scaling analysis for concentrated vortices (Morton 1969) which shows that away from rigid boundaries normal to the vortex axis, axial velocity gradients are an order of magnitude smaller than radial gradients and that radial velocities are weak compared with axial and azimuthal components. Moreover, we remind Eskridge and Das that \( u = 0 \) is effective as a no-slip condition only if significant radial velocities occur just below the upper boundary; this is not the case here as can be verified \textit{a posteriori} from our solutions. In any case, any azimuthal vorticity generated on account of this condition is immediately advected out of the flow domain and does not contaminate the solutions.

With some obvious qualifications, the idea of a 'hot vacuum cleaner in the sky' has much to commend it!

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R. K. Smith, Department of Mathematics, Monash University, Clayton 3168, Australia.

L. M. Leslie, Australian Numerical Meteorology Research Centre, P.O. Box 5089 AA, Melbourne 3001, Australia.

1 August 1978

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**COMMENTS ON THE PAPER 'RELATIONS BETWEEN SURFACE FLUXES AND MEAN PROFILES OF VELOCITY, TEMPERATURE AND CONCENTRATION, DOWNWIND OF A CHANGE IN SURFACE ROUGHNESS' BY P. J. MULHEARN (Q.J. 1977, 103, 785–802)**

By R. A. ANTONIA and L. W. B. BROWNE

We would like to make two main comments. The first relates to the claim by the author (bottom of p. 785) that analytical forms for non-dimensional changes in velocity, temperature and concentration are \textit{derived}, whereas they are \textit{assumed} by Townsend (1965a, b). This claim is not justifiable since Eqs. (14) and (25) are assumptions which are equivalent to expressions, as assumed by Townsend, for the mixing length transfer relation. In particular, while the author may be correct in asserting that the forms of \( f \) and \( F \) should be true for any change of surface roughness, Eq. (14) is true only for an infinitesimal change in surface roughness. Laboratory investigations (Antonia and
Luxton 1971 1972; Antonia et al. 1977) of the effects of surface roughness or heat flux changes on boundary layers have shown that Eqs. (14) and (25) are not valid immediately downstream of the step.

The second comment concerns the very good agreement between the theoretical prediction and the experimental data \((x > 189 \text{ mm})\) of Blom (1970), as shown in Fig. 6, for a step change in surface temperature. Mulhearn’s assertion that Blom’s experimental configuration is an exception to the usual situation where the internal layer becomes deeper than the upstream constant stress region before a self-preserving region can be established, is not supported by the experimental evidence. At Blom’s stations 4 \((x = 133 \text{ mm}, \text{ not } 189 \text{ mm})\) and 6 \((x = 433 \text{ mm}, \text{ not } 489 \text{ mm})\), the thermal layer edges, \(\delta_x\) (defined as the height above the surface where the values of \(C - C_1 = 0.01(C(0) - C_1)\), are approximately 11 and 24 mm, respectively, while the total thickness, \(\delta_o\), of the boundary layer, at the position of the step, is approximately 27 mm. The data of Blom have been replotted (Fig. 1) because of an error in the value of \(x\) used by Mulhearn. The behaviour of \(g\)

![Figure 1](image-url)

**Figure 1.** Comparison between theory for the temperature difference function and the experiments of Blom (1970), Fulachier (1972) and Perry and Hoffmann (1976).
Figure 2. Comparison between theory for the heat flux distribution function and the experiments of Blom (1970) and Fulachier (1972).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value of $x$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blom: $6.13 \text{ ms}^{-1}$</td>
<td>$x = 683$</td>
</tr>
<tr>
<td>Blom: $10.1 \text{ m s}^{-1}$</td>
<td>$x = 683$</td>
</tr>
<tr>
<td>Fulachier: $11.9 \text{ m s}^{-1}$</td>
<td>$x = 50$</td>
</tr>
<tr>
<td>$\bigcirc$</td>
<td>$x = 150$</td>
</tr>
<tr>
<td>$\bigcirc$</td>
<td>$x = 250$</td>
</tr>
<tr>
<td>$+$</td>
<td>$x = 350$</td>
</tr>
<tr>
<td>$\bigcirc$</td>
<td>$x = 500$</td>
</tr>
<tr>
<td>$\bigtriangleup$</td>
<td>$x = 640$</td>
</tr>
<tr>
<td>$\triangledown$</td>
<td>$x = 800$</td>
</tr>
</tbody>
</table>

- - -: Theory using $m = -0.2$

- - - -: Theory using $G = e^{-\xi}$

at Blom's station 3 ($x = 33 \text{ mm}$, not $89 \text{ mm}$) is quite different from that reported by Mulhearn while, for larger values of $x$, the theoretical prediction using $m = -0.2$ underestimates slightly the experimental results. An $m$ value of $-0.3$ would give better agreement with the data but a best-fit plot indicates that $m$ is more likely to be $-0.14$. Also included in Fig. 1 are experimental data, for a step change in temperature, of Fulachier (1972) and of Perry and Hoffmann (1976). The value of $m$ from Fulachier is $-0.2$ while from Perry and Hoffmann it is $-0.1$. Since values of $\delta_x/\delta_0$ for all the data shown in Fig. 1 are comparable (at comparable $x/\delta_0$), and in clear violation of the requirements of the theory, the agreement, such as it is, between experiment and theory of Fig. 1 can at best be regarded as fortuitous (note that, in any case, Eq. (25) was not derived from a step change in temperature). The comparison in Fig. 1 (or Fig. 6 of Mulhearn) may also be thought not to be a stringent test of the analytical form of $g$ since $\ell_c$ is supplied by the theory rather than independently provided by the experiments.

Agreement between experimental heat flux distributions and theory ($G = \xi g'$, with $m = -0.2$), shown in Fig. 2, is not as good, even at relatively large values of $x$, as that exhibited in Fig. 1. The relative agreement between theory and data in Fig. 2 is similar to that reported in Antonia et al. (1977) for a step change in surface heat flux.

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Department of Mechanical Engineering, University of Newcastle, New South Wales 2308, Australia.
26 June 1978

**Reply**

By P. J. Mulhearn

I was very interested in Antonia and Browne's comments, especially their second, on the relevance of the theory of Mulhearn (1977) to wind tunnel data. On their first comment I agree that Townsend (1965a, b) derived my Eq. (14) and (25) by mixing-length arguments, but these are no more mixing-length relations than is the 'classical' log-law, which can also be derived by mixing-length arguments. I do not accept that Eq. (14) is only true for an infinitesimal change in surface roughness. The arguments in its favour are set out in Mulhearn (1977). Laboratory data cannot be cited to disprove it, because, as Antonia and Browne point out, the assumptions on which the theory is based are always violated in laboratory experiments. At short fetches ln(l_u/l) is too small, while at large fetches the internal layer is deeper than the upstream constant stress region.

I agree with their second comment that the experiment of Blom (1970) is not an exception to this rule. In looking at Blom's data I made the incorrect assumption that the outer edge of the upstream log-layer, at y_u/u of 300, coincided with the edge of the constant stress region. In addition, there is some ambiguity in Blom's thesis with regard to the length of the unheated, upstream section of the plate on which his boundary layer grew, and hence in the origin for x. Antonia and Browne (private communication) pointed out that in one place Blom gives 1 m and in another 1.0556 m for this length. The precision of the latter figure indicates that it is the correct one and hence the x values used by Antonia and Browne are correct. Antonia and Browne's general point that the assumptions on which the theory of Mulhearn (1977) is based are violated in laboratory experiments is one I support. In the light of this, the theory does better than it should, as the data for g(ξ) in Antonia and Browne's Fig 1. show. Also the results for G(ξ) from Fulachier (1972) in their Fig. 2 are not too different from the theory for x < 250 mm.

However, the inapplicability of the theory to laboratory experiments is a secondary point because it was primarily aimed at perturbations to the atmospheric boundary layer in which the assumptions are often well satisfied.

One small point remains. Antonia and Browne's comment that Eq. (25) was not derived for a step change in temperature is misleading. This equation will apply to any surface heat flux variation, downstream of an abrupt change, provided that c_0 \propto \tilde{u} or c_0 \frac{d\theta_0}{dx} \ll \tilde{u} \frac{d\theta_0}{dx}, and so can include the case for a step change in temperature.

**References**

Muller, P. J.


Townsend, A. A.


R.A.N. Research Laboratory,
P.O. Box 706,
Darlinghurst,
N.S.W. 2010,
Australia.
10 July 1978