Conditional symmetric instability – a possible explanation for frontal rainbands

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SUMMARY

In order to study the possible importance of symmetric baroclinic instability in the formation of frontal rainbands, the existing theory is reviewed and the inclusion of the effects of latent heat release attempted. When an atmosphere is rendered symmetrically unstable by latent heat release, it may be said to be conditionally symmetrically unstable. Simple numerical experiments support the extended theory and describe the structure of the finite amplitude cells. These exhibit conditional gravitational instability in preferred linear regions. It is shown that such a gravitational destabilization is possible only when a wet bulb potential vorticity is initially negative. This latter is a necessary and possibly sufficient condition for ‘conditional symmetric instability’ (CSI). Limited comparison with observed frontal rainbands lends some support to the hypothesis that CSI can be a dominant formative mechanism, though more sophisticated numerical modelling and observational studies are required.

1. INTRODUCTION

The fact that frontal cloud and precipitation are frequently concentrated in bands approximately parallel to the front has been highlighted in many observational studies (e.g. Elliott and Hovind 1964, 1965; Nozumi and Arakawa 1968; Browning and Harrold 1969; Kreitzberg and Brown 1970). One occurrence of rainbands has been the object of a radar–synoptic study by Browning et al. (1973) and a dropsonde investigation below 4 km by Roach and Hardman (1975). The presence of mid-level convection, near 600 mb, has been commented on in the above papers and in Houze et al. (1976). Kreitzberg and Brown, and Browning et al., have stressed the importance of this convection and stated that it occurs due to differential advection.

The observational studies have indicated that the spacing of bands is of the order of 80–300 km and that the length scale along the bands is much larger. The bands make only a small angle with temperature contours and, indeed, Elliott and Hovind related their orientation to the vertical shear in the region of convection. Theoretical possibilities for the origin of the bands include Ekman layer instability in the frontal region, a gravity wave generated at the front, and a natural scale for the convection which is itself caused by the differential advection (Browning et al. 1973). Another attractive possibility, which we wish to study in this paper, is that the bands are a manifestation of symmetric baroclinic instability. Historically, this instability was studied in relation to the general structure of planetary atmospheres and arose from symmetric meridional perturbations of a circular vortex. Loosely, the criteria for the instability are that the horizontal temperature gradient is large and the Richardson number small. In a two-dimensional situation the instability takes the form of rolls orientated along the thermal wind. In an infinite, hydrostatic atmosphere the most unstable mode may be described as inertial instability on isentropic surfaces.

The theory of symmetric baroclinic instability is summarized in section 2, closely following the work of Eliassen (1962), Ooyama (1966) and Hoskins (1974). However, as pointed out by Eliassen and Kleinschmidt (1957), the condition for this instability is rarely
satisfied in frontal regions. Following their suggestion and that of Hoskins, it is of interest to consider the possibility of symmetric instability in a moist atmosphere where the effective static stability is reduced. In section 3 an attempt is made to include a crude model of latent heat release in the theory, leading to the concept of ‘conditional symmetric instability’ (CSI). In sections 2 and 3 severe restrictions on the production of gravitational instability by simple differential advection are produced.

To remove some of the limitations of the theory simple numerical experiments have been performed and are described in section 4. The dynamical structure of finite amplitude CSI modes are exhibited and attention drawn to the production of gravitational instability in preferred linear regions. The limitation of band size for different dissipation parameters is also presented. In section 5 a limited comparison of the theoretical CSI results with observations of rainbands is made. Concluding remarks are presented in section 6.

2. CLASSICAL SYMMETRIC INSTABILITY

(a) Non-hydrostatic

For the subsequent analysis we use the following non-hydrostatic, but Boussinesq, equations:

\[ Du/Dt - f v + \partial p/\partial x = 0 \]  \hspace{1cm} (1)
\[ Du/Dt + f u + \partial p/\partial y = 0 \]  \hspace{1cm} (2)
\[ Dw/Dt - g \theta/\theta_0 + \partial p/\partial z = 0 \]  \hspace{1cm} (3)
\[ \partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 0 \]  \hspace{1cm} (4)
\[ D\theta/Dt = 0 \]  \hspace{1cm} (5)

The notation is, for the most part, standard. The Coriolis parameter, \( f \), is taken to be constant, and \( \theta_0 \) is a typical value of the potential temperature, \( \theta \).

We consider a basic flow \( \bar{V}(x,z) \) in the \( y \) direction, but having no \( y \) dependence, in thermal wind balance with a potential temperature distribution, \( \Theta(x,z) \):

\[ f \partial \bar{V}/\partial z = (g/\theta_0)(\partial \Theta/\partial x). \]

We consider a perturbation, also independent of the downstream coordinate \( y \). We denote perturbation quantities by primes, and note that the continuity equation (from Eq. (4)) implies that we may introduce a streamfunction, \( \psi \), for the perturbation velocities normal to the basic flow: \( u' = \partial \psi/\partial z, w' = -\partial \psi/\partial x \). From the perturbation forms of Eqs. (1) and (3), the \( y \) component of the vorticity equation may be written

\[ \partial (\partial^2 \psi/\partial x^2 + \partial^2 \psi/\partial z^2) = f \partial u'/\partial z - (g/\theta_0)(\partial \theta'/\partial x). \]  \hspace{1cm} (6)

Circulation in an \( x-z \) section across the basic flow is forced by a departure from thermal wind balance in the perturbation.

We now introduce basic flow frequencies, \( F \) and \( N \):

\[ F^2 = f(f + \partial \bar{V}/\partial x), \quad S^2 = f \partial \bar{V}/\partial z = (g/\theta_0)(\partial \Theta/\partial x), \quad N^2 = (g/\theta_0)(\partial \Theta/\partial z). \]  \hspace{1cm} (7)

Typical atmospheric values are \( F \approx 10^{-4} \text{s}^{-1}, S \approx 0.5 \times 10^{-3} \text{s}^{-1}, N \approx 10^{-2} \text{s}^{-1} \). It should be noted that \( F^2/S^2 \) is the slope of the absolute vorticity vector and of the surfaces \( M = f x + V = \text{const.} \), and \( S^2/N^2 \) that of the potential temperature surfaces in the basic flow. The perturbation forms of Eqs. (2) and (5) may be written:

\[ \partial (fu')/\partial t = -F^2 u' - S^2 w' \]  \hspace{1cm} (8)
\[
\frac{\partial}{\partial t}(g\theta'/\theta_o) = -S^2u' - N^2w'.
\]  
\[\text{(9)}\]

For the purposes of this analysis we assume that the length scales over which the basic flow frequencies vary are much larger than the length scales of the perturbation. Then substituting Eqs. (8) and (9) in Eq. (6) gives

\[
\frac{\partial}{\partial t}(\partial^2\psi/\partial x^2 + \partial^2\psi/\partial z^2) = -N^2 \partial^2\psi/\partial x^2 + 2S^2 \partial^2\psi/\partial x\partial z - F^2 \partial^2\psi/\partial z^2.
\]  
\[\text{(10)}\]

In an unbounded domain, we may seek solutions proportional to \(\exp(i\sigma t)\exp ik(x\sin\phi + z\cos\phi)\), where \(\phi\) is the angle between the perturbation displacement and the horizontal. The frequency, \(\sigma\), is then given by

\[
\sigma^2 = N^2\sin^2\phi - 2S^2\sin\phi\cos\phi + F^2\cos^2\phi.
\]  
\[\text{(11)}\]

As shown by Ooyama (1966) and Hoskins (1974), the square of the frequency has its minimum value given by

\[
2\sigma^2_{\text{min}} = N^2 + F^2 - (N^2 + F^2)^2 - 4q)^{\frac{1}{4}},
\]  
\[\text{(12)}\]

where

\[
q = F^2N^2 - S^4.
\]  
\[\text{(13)}\]

is proportional to the Ertel potential vorticity of the basic flow (Eliassen and Kleinschmidt 1957). There is instability \((\sigma^2_{\text{min}} < 0)\) if and only if either \(N^2 + F^2 < 0\) or \(q < 0\). For \(N^2\) and \(F^2\) positive the latter criterion may be interpreted as that there is stability if the absolute vorticity vector is more vertical than the potential temperature surfaces, and instability if the potential temperature surfaces are more vertical. The orientation of the perturbation with the lowest frequency in the stable case or the maximum growth rate in the unstable case lies between the vorticity vector and the potential temperature surfaces.

We may note that the above analysis includes as special cases:

(i) Pure gravitational instability: \(S^2 = 0, N^2 < 0, F^2 > N^2\); maximum instability in the vertical with growth rate \((-N^2)^{\frac{1}{4}}\).

(ii) Pure inertial instability: \(S^2 = 0, F^2 < 0, N^2 > F^2\); maximum instability in the horizontal with growth rate \((-F^2)^{\frac{1}{4}}\).

Let us consider an atmosphere which is initially stable with \(N^2, F^2\) and three-dimensional Ertel potential vorticity, \(q\), everywhere positive and suppose that in the subsequent motion frictional effects and heat sources and sinks may be neglected. With these restrictions, in three-dimensional motion \(q\) is conserved and so it remains positive everywhere. Provided there is approximate thermal wind balance (from Eq. (13) and its three-dimensional analogue), \(q < F^2N^2\) and so \(F^2N^2\) remains positive. Therefore the atmosphere must retain the property that \(N^2, F^2\) and \(q\) are everywhere positive. As stressed in Hoskins (1974), in the absence of frictional effects and heat sources and sinks, and with the above quasi-geostrophic assumption, an atmosphere cannot become symmetrically unstable. In particular, under these conditions gravitational instability cannot be generated by differential motions in a stable atmosphere.

(b) Hydrostatic

Making the hydrostatic assumption by neglecting \(Dw/Dt\) in Eq. (3) gives only minor modifications to the above analysis. In Eq. (10), the term in \(\psi_{xx}\) is omitted from the left hand side and the frequency-squared equation (Eq. (11)) has the right hand side divided by

* If thermal wind balance in the basic flow is not assumed, then Eqs. (11–13) are obtained with \(S^2\) replaced by \(1/ \partial /\partial z (\partial \theta /\partial z) \partial \theta /\partial z\). The simple analysis of Lilly (1966) for the longer wavelength mode of Ekman layer instability is the special case \(F^2 = N^2\). \(\partial \theta /\partial z = 0\), for which the most unstable mode has a 45° orientation and growth rate \((|\partial \theta /\partial z|^{\frac{1}{2}})^{\frac{1}{2}}\).
\[ \cos^2 \phi. \] For almost horizontal perturbation orientations, \( \cos \phi \sim 1 \) and the approximation is good. The least stable direction is now exactly along the potential temperature surface which for consistency must have a small slope. For \( N^2 > 0 \), there is again instability if and only if \( q < 0 \), i.e. \( \Theta \) surfaces more vertical than the absolute vorticity vector.

The most unstable mode is dynamically very simple. As indicated above, it is composed of particles ascending and descending \( \Theta \) surfaces. Thus the perturbation potential temperature and therefore the perturbation pressure are identically zero. The perturbation momentum equations may then be written

\[
\frac{\partial u'}{\partial t} = f v', \quad \frac{\partial v'}{\partial t} = -\zeta_\Theta u'.
\]

\[ (14) \]

where \( \zeta_\Theta = (1/f)(F^2 - S^4/N^2) \), is the absolute vorticity on \( \Theta \) surfaces. Therefore \( \partial^2 u'/\partial t^2 = (-f \zeta_\Theta) u' \). If \( f \zeta_\Theta < 0 \), which is equivalent to \( q < 0 \) for \( N^2 \) positive, inertial forces which usually act to give stability here act to increase any perturbation. Under the hydrostatic approximation, the most unstable mode is a case of inertial instability on \( \Theta \) surfaces. It is easily shown that the equation for the kinetic energy of this mode is

\[
\frac{\partial}{\partial t}(\frac{1}{2}(u'^2 + v'^2)) = -u'v' \partial V/\partial x - v'w' \partial V/\partial z.
\]

In the absence of horizontal shear in the basic wind, the eddy kinetic energy grows at the expense of 'zonal' kinetic energy through a \( v', w' \) correlation with the opposite sign to that of the basic vertical wind shear.

Finally, we note that the instability criterion may also be written

\[ 1/Ri < 1, \]

\[ (15) \]

(cf. Stone 1966), where \( Ri = F^2 N^2 / S^4 \) is the usual Richardson number apart from the use of \( F^2 \) rather than \( f^2 \). Also, the least stable mode has

\[
\sigma_{\text{min}}^2 / F^2 = 1 - 1/Ri.
\]

\[ (16) \]

3. Instability in a moist atmosphere

(a) General constraints

Since the criterion for symmetric instability in a dry atmosphere is rarely satisfied on the 100 km scale, we now consider the influence of moist processes. In three-dimensional motion in a moist atmosphere the wet bulb potential temperature \( (\theta_w) \) is changed only by the diabatic processes other than latent heat release:

\[
D\theta_w / Dt = Q.
\]

\[ (17) \]

Making the hydrostatic approximation we introduce a reduced static stability

\[
N_w^2 = (g/\theta_o)(\partial \theta_w/\partial z)
\]

and a wet bulb potential vorticity

\[
q_w = f(g/\theta_o)\zeta \nabla \theta_w = f\zeta N_w^2 + (g/\theta_o)\{k \wedge (\partial v/\partial z)\} \cdot \nabla \theta_w.
\]

\[ (18) \]

where \( \xi \) is the three-dimensional absolute vorticity vector, \( \zeta \) its vertical component, \( k \) the unit vertical vector, \( \nabla \) the horizontal gradient operator and \( v = (u, v, 0) \). We assume that the horizontal angle between \( -k \wedge (\partial v/\partial z) \), which if there is thermal wind balance is parallel to \( \nabla \theta \), and \( \nabla \theta_w \) is less than 90°. Then Eq. (18) implies that

\[
q_w < f\zeta N_w^2.
\]

\[ (19) \]

If an atmospheric state initially has \( q_w, f\zeta \) and \( N_w^2 \) positive, then conditional gravitational
instability \((N^2_w < 0)\) or inertial instability \((f_s < 0)\) can be generated only if the wet bulb potential vorticity is made negative.

From Eqs. (1)-(4), the vorticity equation is

\[
\frac{D\zeta}{Dt} = (\zeta \nabla)u - \mathbf{k} \wedge (g/\theta_0)\nabla\theta + \mathcal{F},
\]

where \(\mathcal{F}\) is a frictional term and \(u = (u,v,w)\). Combining with the gradient of Eq. (17) gives

\[
\frac{Dq_w}{Dt} = f(g^2/\theta_0^2)\mathbf{k} \cdot (\nabla\theta_w \wedge \nabla\theta) + f(g/\theta_0)\zeta \cdot \nabla Q + f(g/\theta_0)\mathcal{F} \cdot \nabla\theta_w. \tag{20}
\]

The last two terms are due to frictional and diabatic effects. The first term indicates a change in \(q_w\) when there is an angle between \(\theta\) and \(\theta_w\) surfaces in the horizontal. As shown in Fig. 1, if the water vapour content increases in the direction of the thermal wind, the wet bulb potential vorticity decreases. If the air is saturated, \(\theta_w\) is a function of \(\theta\) and \(\rho\) only so that the production term is negligible. Thus with the entirely reasonable assumption mentioned in the above paragraph, \textit{conditional gravitational instability cannot be generated by purely differential motion in a saturated or two-dimensional quasi-geostrophic atmosphere unless the wet bulb potential vorticity, or equivalently the vorticity on a wet bulb potential surface, is initially negative.}

![Figure 1. A possible horizontal section showing potential temperature (\(\theta\)) contours with \(\nabla\theta\) pointing towards the warm air. Since moister air has a higher wet bulb potential temperature (\(\theta_w\)) than dry air at the same temperature, for this case the \(\theta_w\) contours and \(\nabla\theta_w\) must be approximately as sketched. The circulation tendency implied by the potential temperature gradient is one with warm air rising and cold air descending, i.e. generation of a horizontal vorticity component in the direction of the dry air. Thus there is a generation of negative \(\zeta \cdot \nabla\theta_w\).](image)

A proper diagnostic study is required to elucidate the relative importance of the terms in Eq. (20) in creating negative \(q_w\). Putting in reasonable numbers, including an angle between \(\theta\) and \(\theta_w\) surfaces of the order of 0.1°, suggests that the first term could make \(q_w\) negative on a fluid particle in 1–2 days, which is consistent with the time that particle would take to traverse the synoptic system.

\[(b) \textit{Symmetric instability} \]

We now consider the possibility of symmetric instability for two-dimensional flow in an atmosphere which is assumed to be saturated everywhere. Prompted by the simple picture of the most unstable mode in a dry hydrostatic atmosphere we consider upward motion along a \(\theta_w\) surface. Again there is no pressure perturbation and, if the \(\theta_w\) surface is more vertical than the absolute vorticity vector, there is a negative restoring force. This condition may also be written in the three forms:
\begin{equation}
f \zeta \Theta_w < 0, \quad q_w < 0, \quad 1/Ri > N^2_w/N^2. \tag{21}
\end{equation}

Note that \( q_w = F^2 N_w^2 - S^2 S_w^2 \), where \( S_w^2 = (g/\Theta) (\partial \Theta_w / \partial x) \), and that the last form in Eq. (21) should include a factor \( S^2 / S_w^2 \) on the right. Since this factor is unity for saturated air near the surface and typically 1.5 near 700 mb, it has been neglected.

In an infinite atmosphere in which there does not have to be compensating downward motion, we therefore expect an instability which we may naturally call 'conditional symmetric instability' (CSI). The necessary condition that differential motions in a saturated atmosphere can generate gravitational instability thus appears as a sufficient condition for CSI in an infinite atmosphere.

In a finite atmosphere, the downward motion with no latent heat release has a positive restoring force and one may expect the above instability conditions to be necessary but not sufficient. Rigorous theory in this case is difficult and the arguments to be given are only meant to be suggestive. In this spirit we consider the possible motion around a hypothetical streamtube as shown in Fig. 2 with the downward branch AB in the direction of least stability along \( \Theta \) surfaces and the upward branch BC in the direction of maximum instability along \( \Theta_w \) surfaces. For a displacement of volume \( \delta \tau \) per unit cross-section there is an x displacement \( \delta x_u = \delta \tau / h_u \) of the branch BC and hence a negative restoring force \( f \zeta \Theta_w \delta x_w \).

If this destabilizing force is to be overcome, the fluid must sustain a perturbation pressure difference between C and B:

\[
\Delta p' \sim \left( -f \zeta \Theta_w \right) \delta x_u L_u = \left( -f \zeta \Theta_w \right) \delta \tau L_u / h_u.
\]

The narrower the updraught, the greater the pressure difference. Similarly, the positive restoring force on AB requires for balance only a perturbation pressure difference.

Figure 2. Surfaces of constant potential temperature, \( \Theta \), wet bulb potential temperature, \( \Theta_w \), and \( M = fx + V \), the latter being parallel to the absolute vorticity vector, \( \zeta \). A possible streamtube with upward branch BC along a \( \Theta_w \) surface and downward branch AB along a \( \Theta \) surface is shown.
SYMMETRIC INSTABILITY

\[ \Delta p' \sim (f'_{\phi}) \delta \tau L_l/h_l. \]

Since the flow CA is so broad we neglect the difference between \( p'_c \) and \( p'_h \) and then we may expect that instability of the pressure difference required to balance the destabilizing force on BC is more than that required to balance the stabilizing force on AB, i.e.

\[ (-f'_{\phi w}) \delta \tau L_w/h_w > (f'_{\phi}) \delta \tau L_l/h_l. \]

Thus we expect an unstable mode to have broad downdraughts and narrow updraughts and may postulate a sufficient condition for instability of the form

\[ \alpha f'_{\phi} + f'_{\phi w} < 0, \quad (22) \]

where \( \alpha = (h_uL_l)/(h_lL_u) = O(1) \). Three other forms of this condition are

\[ \alpha(N^2_w/N^2)q + q_w < 0, \quad Ri^{-1} > (\alpha + 1)/(\alpha + N^2/N^2_w), \quad \alpha_0 < \phi_w, \quad (23) \]

where the second form again requires the neglect of a factor \( S^2_w/S^2 \) and the last form, in terms of the angles shown in Fig. 2, uses, additional, small trajectory slope approximations.

The dry symmetric instability (SI) condition is then the limit \( \alpha \to \infty \), and the infinite saturated atmosphere CSI condition is obtained by setting \( \alpha = 0 \). These conditions are shown in Fig. 3. Also shown are the neutral curves \( \alpha = 1 \) and \( \alpha = 2 \). Clearly, sensitivity to the value of \( \alpha \) is not very great. A basic state with \( N^2_w/N^2 = 0:2, Ri^{-1} = 0:8 \), say, would be neutral for \( \alpha = 15 \) and may with confidence be expected to exhibit CSI. Marked also in Fig. 3 are the stability characteristics of each region. The domain \( N^2_w < 0 \) is one of conditional gravitational instability (CGI).

![Figure 3](image-url)  
**Figure 3.** Stability diagram for CSI. The abscissa is \( N^2_w/N^2 \) and the ordinate \( Ri^{-1} = S^2/F^2N^2 \) (see Eq. (7)). The neutral curves are as given by Eq. (23) for \( \alpha = 0, 1, 2 \) and \( \infty \); that given by the Lindzen-type analysis of section 3 (c) is denoted by L. SI indicates the region \( Ri^{-1} > 1 \) which is unstable to dry symmetric instability, CSI the extra region unstable to conditional symmetric instability if the atmosphere is saturated, and CGI in the region \( N^2_w < 0 \) stands for conditional gravitational instability.

The curves \( \alpha = 0, 1, 2 \) and \( \infty \) and the line L also give the growth rate curves \( \gamma = F \) for the analyses in sections 3 (b) and 3 (c) provided the ordinate is replaced by \( Ri^{-1}/2 \).

To produce an idea of possible growth rates, we again refer to the branches BC and AB in Fig. 2, ignoring branch CA. The simple equation of motion for BC is

\[ \delta \hat{x}_u = -f'_{\phi w} \delta x_u - \Delta p'/L_u. \]
Multiplying by $h_u$ gives an equation for the volume perturbation:
\[
\delta \ddot{c} = -f\zeta_\omega \delta \tau - (h_u/L_w)\Delta p'.
\]
Similarly, from AB,
\[
\delta \ddot{c} = -f\zeta_\omega \delta \tau + (h_1/L_1)\Delta p'.
\]
Eliminating $\Delta p'$ gives $\ddot{c} = \gamma^2 \delta \tau$, where the growth rate $\gamma$ is given by
\[
\gamma^2 = -(\alpha + 1)^{-1}(\alpha f\zeta_\omega + f\zeta_\omega),
\]
(24a)
where $\alpha$ has again been substituted for the ratio $h_uL_1/h_1L_w$. Again the limits $x \to \infty$ and $\alpha = 0$ give the infinite atmosphere dry SI and CSI expressions. Analogues to the stability conditions, Eq. (23), are the alternative forms of Eq. (24a):
\[
\gamma^2 = -(\alpha + 1)^{-1}(aq/N^2 + q_u/N^2_{w}), \quad \gamma^2 = F^2(\alpha + 1)^{-1}R\gamma^{-1}(\alpha + N^2/N^2_w - 1),
\]
(24b)
For mid-latitude situations, $\gamma = F$ corresponds to an e-folding time $T_e = F^{-1}$ of around 3 hours and a doubling time of about 2 hours. This contour for $\alpha = 0, 1, 2$ and $\infty$ is indicated in Fig. 3 (see caption). Again the sensitivity to $\alpha$ is not large provided that $\alpha$ is of order unity. The point (0-2, 0-8) considered previously has a growth rate $\sqrt{3}F$ for $\alpha = 0$. The value is $F$ for $\alpha = 1.67$ and $\sqrt{3}F$ for $\alpha = 6.11$. Thus we may expect that an atmospheric state corresponding to this point would exhibit CSI with a doubling time of the order of hours.

(c) A different approach

A rather different treatment of the heating only in upward motion may be made in a manner similar to that of Lindzen (1974). As will be discussed below, we model the effect of latent heat release by a reduction of the static stability from $N^2$ to $N^2_w$ for upward moving air. Suppose that $w = \hat{w} \text{sinkx}$ then $w$ times the static stability is $\hat{w}N^2_w \text{sinkx}$ for $0 \leq x < \pi$, and $\hat{w}N^2_w \text{sinkx}$ for $\pi \leq x < 2\pi$.

In a linear analysis we neglect the wavenumber zero and higher harmonic components and retain only the wavenumber $k$ component whose contribution is $\hat{w}(\text{sinkx})\{1/2(N^2 + N^2_w)\}$. Thus we obtain CSI results by substituting the average static stability $1/2(N^2 + N^2_w)$ for $N^2$ in the dry symmetric instability theory. The neutral curve and the $\gamma = F$ curve are indicated by $L$ in Fig. 3 (see caption). For $N^2_w < 1/2N^2$, this theory predicts less instability than that described with $\alpha = 2$. However, the point (0-2, 0-8) still has a growth rate $0.58F$, giving a doubling time of 4 hours.

4. NUMERICAL EXPERIMENTATION

(a) Introduction

The treatment of symmetric instability in a moist atmosphere described in the previous section is suggestive but not rigorous. In order to examine the phenomenon more closely in a few specific cases, a numerical model was developed. At this stage it was deliberately kept simple. The principal aims were to include the finite vertical extent of the domain implied by the lower boundary and by the decrease in water vapour with height and to study the finite amplitude development of the instability.

The equations used are the primitive equations with the hydrostatic approximation and with $z = \{1-(p/p_0)^k\}H_s/k$ as vertical coordinate. Here the scale height $H_s = p_0/p_{00} =$
$R\theta_0/g$, where zero suffices refer to typical 1000 mb values and $\kappa = R/c_p$. Since $\theta dz = \theta_0 d(\text{height})$, $z$ is almost identical with physical height in the troposphere. Introducing a pseudo-density $r$ which is a function of $z$ only:

$$r(z) = \rho_0(p/p_0)^{1/7},$$

then

$$rdz = -dp/g = \rho \, d(\text{height})$$

and

$$\rho \omega = -(1/g)(Dp/Dt) = \rho \times (\text{physical vertical velocity}).$$

With $\partial/\partial y \equiv 0$ the dry equations in flux form are

$$\partial u/\partial t = -\partial(uu)\partial x - (1/r)\partial(rw\omega)/\partial z + fu - \partial P/\partial x + v_x \partial^2 u/\partial x^2 + v_z \partial^2 u/\partial z^2,$$

$$\partial v/\partial t = -\partial(uv)\partial x - (1/r)\partial(r\omega\omega)/\partial z - fu + v_x \partial^2 v/\partial x^2 + v_z \partial^2 v/\partial z^2,$$

$$\partial P/\partial z = (g/\theta_0)\theta,$$

$$\partial \theta/\partial t = -\partial(\theta \theta)\partial x - (1/r)\partial(r\omega\theta)/\partial z + v_x \partial^2 \theta/\partial x^2 + v_z \partial^2 \theta/\partial z^2,$$

$$\partial u/\partial x + (1/r)\partial(rw)/\partial z = 0.$$

$P$ is the geopotential and $f$ the Coriolis parameter, taken as constant. The bottom boundary condition of zero vertical motion was approximated by imposing $\omega = 0$ at $z = 0$. A lid with $\omega = 0$ was taken at $z = H$. The use of these equations is entirely equivalent to the use of the more usual form in which $p$ is the independent variable in the vertical. The present form is useful both for the implied linear thermal wind relation and the obvious approach to the Boussinesq equations (1)–(5) when the scale height of the motion is much less than $H$. Simple diffusion terms were included in the momentum and thermodynamic equations, with the Prandtl number taken as unity. We also used no-slip boundary conditions $u = 0, v = 0$ at $z = 0$, and stress-free conditions $\partial u/\partial z = 0, \partial v/\partial z = 0$ at $z = H$. $\theta$ is taken to be specified constant functions on $z = 0, H$. Far away from the region of interest, vertical boundaries with $u = 0$ were assumed.

### Table 1. Change in potential temperature, $\theta$, in 100 mb ascent of saturated air (K)

<table>
<thead>
<tr>
<th>Pressure (mb)</th>
<th>$\Theta_m = 5^\circ C$</th>
<th>$\Theta_m = 10^\circ C$</th>
<th>$\Theta_m = 15^\circ C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000–900</td>
<td>3·2</td>
<td>3·6</td>
<td>4·1</td>
</tr>
<tr>
<td>800–700</td>
<td>2·8</td>
<td>3·6</td>
<td>4·4</td>
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<tr>
<td>600–500</td>
<td>1·8</td>
<td>2·4</td>
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<tr>
<td>500–400</td>
<td>0·8</td>
<td>1·5</td>
<td>3·2</td>
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<tr>
<td>400–300</td>
<td>0·1</td>
<td>0·4</td>
<td>1·6</td>
</tr>
</tbody>
</table>

Rather than introducing a water vapour variable and treating the condensation process explicitly, at this stage the simpler approach of using a heating function for upward motion was retained. Assuming that the rising air is saturated the approximate changes in potential temperature as a function of $\theta_m$ and pressure are shown in Table 1. It was considered that the most important feature of the heating was the decrease above 700 mb, in that this provides a natural upper limit for the height scale of symmetric instability. The variation with $\theta_m$ was neglected. Thus the potential temperature equations for $\omega > 0$ included an extra term $\rho \omega A$, where

$$A = 3·6 \times 10^{-4} \text{ for } p > 700 \text{ mb} = 7 \times 10^4 \text{ Pa}$$

$$= 3·6 \times 10^{-4}(p - 3 \times 10^4)/4 \times 10^4 \text{ for } 7 \times 10^4 > p > 3 \times 10^4.$$
With this approximation, \( \theta_w = \theta - \int_0^z g r A \, dz \). Then \( \partial \theta_w / \partial x = \partial \theta / \partial x \) and \( N^2 = N^2 - (g/\theta_0)(g r A) \) is the effective static stability for ascending air. Both these approximations were used in the previous section. It will sometimes be useful to refer to the level \( z = 2\cdot39 \text{ km} \) (\( p = 732 \text{ mb} \)) where \( g r = 10 \text{ Pa km}^{-1} \). There the reduction in \( \partial \theta / \partial z \) is \( 3\cdot6 \text{ K km}^{-1} \).

The finite difference scheme was taken directly from Williams (1967). It used standard second-order finite differences with no staggering except for specifying the vertical velocity variable at vertical levels midway between those at which other variables are stored.

The basic states for most integrations were given by specifying uniform values for \( \partial \theta / \partial x \) and \( \partial \theta / \partial z \). Taking zero geopotential gradient at \( z = 0 \), the geopotential at other levels was determined hydrostatically. The velocities were determined geostrophically and adjusted to fit the upper boundary condition. An additional velocity in the cross-flow direction was specified using the stream function \( \psi \) such that \( ru = \partial \psi / \partial z \), \( rw = -\partial \psi / \partial x \). \( \psi \) is taken to be zero except in a region \( x_0 < x < x_0 + L \), \( z_0 - D/2 < z < z_0 + D/2 \), where it has the form \( \{ \sin 2\pi (x - x_0) L^{-1} \} \{ \cos \pi (z - z_0) D^{-1} \} \), with \( L, D \) and \( z_0 \) chosen as 50, 1 and 2 km, respectively.

Constants used in the numerical model were: \( H_s = 8 \text{ km} \), \( R = 287 \text{ m}^2\text{s}^{-2}\text{K}^{-1} \), channel length 1000 km (usually), \( \Delta z = 100 \text{ m} \), \( \Delta x = 10 \text{ km} \) (usually), \( \Delta t = 14 \text{ min} \) (usually). Because the model is simple and two-dimensional we were able to use this high resolution.

(b) Two model runs

In order to illustrate the sort of results given by the numerical model, we now describe two specific integrations. Experiment A used \( \partial \Theta / \partial z = 4\cdot8 \text{ K km}^{-1} \) and \( \partial \Theta / \partial x = 1 \text{ K/30 km} \) everywhere, and \( f \) was taken to be \( 0\cdot77 \times 10^{-4} \text{ s}^{-1} \). With these values, potential temperature surfaces have a uniform slope 1/144 and the vorticity vector (\( M \) surfaces) everywhere slopes at 1/201.7. Thus in an infinite domain the basic state would be dry symmetrically unstable, the e-folding time being 5.7 hours. In a finite domain with explicit dissipation and finite difference smoothing a numerical experiment showed that the actual growth rate is much smaller, with \( T_e \sim 18 \text{ hours} \). The \( \Theta_w \) surfaces have a slope varying from 1/144 near the top to 1/9 near \( z = 0 \). At \( p = 732 \text{ mb} \), the \( \Theta_w \) slope is 1/36. The model diffusion was \( V_x = 2 \times 10^4 \text{ m}^2\text{s}^{-1} \), \( V_z = 2 \text{ m}^2\text{s}^{-1} \).

A single perturbation was introduced at \( t = 0 \) as described above. As shown in Fig. 4, this perturbation died away initially, but between \( t \) and 1 day, there was almost exponential growth. The structure changes little during this latter period and so it is sufficient to describe the motion at day 1. Fig. 5(a) shows the cross-band stream function. There is concentrated slant-wise ascent, the slope being about 1/80, and broad descent. The maximum vertical velocity is about 12 cm s\(^{-1}\). The circulation cells are centred near 700 mb. At day \( t \) the amplitude was 1/25 of the day 1 value. Otherwise the main differences were that the upper and lower cells had similar magnitude, the ascent was at a slope nearer 1/70, and the vertical and horizontal scales were about \( t \) of their value at day 1. In Fig. 5(b) is shown the wind perturbation along the band relative to the basic flow at day 1. As the air ascends and moves in the negative \( x \) direction at up to 10 m s\(^{-1}\), it also moves in the negative \( y \) direction at up to 12 m s\(^{-1}\). The descending air moves slowly in the positive \( y \) direction. The \( v'w' \) correlation mentioned in section 2 is clearly present. The vertical component of relative vorticity, \( \partial \nu' / \partial x' \), (not shown) has maximum and minimum values about 2.9 \times 10^{-4} \text{ s}^{-1}. Thus the vertical component of absolute vorticity is strongly negative in a band running along the upward branch of the lower circulation cell. The potential temperature perturbation is given in Fig. 5(c) and the total wet bulb potential temperature in Fig. 5(d). Although the maximum temperature perturbation is only 1.3 K, the gradient is strong enough to destabi-
Figure 4. Quarter-day values of the maxima of cross-band streamfunction, \(\psi\), and perturbation velocity, \(u'\), for the first experiment. The initial value of \(u'\) is zero. The amplitude is shown on a logarithmic scale. \(\psi\) is in units of kg m\(^{-1}\) s\(^{-1}\) and \(u'\) in units of 10\(^{-2}\) m s\(^{-1}\).

lize the atmosphere gravitationally in a region near 660 mb. The model is not capable of describing the convective processes important in such a region, but if the run is continued, the destabilization continues at a rapid rate.

To help understand the dynamics of the mode we have obtained, some streamlines and other information are given in Fig. 5(e). We introduce the hydrostatic, non-linear, non-Boussinesq perturbation \(y\) momentum equation, potential temperature equation and \(y\) vorticity equation (analogous to Eqs. (8), (9) and (6), respectively):

\[
\begin{align*}
Dv'/Dt & = -u' \partial M/\partial x - w' \partial M/\partial z \quad . \quad . \quad (25) \\
D\theta'/Dt & = -u' \partial \Theta/\partial x - w' \partial \Theta/\partial z \text{ for descent} \quad . \quad . \quad (26a) \\
& = -u' \partial \Theta_w/\partial x - w' \partial \Theta_w/\partial z \text{ for ascent} \quad . \quad . \quad (26b) \\
r \frac{D}{Dt}[(1/r)(\partial u'/\partial z)] & = f \partial v'/\partial z - (g/\Theta_0)(\partial \theta'/\partial x). \quad . \quad . \quad (27)
\end{align*}
\]

The surfaces \(M (=fx + \nu)\), \(\Theta\), \(\Theta_w\) constant are shown in Fig. 5(e). The slant-wise ascent is more steep than \(M\) surfaces and so (Eq. (25)) produces strong negative \(u'\) centred on A. The
Figure 5. Day 1 of experiment A described in the text. Shown are (a) cross-band streamfunction, (b) wind perturbation along the band, (c) potential temperature perturbation, (d) wet bulb potential temperature and (e) composite figure. The contour spacing is shown below each figure. The zero contour is not drawn. Negative contours are dashed. In (b), (c) and (d) the ±3 streamfunction contours are indicated by long dashes. The information in Fig. 5 (e) is explained in the text. Only 700 km of the total 1000 km horizontal domain is shown.
SYMmetric INSTABILITY

descent steeper than \( M \) surfaces, gives small positive \( v' \) centred near E and F. The elongated minimum in \( v' \) near A, from Eq. (27), accelerates the two circulation cells by giving \( \partial v'/\partial z \) positive above and negative below.

The ascent is mostly shallower than \( \Theta_w \) surfaces and leads to a strong warming near A. Near B the ascent, though weaker, is more vertical and leads to significant cooling. Near D the ascent is very shallow and there is warming. The descent is generally weak but near and above E is sufficient to give some warming. Elsewhere it is approximately parallel to \( \Theta \) surfaces, except near C where it gives strong cooling. Reference to Fig. 5(c) and Eq. (27) shows that the temperature perturbation forces vorticity accelerations of the sign shown in Fig. 5(e). The temperature gradient between C and D acts so as to accelerate the lower cell. The mid-level gradient acts in the sense of accelerating the upper cell but its position tends to be rather too close to the maximum updraught. This may well explain why the growth of the upper cell is slightly less.

Thus the dynamics of this almost exponentially growing mode are quite easily understood. Unlike the fastest growing mode of dry symmetric instability in an infinite atmosphere, the temperature field and conversions from potential energy do play a positive role. However, the dominant feature is still the forcing of circulation through the wind shear term and the conversion from zonal kinetic energy by the negative \( v'v'' \) correlation.

The period 1-2 day gives an approximate e-folding time \( T_e = 3.7 \) hours. For comparison with the analysis presented in section 3, we have to choose a single value of \( \partial \Theta_w/\partial z \) which in the model varies from 0.3 \( K \) \( \cdot \) \( km^{-1} \) at \( z = 0 \) to 4.8 \( K \) \( \cdot \) \( km^{-1} \) at \( z = H_v \). Since the \( p = 732 \) mb level is close to the middle of the domain occupied by the cells, it is convenient to use its value of 1.2 \( K \) \( \cdot \) \( km^{-1} \). Then \( (N_2^2/N_1^2, Ri^{-1}) = (0.25, 1.4) \) and the values of \( T_e \) given by theory in section 3(b) (Eq. (24)) are 1.7, 2.3, 2.7 hours for \( \alpha = 0, 1, 2 \). The theory of section 3(c) gives \( T_e = 3.2 \) hours. We may note that a run which was identical apart from having diffusions set to zero produced a very similar mode but with the updraught on the scale of a few gridpoints and an e-folding time of 2.8 hours.

The second example, experiment B, had \( \partial \Theta/\partial z = 3.8 \) \( K \) \( \cdot \) \( km^{-1} \), \( \partial \Theta/\partial x = 1 \) \( K \) \( \cdot \) \( 30 \) \( km \) again, and \( f = 1.2 \times 10^{-4} \) \( s^{-1} \). Thus the moist gravitational stability was weaker and the inertial stability stronger than in the first case. The \( \Theta \) surfaces have a slope 1/114 and the \( M \) surfaces a steeper slope of 1/83. The flow is therefore dry symmetrically stable. The slope of the \( \Theta_w \) surfaces varies from 1/114 at the top to 1/6 at \( p = 732 \) mb. At 790 mb they become vertical, there being a decrease in \( \Theta_w \) of 0.6 \( K \) between the surface and this level. A further experiment, in which this reversal was removed, showed this feature not to be of importance. This time, to show how two bands may coexist, two perturbations of the stream function were imposed initially 225 \( km \) apart.

As in experiment A, there is initial decay and then approximate exponential growth after 1 day. The pictures of experiment B are shown for 1 day. Fig. 6(a) shows the cross-band streamfunction. There are two regions of concentrated slant-wise ascent at an angle of about 1/40 and still about 225 \( km \) apart. The maximum ascent is about 9 \( cm \) \( s^{-1} \). The descent is again broad and approximately along \( \Theta \) surfaces for much of the region. Allowing for growth, the main difference from 1 day earlier is that the upper cell of the left-hand band is comparatively much weaker. Other fields are very much as suggested by experiment A. In Fig. 6(b) are shown the wind perturbation along the band and the total \( \Theta_w \) field. Negative \( v' \) up to 4.2 \( m \) \( s^{-1} \) is associated with the ascent and negative \( u' \) up to 3.5 \( m \) \( s^{-1} \). The negative relative vorticity maximum is about 0.9 \( \times \) \( 10^{-4} \) \( s^{-1} \) so that the vertical component of absolute vorticity is nowhere negative at this time. The \( \Theta_w \) field shows destabilization near 700 mb just above the maximum ascent. Again if the integration is continued, this destabilization continues at a rapid rate.

The dynamical structure of the modes is very similar to that described in detail for
experiment A. The e-folding time, \( T_e \), is about 3·4 hours. Using the value of \( \partial \Theta_w / \partial z \) at \( p = 732 \text{ mb} \), this run is for \( (N_w^2/N^2, Ri^{-1}) = (0·053, 0·728) \). The section 3(b) model growth rate equation (Eq. (24)) for \( \alpha = 2 \) gives \( T_e = 33 \text{ min} \). The choice of reference level is important enough that Eq. (24) with \( \alpha = 2 \) gives the observed value if \( \partial \Theta_w / \partial z \) is evaluated at 650 mb. The theory of section 3(c) gives \( T_e = 3·7 \text{ hours} \).

(c) Summary of numerical results

Many numerical experiments have been performed to study the structure and growth rate of CSI as a function of model parameters. The structures described above are typical of those found. In order to present a picture of the growth rates obtained we assume, following the theory in section 3, that \( FT_e \) is approximately a function of \( Ri^{-1} (=S^4/N^2F^2) \), \( N_w^2/N^2 \) and the dissipation. By fixing \( \partial \Theta / \partial x \) at 1 K/30 km and varying \( N^2 \) and \( F^2 \), 17 values of \( FT_e \) have been obtained for zero explicit diffusion and 14 values for \( \nu_s = 2 \times 10^{-4} \text{ m}^2 \text{s}^{-1} \), \( \nu_z = 2 \text{ m}^2 \text{s}^{-1} \). From these values the contours \( FT_e = 1·5 \) have been drawn in Fig. 7 for the two diffusions. It should be noted that the first contour typically corresponds to an e-folding time of 4 hours and growth by a factor of 20 in 12 hours. Growth rates decrease so rapidly below the \( FT_e = 3 \) contours that these may be interpreted as stability boundaries for the numerical model. The crosses indicate the points corresponding to the experiments described above. For experiment A the values of \( FT_e \) were 0·77 and 1·0 without and with explicit dissipation, and experiment B gives \( FT_e = 1·5 \) with dissipation.

Fig. 7 was plotted using the value of \( N_w^2/N^2 \) at \( p = 732 \text{ mb} \), but the choice of this level is somewhat arbitrary. Considering this and the simple nature of the theory in section
3, the agreement of the numerical results with the theory is mostly satisfactory. At small values of $N^2_0/N^2$ the theory of section 3(b) appears to overestimate the instability. However, it is possible that the numerical model underestimates the growth due to lack of resolution of the concentrated updraughts in these steep modes.

As a partial test of the hypothesis that $FT_e$ depends only on $N^2_0/N^2$, $Ri^{-1}$ and the dissipation, the points (0.1, 0.69) and (0.89, 0.51) with zero explicit dissipation were investigated with $\partial \Theta/\partial x = 1 K/18.75 km$ as well as the standard $1 K/30 km$. Since the value of $S^4$ was increased by a factor 2.56, $F^2$ was decreased by this factor in each case. For the first point the basic value of $FT_e$ was 1.2 and the new value 1.4. For the second point the values were 2.2 and 2.5. Thus the dependence on $S^2$ does indeed appear to be mostly of the form $S^4/F^2$.

Some experiments with two initial disturbances as in experiment B described above have been designed to show the minimum distance between bands. This distance has been taken to be the smallest separation such that gravitational instability is created before the bands strongly interact. For the same basic $N^2$, $S^2$ and $F^2$ the distance is a function of
dissipation. With the values used in the second experiment above the distances were approximately 100, 200, 300 and 350 km for $v = 1$, 2, 5-5, 6-5 m$^2$s$^{-1}$ and $v = 10^4 v_z$.

Following the work of McIntyre (1970), the Prandtl number was varied for a few experiments. The growth rates did not change significantly although the ratio $v'/\theta'$ varied in the expected manner.

5. Observations

The information presented in observation studies is not sufficient to enable a detailed comparison of the structure of observed rainbands with that of the conditional symmetric instability modes described here. However, since Figs. 4 and 7 of Roach and Hardman (1975) suggested the presence of a $v, w$ correlation, their data were reanalysed using orthogonal polynomial fits. Fig. 8 shows $v$ and $w$ at 3 km across the rainbands. The basic vertical shear in $v$ is negative and so the good positive correlation between $v$ and $w$ strongly suggests that the kinetic energy of the bands comes at least partly from the kinetic energy of the large-scale flow. This is consistent with CSI theory. The correlation is good also at 4 km but not at 1.5 km which argues against the importance of Ekman layer instability. Many authors have drawn attention to the presence of mid-level convection. This is in agreement with the theoretical and numerical results presented above. A finite amplitude CSI mode may be expected to produce such mid-level gravitational destabilization.

An attempt has been made to fit observed baroclinic systems into the $N^2_v/N^2, R^{-1}$ framework suggested theoretically. Two sources of data were available. The first was a radar study undertaken by the Meteorological Office Research Unit, Malvern during the period June 1973 to June 1975, and the second is from good quality satellite pictures; the results from both are shown in Fig. 9.

During the period June 1973 to June 1975 all frontal regions that crossed the west coast of the British Isles were analysed from the information available in the Daily Weather Reports, supplemented in a few cases by data from numerical forecasts. Representative lower tropospheric values of $N^2_v/N^2$ and $R^{-1}$ were obtained and information on whether or not bands were present was provided by the radar pictures. These were available only for

![Figure 8](image-url)  
**Figure 8.** The data obtained from the Scillonia project, Roach and Hardman (1975), were re-analysed objectively using orthogonal polynomials. The figure shows $v$ and $w$ at a height of 3 km along a line almost perpendicular to the rainbands. (In the paper by Roach and Hardman the section is at $y = -10$ km; near the centre of the region.) The $v$ curve has a root-mean-square deviation from the measurement of less than 0.5 m$^2$s$^{-1}$ over the region indicated and in this region there is a positive correlation between $v$ and $w$. 
Figure 9. Observed baroclinic systems plotted in the parametric space \((N^2/N^2, Ri^{-1})\).

Data from the radar study June 1973–June 1975: ■ banded structure in the rainfall, ● no banded structure in the rainfall.

Data from the satellite study July 1977–December 1978: □ banded structure in the clouds, ○ no banded structure in the clouds. N.B. \(Ri^{-1}\) is underestimated in the satellite study, see text.

The stability boundaries for \(\alpha = 0, 2\) and \(L\) are indicated.

some of the cases and frequently they were insufficient to distinguish banded and non-banded precipitation. Positively identified cases are shown in Fig. 9 (see caption).

The satellite pictures were analysed during the period July 1977 to December 1978. It was found difficult to distinguish between a uniform cirrus sheet and a non-banded frontal zone, consequently non-banded cases were most often young wave depressions. The sample region was the North Atlantic, and representative parametric values were obtained from numerical forecasts. The resolution of the forecast data is 100 km; the most serious consequence of this is a smoothing of horizontal gradients and hence an underestimation of \(Ri^{-1}\). The results are shown alongside the radar study in Fig. 9. It is difficult to relate the two studies but in the few cases of the second study where radiosonde data were available, \(Ri^{-1}\) was underestimated by between 25 and 50% but this correction has not been applied.

The possible error in the positioning of each point is certainly as large as 0.1 in abscissa and ordinate. There are some points showing bands and some not showing bands which are consistent with CSI theory and few real disagreements when the underestimation of \(Ri^{-1}\) in the second study is taken into account.

6. Conclusion

The picture of rainbands which we suggest on the basis of the work described in this paper may be thought of as a three-stage process. First, as air moves northwards and rises through a baroclinic wave, its wet bulb potential vorticity may be made negative because of moisture gradients in the direction of the thermal wind and perhaps also because of diabatic effects. The second stage occurs when this air is lifted sufficiently to become saturated and thus becomes conditionally symmetrically unstable. This instability manifests itself as rolls approximately along the thermal wind, leading to a banded cloud structure. As the rolls
grow, the third stage occurs, when their motion causes conditional gravitational instability at mid-levels in preferred linear regions. The resulting convection leads to the strongly banded nature of the rainfall.

None of the analytical, numerical or observational evidence presented here conclusively confirms the hypothesis that CSI is important in the formation of rainbands. However, together they present a coherent picture which needs to be tested further. Numerical experimentation with more sophisticated mesoscale models should be undertaken. These models should, in the first place, be two-dimensional so that sufficient resolution may be obtained. It is possible that a model with a better inclusion of water vapour might give an indication of preferred length scales between the bands. At present we can only suggest a lower limit based on turbulent dissipation and an upper limit based on the depth over which significant water vapour is available and the slopes of $\Theta$ and $\Theta_w$ surfaces.

On the observational side, detailed study of a few rainband occurrences are required so that comparison with the quite simple dynamical framework suggested in section 4 can be made.

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**REFERENCES**


