Notes and Correspondence

Comparison and combination of two recent proposals for a generalized Penman equation

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SUMMARY

Two recent proposals for the generalization of the Penman equation based on completely different calibration attempts appear to yield a strong basis for such a generalization, if combined. Consequences of the combination and its use are discussed in the light of each of the original proposals.

1. INTRODUCTION

The continuing attention given to Penman's evaporation equation, after thirty years of research, shows clearly its importance in agricultural and hydrological water management. Recently there have been proposals based on experiment (Doorenbos and Pruitt 1977) and on theory (Thom and Oliver 1977) to generalize the equation in the interests of greater physical realism and higher accuracy. Although both these proposals are limited in the validity and scope of their generalization (Stigter 1979), combination of their features is here shown to yield a much firmer foundation.

2. COMPARISON OF PROPOSALS

In the aerodynamical term of the Penman equation, the wind function proposed by Doorenbos and Pruitt is based on thorough experimental calibration by grass lysimetry. That implied in the equations derived by Thom and Oliver is theoretically well founded but is empirically based only on Penman's original set of data. If we wish to use, for calibration, daily wind run (or average wind speed) data obtained from ordinary meteorological stations at 2 m height, a theoretically and experimentally founded reference should be obtained regarding the roughness length, $z_0$, and the displacement height, $d$, of the ideal average vegetated surface of such a meteorological station and its surroundings. For that purpose we derived values of $z_0 = 0.85 \text{cm}$ and $d = 4 \text{cm}$ (Stigter 1979), applying to the generalized logarithmic wind profile that is commonly used to convert wind speed measured at any height to wind speed at standard height.

Using these values we found, at high 24-hour average wind speeds of 4 to 5 m s$^{-1}$, for which values stability corrections are negligible, differences of 25 to 30% between an expression for the wind function derived from theory of momentum transfer over a rough surface under neutral atmospheric conditions and the proposal of Doorenbos and Pruitt. However, making use of a proposed lower value of the von Kármán constant (Businger 1975), both expressions could be brought into acceptable agreement at these high wind speeds (Stigter 1979). This yielded support for the wind function proposed in units applicable to saturation deficits in the aerodynamical term expressed in mb and wind speeds, $u$, expressed in km day$^{-1}$, namely:

$$f(u) = 0.27(1 + u/100),$$

Comparing Eq. (1) with the theoretical expression (with the lower von Kármán constant) for the neutral case at low 24-hour average wind speeds, below 2 m s$^{-1}$, we found the former to be appreciably higher. This may be accounted for by stability corrections, but these could not be assessed numerically, lacking a theoretical model.

Thom and Oliver have also proposed a generalized expression for the aerodynamical term in the Penman equation. They proved that their proposal includes corrections for non-neutral atmospheric stability conditions and they used a term for the degree of surface roughness. From their
propose that we can derive with our values for $z_0$, $d$, and the von Kármán constant as derived from Businger’s paper ($k = 0.37\pm0.02$):

$$f(u) = 0.37(1-u/160). \quad (2)$$

Values for Eqs. (1) and (2) at several average 24-hour wind run totals (or average wind speeds) are given in Table 1. Differences appear to be smaller than 5% at wind speeds from 1.8 to 5 m/s. At 1 m/s the difference is 12%. This should be compared with an inaccuracy due to the underlying assumptions in the derivation of the equation on which Eq. (2) is based, of at least ±10% at such wind speeds; comparable error limits arise from the spread of the experimental results underlying the choice of Eq. (1).

### Table 1. Numerical Values of Generalized Wind Functions $f(u)$, Determined from Eqs. (1) and (2)

<table>
<thead>
<tr>
<th>Wind speeds m/s</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>86</td>
<td>0.50</td>
<td>0.74</td>
<td>0.97</td>
<td>1.20</td>
<td>1.44</td>
</tr>
<tr>
<td>173</td>
<td>0.57</td>
<td>0.77</td>
<td>0.97</td>
<td>1.17</td>
<td>1.37</td>
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<tr>
<td>259</td>
<td></td>
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<tr>
<td>346</td>
<td></td>
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<tr>
<td>432</td>
<td></td>
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</tbody>
</table>

3. Consequences of the Results

From the fine analysis given by Thom and Oliver it can now be deduced that not only their wind function but also Eq. (1) largely include stability corrections. On the other hand their modified Eq. (18) is now also calibrated by world-wide collected experimental results, tested in different climatological zones, published by Doorenbos and Pruitt (1977), if we use the lower $k$ value for which evidence has been reviewed (Businger 1975). In that case Eqs. (1) and (2) can be seen as almost equivalent and both can be applied over a wide range of conditions.

Two more consequences of the allowed combination of the two approaches should be mentioned:

(i) Using $k = 0.37$ instead of 0.41 would change, in Eq. (18) of Thom and Oliver, the constant 13.8 to 11.2. Identity with the original Penman equation would now be derived for $z = 2 m$ above $d$, when $z_0 = z_{00} = 0.28 cm$ ($z/z_0 = 71.5$). The latter change has no consequences because the new as well as the lower, old values of $z_{00}$ are far too low. Taking account of the fact that we prefer $z$ to be measured from the soil surface upwards, we may write their Eq. (18) for the aerodynamical term as:

$$E_n = 30f(u)(e_0 - e_0)/(\ln(z-d)/z_0)^2 \quad (3)$$

applying equally well to $f(u)$ of Eqs. (1) and (2). Another numerical consequence of the above is the fact that the value of 2.5 for the parameter $m$ used in Thom and Oliver’s paper now applies to a $z/z_0$ ratio of 65 rather than 100. For a $z/z_0$ ratio of 100, $m$ would have the value 2.0. This nowhere invalidates the principles of their preliminary deductions and implications.

(ii) A more radical consequence to be added to the analysis of Thom and Oliver, from the experimental results used by Doorenbos and Pruitt, stems from consideration of differences between daytime and nighttime weather conditions. Experimental evidence (Tanner and Pelton 1960; Doorenbos and Pruitt 1977) shows that corrections should be applied under some climatological conditions if 24-hour averages of wind speed and atmospheric humidity are used in the Penman equation. From the experiments also used to support Eq. (1), such correction factors to the Penman equation appear to be negligible in the use of Eqs. (1), (2) and (3) under the most common conditions, where radiation is moderate to high, maximum relative humidity is medium to high, and moderate daytime wind is about double the nighttime wind (Doorenbos and Pruitt 1977).

For other conditions, underprediction and (more frequently) overprediction occur. Of course, the empirical statistical relationships used in the radiation term of the Penman formula play a role in calibrating the correction factors concerned. It is important that in this respect a generalization by Doorenbos and Pruitt of the relationship between hours of bright sunshine and solar radiation received is confirmed for East African conditions, using a recent more detailed generalization by Rietveld (1978). The latter can be of help in determining local or seasonal correction factors, where needed. The weakest link of the chain appears to be the relationship for the estimation of net longwave radiation. Pressure dependence of the wind function is fully incorporated by a pressure dependence of the psychrometric constant (Stiger 1978).
4. DISCUSSION AND CONCLUSIONS

It may be concluded that the combination of the two approaches discussed above yields a firm basis for a well calibrated generalization of the Penman equation. It is clear that the use of Eq. (3) in combination with daytime/nighttime adjustment factors does not include any stomatal resistance, integrated over the vegetation to which the Penman equation is applied (Stigter 1974), apart from the average crop stomatal resistance of the grass lysimeters used for calibration of Eqs. (1) and (2).

It may be argued that the discrepancy between the Doorenbos and Pruitt results and a rigorously momentum-oriented wind function, using the classical $k$ value, is only due to the fact that the reference crop stomatal resistance is not zero. However, in this note we wish to defend the stand that, as long as sound reviews suggest lower $k$ values as well as small reference crop stomatal resistances, it is preferable to take the latter as zero for practical purposes (Stigter 1979). Moreover, this is in line with definitions of potential evaporation that are in common use and with some existing simple models in use for transpiration modeling. The wind function to be used is now appreciably improved and physically much better founded.

From here one can proceed in different ways. In the case of the determination of crop coefficients in agricultural water management problems (Doorenbos and Pruitt 1977) one may include higher actual crop stomatal resistances into these coefficients under ‘water not-limiting’ conditions. ‘Limiting soil water’ conditions can then be incorporated separately. In hydrometeorological practice one may include the full stomatal resistance of the vegetation directly into an again more general form of the Penman equation (cf. parameter $n$ of Thom and Oliver, whose Eq. (19) using our Eq. (3) can be recommended).

In both cases the calibration problem remains. In the former, only lysimeter experiments and a more physical approach to crop coefficient values from field studies of soil and crop water status can solve the problem. In the latter, catchment area water balance studies may contribute to the solution. In both cases, before numerical values gain wide acceptance, all factors discussed in the combination of approaches outlined in this paper should be studied. We believe that Eq. (3) can now be used as a starting-point.

REFERENCES


