen. The wave pattern generated at Bear Island is difficult to classify. Other photographs show that the topography of this island seems to favour a wave pattern of this special form.

3. The lee-wave pattern from a point source. Wave kinematics

Whitham (1974) surveys the theory and gives some illustrative examples on how wave patterns can be obtained by kinematic arguments. We will use the same technique to obtain the form of the lee-wave pattern for atmospheric models. We describe the motion in a fixed Cartesian coordinate system. Axes $x_1$ and $x_2$ are horizontal, $x_1$ being along the direction of the wind. The wave field is assumed to be steady and the wave parameters almost periodic functions of $x_1$ and $x_2$. We denote the phase function by $\psi(x)$ and define a local wavenumber vector by

$$ k = \nabla \psi \quad . \quad . \quad . \quad . \quad . \quad (3.1) $$

The vector $k$ is therefore normal to curves of constant phase. The dispersion relation for linear waves is a functional relation between $k$ and the physical parameters defining the state of the atmosphere. We write the functional relation in the form

$$ D(k, U, N, H) = 0 \quad . \quad . \quad . \quad . \quad (3.2) $$

where $U, N, H$ denote respectively the wind velocity, the Väisälä–Brunt frequency, the thickness of the layers in the atmosphere (see appendix) With the definition (3.1), Eq. (3.2)
becomes a differential equation for $\psi$. This equation can be written in characteristic form

$$
\frac{dx}{d\lambda} = \frac{\partial D}{\partial k}; \quad \frac{dk}{d\lambda} = -\frac{\partial D}{\partial x}; \quad \frac{d\psi}{d\lambda} = k \frac{\partial D}{\partial k} \quad . \quad (3.3)
$$

This set of equations determines the characteristics $x = x(\lambda)$, the phase function $\psi$, and the wavenumber vector $k$ in the horizontal plane. With horizontally uniform layers it follows that $k$ is constant on the characteristics and for a point source the characteristics pass through this point. With the point source as origin, it follows from Eq. (3.3) that

$$
\frac{x_2}{x_1} = \frac{\partial D}{\partial k_2} \frac{\partial k_1}{\partial D} \quad . \quad (3.4)
$$

and

$$
\psi = k \cdot x \quad . \quad (3.5)
$$

Eqs. (3.4)-(3.5) determine the phase lines.

The discontinuous model leads to a relatively simple expression for $D$ (Eq. (A1)), and equations for the lines of constant phase can be obtained easily from Eqs. (3.4) and (3.5). We find

$$
x_1/H_0 = p F^2 \cos^3 \phi (1 - F^2 \cos^2 \phi + 2 \tan^2 \phi)(1 - F^2 \cos^2 \phi)^{-2}
$$

$$
x_2/x_1 = -\tan \phi (1 + F^2 \cos^2 \phi)(1 - F^2 \cos^2 \phi + 2 \tan^2 \phi)^{-1} \quad . \quad (3.6)
$$

where $p = 2\pi n U_1/U_0^2$, $(n = 1, 2 \ldots)$ and $-\frac{1}{2} \pi < \phi < \frac{1}{2} \pi$. $F$ is the Froude number. In the limit $H_0 \to \infty$, Eq. (3.6) reduces to the well-known equation for the phase lines for ship waves in deep water. For $F$ and $H_0$ finite, the phase lines obtained from Eq. (3.6) are similar to the phase lines for ship waves in shallow water. For $F < 1$ both transverse and diverging waves are possible. For $F > 1$ there will be only diverging waves. The waves are confined within a wedge-shaped region behind the wave source and the wedge angle varies in a similar way as for ship waves in shallow water. Subsequently we denote the half wedge angle by $\theta$. For $F = 0 (H_0 \to \infty)$, $\theta = 19.47^\circ$, for $F = 1$, $\theta = 90^\circ$ and for $F \to \infty$, $\theta \to 0$.

Although the discontinuous model reveals characteristic features of the wave pattern, the model is too simple for direct comparison with the observational result in section 2. We have therefore investigated the properties of different four-layer models. The dispersion relation for a four-layer model is given in Eq. (A2). This equation together with Eqs. (3.4) and (3.5) determine the phase lines, which are evaluated by numerical methods.

In Figs. 4(a) and (b) the computed values of $\theta$, as a function of the Score parameter, $\gamma$, for the most stable layer are depicted ($\gamma = N/U$, the ratio between the Viisälä-Brunt frequency and the wind speed within the layer). For these computations we have used layer thicknesses corresponding, respectively, to case III and case IV (see Table 1). In Figs. 4(a) and (b) these models are marked by dots.

The curves labelled A (full line, 1. mode; dotted line, 2. mode) correspond to a model with $\gamma_0 = \gamma_2 = \gamma_3 = 0$, i.e. a three-layer model where a stable layer is embedded between two neutrally stable layers. For a certain value, $\gamma_1 = \gamma_1^w$, say, the angle $\theta$ for the 1. mode approaches 90°. The diverging wave type is the only possible wave form for $\gamma_1 < \gamma_1^w$. For $\gamma_1 > \gamma_1^w$, both diverging and transverse waves are possible.

If the stability of the two upper layers is increased, the value of $\theta$ decreases. This effect is illustrated by the curves labelled B and C. The stable stratification of the upper layers allows long waves to propagate vertically and trapped waves are possible only for $\gamma_1$ above a certain value and for wavelengths shorter than a certain cutoff. For simplicity, we denote $\gamma_1 \times 10^3$ m by $\tilde{\gamma}_1$. In case B, Fig. 4(a), the diverging waves are the only possible wave form when $0.3 < \tilde{\gamma}_1 < 0.6$. For $0.6 < \tilde{\gamma}_1 < 1.1$ both diverging and transverse waves are possible. The transverse waves disappear in a sector which includes the centre of the
Figure 4. (a) The half wedge angle, $\theta$, as function of $\gamma_1$ for models with $U/U_0 = 0.8$, $H_0 = 1700 m$, $H_1 = 900 m$ and $H_2 = 5800 m$. A: $\gamma_0 = \gamma_2 = \gamma_3 = 0$; B: $\gamma_0 = 0, \gamma_2 = 0.2 \times 10^{-3} m^{-1}$, $\gamma_3 = 0.3 \times 10^{-3} m^{-1}$; C: $\gamma_0 = 0, \gamma_2 = 0.4 \times 10^{-3} m^{-1}$ and $\gamma_3 = 0.5 \times 10^{-3} m^{-1}$. Solid lines and dashed lines correspond, respectively, to the 1. mode and the 2. mode. Case III is marked by a dot. (b) The half wedge angle, $\theta$, as function of $\gamma_1$ for models with $U/U_0 = 1.0$ for models with $H_0 = 700 m$, $H_1 = 800 m$, $H_2 = 900 m$. A: $\gamma_0 = \gamma_2 = \gamma_3 = 0$; B: $\gamma_0 = 0, \gamma_2 = 0.2 \times 10^{-3} m^{-1}$, $\gamma_3 = 0.3 \times 10^{-3} m^{-1}$; C: $\gamma_0 = 0, \gamma_2 = 0.3 \times 10^{-3} m^{-1}$ and $\gamma_3 = 0.5 \times 10^{-3} m^{-1}$. Solid lines and dashed lines correspond, respectively, to the 1. mode and the 2. mode. Case IV is marked by a dot.

wake. For $\tilde{\gamma}_1 > 1.1$ there is no cutoff and both wave types are possible. Case C, Fig. 4(a), is similar. For $0.5 < \tilde{\gamma}_1 < 1.0$, only diverging waves are possible, while for $1.0 < \tilde{\gamma}_1 < 1.2$ both diverging and transverse waves are possible but the latter disappear near the centre of the wake. For $\tilde{\gamma}_1 > 1.25$, there is no cutoff.

In cases B and C, Fig. 4(b), diverging and transverse waves are possible for $\tilde{\gamma}_1 > 1.25$ and $> 1.45$, respectively. For $0.4 < \tilde{\gamma}_1 < 1.2$, and $0.7 < \tilde{\gamma}_1 < 1.4$, respectively, in case B and C, both types are possible and the transverse waves are cut off.

For the range of parameters treated in Figs. 4(a) and (b) the diverging wave type is the only possible wave form for the 2. mode. Graphs corresponding to this mode are shown dotted.

For other four-layer models we have found quantitatively similar results as summarized in Figs. 4(a) and (b).

### 4. COMPARISON BETWEEN THEORY AND OBSERVATION

It should be stressed that kinematic arguments give possible steady wave patterns due to a point source.

In order to determine uniquely which wave type is excited, the amplitudes of the waves have to be evaluated. This is a formidable problem which also requires a realistic representation of the topography of the wave source. Photographs of the wave patterns in cases I–IV indicate that at least in some of these situations the waves appear to be generated by a point source. We have therefore evaluated possible steady wave patterns for atmospheric models corresponding to cases I–IV by the methods of section 3.

Since the wave motion is confined to the lower part of the atmosphere, we have disregarded the strong stability of the stratosphere. If this effect had been included in the
models, the main effect would be a weak damping of the wave field in the horizontal direction (slightly leaky waves).

Values of the parameters for the four models are given in Table 1. The phase-line patterns corresponding to these four models are displayed in Figs. 5(a)–(d).

**Case I.** The computed wave pattern consists mainly of diverging waves (Fig. 5(a)) and the half wedge angle is 17.8°. Also the photograph (Fig. 2(a)) shows a well-developed train of diverging waves especially on the western side of the central axis of the wake. The part of the wave train generated by Beerengaerg seems to be confined within a wedge (see under case I, section 2) and the wedge angle and wavelength are in close agreement with the waves in Fig. 5(a). The part of the wave train generated by the mountains in the western part of the island is also confined within a wedge of similar width.

![Figure 5](image-url)

**Figure 5.** Phase lines for cases I–IV displayed, respectively, in (a)–(d). The phase differences between corresponding points on neighbouring lines are 2π.

Fig. 5(a) shows that all the linear wave components of this model propagate relatively slowly and the model is therefore unable to explain the existence of the single-crested wave east of Jan Mayen (Fig. 2(a)). The wave may therefore either be an internal solitary wave or an internal bore.

**Case II.** The steady wave pattern (Fig. 5(b)) consists mainly of diverging waves.

<table>
<thead>
<tr>
<th>Case</th>
<th>(U/U_0) (km)</th>
<th>(H_0) (km)</th>
<th>(H_1) (km)</th>
<th>(H_2) (km)</th>
<th>(\gamma_0 \times 10^3) (m(^{-1}))</th>
<th>(\gamma_1 \times 10^3) (m(^{-1}))</th>
<th>(\gamma_2 \times 10^3) (m(^{-1}))</th>
<th>(\gamma_3 \times 10^3) (m(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.1</td>
<td>0.9</td>
<td>1.7</td>
<td>0.3</td>
<td>0.2</td>
<td>0.5</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>II</td>
<td>0.8</td>
<td>0.8</td>
<td>0.5</td>
<td>0.3</td>
<td>1.5</td>
<td>0.6</td>
<td>0</td>
<td>0.8</td>
</tr>
<tr>
<td>III</td>
<td>0.8</td>
<td>1.7</td>
<td>0.9</td>
<td>5.8</td>
<td>0</td>
<td>0.9</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>IV</td>
<td>1.0</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
<td>1.5</td>
<td>0.3</td>
<td>0</td>
<td>0.55</td>
</tr>
</tbody>
</table>

**TABLE 1**
The half wedge angle is $15.3^\circ$ and the wavelength is shorter than 5 km. The diverging wave type is not observed in this case nor in other similar situations (see under case II, section 2). These results indicate that wave components shorter than 5 km, say, are unlikely to be excited when a relatively thin stable layer intersects Beerenberg well below the summit.

As in case I the model is unable to explain the existence of the single-crested wave which can be seen east of Jan Mayen (Fig. 2(b)).

Case III. In this case the steady pattern (Fig. 5(c)) consists only of diverging waves and the half wedge angle is $10.7^\circ$. This wave type is also observed (Fig. 2(c)). The computed half wedge angle agrees well with the observed angle for the wave train north of the wake. The wave train south of the wake extends over a wider sector but this is probably an effect of the mountains in the western part of the island (see under case III, section 2). We note that the observed wavelength at the outer edge of the wedge compares well with the waves in Fig. 5(c). The observed wavelength near the centre of the wave is, however, somewhat larger than for the waves in Fig. 5(c).

Case IV. In this case the computation leads to a wave pattern which consists both of diverging waves and of well-developed transverse waves (Fig. 5(d)). The half wedge angle is $21.7^\circ$ and the wavelength of the transverse waves is about 10 km. The wavelength of the diverging waves is shorter. The transverse wave train compares relatively well with the observed wave train at Hopen. The straight-crested wave train generated at Bear Island differs, however, from both the wave trains in Fig. 5(d). We note that the second mode for this model leads to straight-crested waves, but other features of the wave train for this mode are in disagreement with the observation. The waves observed at Bear Island may therefore be strongly affected by the topography of the island or by a reduced wind speed in its wake.

Acknowledgments

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References


Scorer, R. S. 1978 *Environmental aerodynamics*, Ellis Horwood, Chichester.

THREE-DIMENSIONAL LEE-WAVE PATTERN


APPENDIX

We assume that the three-dimensional wave motion is steady, adiabatic and frictionless. The governing equation for this motion is given by the authors referred to in the introduction, and need not be recapitulated.

Since the wind velocity, \( U \), changes relatively slowly with height in all the cases we examine, we assume that the Scorer parameter, \( \gamma = N/U \) (\( N \) being the Väisälä–Brunt frequency), is the only parameter needed to determine the motion. In the usual way we define layers with constant \( \gamma \) and the requirement of no vertical motion at the ground, \( x_3 = 0 \), and for \( x_3 \to \infty \), defines a well-known eigenvalue problem which in turn leads to the dispersion relations given below.

Subsequently we denote the length of the horizontal wavenumber vector by \( k \) and we introduce an angle \( \phi \) defined by \( \cos \phi = -k_3/k \).

(i) **Discontinuous model:** a neutrally stable atmosphere above and below an infinitely thin inversion.

We set \( N = 0 \), \( U = U_0 \) for \( 0 < x_3 < H_0 \) and \( N = 0 \), \( U = U_1 \) for \( x_3 > H_0 \) and denote the relative density difference at the inversion layer by \( \varepsilon \) (\( \varepsilon \ll 1 \)). The dispersion relation for this model is

\[
D = 1 + (U_1/U_0)^2 \tanh k H_0 - (1/F^2 \cos^2 \phi)(\tanh(kH_0)/kH_0) = 0 \tag{A1}
\]

where \( F = U_0/\sqrt{\varepsilon g H_0} \), is the Froude number. With \( \tanh kH_0 \approx kH_0 \) an approximation to Eq. (A1), valid also for \( kH_0 \) considerably larger than unity, is obtained. This approximation is used for evaluating the phase lines, Eqs. (3.6).

(ii) **4-layer model:** the thicknesses of the three lowest layers are denoted \( H_i \) (\( i = 0, 1, 2 \); numbered from the ground). The fourth, top, layer extends to infinity. The Scorer parameters in the four layers are represented by \( \gamma_i \) (\( i = 0, 1, 2, 3 \)). We allow for a discontinuity in wind velocity at \( x_3 = H_0 \) and set \( U = U_0 \) for \( x_3 = H_0^- \) and \( U = U_1 \) for \( x_3 = H_0^+ \). With this notation the dispersion relation reads

\[
D = q_3(q_0 \tan(q_1 H_1)/q_1 + (U_1/U_0)^2 \tanh q_0 H_0)(q_3 + q_2 \tanh q_2 H_2) + \\
+ (q_0 - q_1(U_1/U_0)^2 \tanh q_0 H_0 \tan(q_1 H_1))(q_3 + q_2 \tanh q_2 H_2) = 0 \tag{A2}
\]

where \( q_i = [(\gamma_i/\cos \phi)^2 - k^2]^i \) and \( q_i = [k^2 - (\gamma_i/\cos \phi)^2]^i \), for \( i = 0, 2, 3 \).

For the trapped wave modes we consider, all the \( q_i \) are real.