Wind direction statistics and lateral dispersion

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SUMMARY

Eulerian observations of wind direction fluctuations in slightly unstable conditions are analysed in terms of Taylor’s statistical theory of dispersion which, with the Hay–Pasquill assumption of similar shapes for Lagrangian and Eulerian spectra, relates the standard deviation of the crosswind material distribution, \( \sigma_v \), to the standard deviation of fluctuations in wind direction, \( \sigma_x \). The dependence of the predicted crosswind dispersion parameter, \( S = \sigma_x \sigma_v \), where \( x \) is the distance downwind from the source, on travel time, \( T \), is examined, and it is concluded that the data are well represented by an empirical form originally proposed by Draxler to fit observed crosswind dispersion.

In contrast to direct dispersion observations, the present data show little scatter and unambiguously demonstrate that the common assumption of an exponentially decaying correlogram results in a less accurate representation. In particular, the exponential form approaches the large-time limit, \( S^2 \propto 1/T \), too rapidly, and thus may be expected to perform poorly in extrapolating from a limited range of data.

The consistency of the Draxler form in representing both the Eulerian measurements presented here, and direct Lagrangian dispersion observations, serves to support not only the form itself, but also, indirectly, the Hay–Pasquill hypothesis.

Evidence of a correlation between the intensity and scale of turbulence is also presented, which, if confirmed over a larger range of atmospheric stability conditions than considered here, will be very useful for estimating dispersion from wind statistics.

1. INTRODUCTION

The crosswind spread of material from a source is often quantified by the lateral standard deviation of the concentration of material, \( \sigma_v \). One of the most fruitful approaches to the estimation of this quantity for a continuous point source has been Taylor’s statistical theory for homogeneous stationary turbulence (Pasquill 1974, p. 123), according to which the variance of the crosswind displacement of an ensemble of particles after time of travel \( T \) is

\[
\sigma^2_v(T) = \sigma_v^2 T^2 \int_0^\infty F_L(n) \left\{ \sin^2(\pi n T)/(\pi n T)^2 \right\} dn,
\]

where \( \sigma_v^2 \) is the variance of the lateral velocity component, and \( F_L(n) \) is the Lagrangian spectrum function for the lateral velocity component and is a function of the frequency \( n \).

The effect of the integral term in Eq. (1) is to reduce the variance by filtering out components of frequency higher than about \( 1/T \). The ensemble statistics are equivalent to the statistics of the time series of the crosswind velocity component of a single particle, and the filtering process in Eq. (1) corresponds to simple linear averaging over time \( T \) of this time series.

It is useful to define the nondimensional lateral dispersion parameter

\[
S^2 = \frac{\sigma^2_v}{\sigma_v^2 T^2} = \frac{\sigma^2_v}{\sigma^2_v,0},
\]

where the second subscript is the averaging time. \( S \) must have limits

\[
S^2 \rightarrow 1 \quad \text{as} \ T \rightarrow 0
\]

and

\[
S^2 \rightarrow 2\tau_f / T \quad \text{as} \ T \rightarrow \infty,
\]
where the Lagrangian integral timescale is defined by

\[ t_L = \int_0^\infty R_L(\xi) \, d\xi, \]  

(6)

and \( R_L(\xi) \) is the Lagrangian correlogram, the inverse cosine transform of \( F_L(n) \). In general, \( S \) depends on both the functional form of \( F_L(n) \) (or equivalently \( R_L(\xi) \)), and on the Lagrangian timescale \( t_L \). However, because of the difficulty in making Lagrangian turbulence measurements, \( F_L(n) \) and \( t_L \) must be determined indirectly.

From a comparison of dispersion calculated for various forms of \( R(\xi) \) ranging from an exponential to a step function, Pasquill (1974, p. 132), in an initial approach to the problem, concluded that dispersion is insensitive to wide variations in the shape of the correlogram and that a good first approximation to \( \sigma_y \) may be obtained from a good estimate of \( \sigma_v^2 \) and a rough estimate of \( t_L \).

Draxler (1975) on the other hand, determined \( S \) as a function of travel time directly from a large body of diffusion measurements, and proposed an empirical form equivalent to

\[ S^{-1} = 1 + (T/2t_L)^4 \]  

(7)
as a satisfactory fit to the data. Pasquill (1975a) noted that this form is considerably different from the range of forms he considered earlier, but pointed out that the large scatter in the diffusion data makes the difference of questionable significance. He (Pasquill 1975b) re-examined data from the ‘Greenglow’ and ‘Hanford 30’ (Fuquay et al. 1964) programmes and concluded that the fit to \( S \) corresponding to the exponential correlogram is poor compared with Eq. (7) and another form corresponding to a spectrum function proposed by Pasquill and Butler (1964).

In a later paper (Pasquill 1976) he presents values of \( S \) estimated by eye from an examination of available diffusion data including more recent data from the St Louis dispersion study. These values are well represented by Eq. (7) and less so by the exponential form, but again the uncertainty due to the scatter of data is large.

Thus, although it is now clear that crosswind dispersion is not insensitive to the shape of the Lagrangian spectrum function, the dimensionless dispersion function \( S \) has not yet been unambiguously determined because of the large scatter inherent in diffusion data.

An alternative approach to the calculation of dispersion from turbulence measurements is due to Hay and Pasquill (1959), who adopted the hypothesis that the Lagrangian correlogram and spectrum function may be obtained by a simple timescale transformation of the corresponding Eulerian or fixed-point quantities. Specifically

\[ R_L(\beta t) = R_E(t) \]  

(8)

and

\[ nF_L(n) = \beta nF_E(\beta n), \]  

(9)

where \( \beta \) is the ratio of Lagrangian to Eulerian integral timescales. According to Pasquill (1974, p. 359) the best estimate of this ratio is

\[ \beta = 0.44/\sigma_{\theta,0}. \]  

(10)

Thus following Hay and Pasquill, Eq. (3) may be rewritten

\[ S^2 = \sigma_v^2 t/\beta / \sigma_{\theta,0}^2, \]  

(11)

where the variances now represent Eulerian measurements.

Finally, Eq. (11) may be written in terms of the variance of wind direction fluctuations using the approximate relation

\[ \sigma_v \approx \bar{u} \sigma_{\theta} \]  

(12)
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which assumes \( \tan \theta' \approx \theta' \) and \( \bar{\omega}' \theta^2 \ll \bar{u} \theta^2 \), where the primes denote eddy quantities, \( \theta' \) is the angle between the instantaneous wind vector and the mean wind direction, and \( u \) is the longitudinal wind component. Then

\[
S^2 = \sigma_{\theta,T/\beta}^2/\sigma_{\theta,0}^2.
\]

(13)

Equation (13) enables \( S \) (and hence from Eqs. (2) and (12) \( \sigma_y \)) to be calculated as a function of travel time \( T \) through the effect of averaging time \( T/\beta \) on Eulerian time series of wind direction.

Values of \( \sigma_y \) calculated in this way have been shown to agree very well on average with direct measurements (Isliker and Dumbauld 1963), and in fact this method is the recommended one for estimating \( \sigma_y \) when Eulerian wind direction records are available (Pasquill 1974, p. 359). The success of the method is due to the fact that \( \sigma_y \) involves an integral of the spectrum function, so that similarity between the 'Eulerian dispersion function', \( S \), calculated from Eq. (13), and the true Lagrangian dispersion function, requires the scaling of Eqs. (8) and (9) to apply only in a broad sense.

This paper takes advantage of this apparent broad similarity to examine the form of the dispersion function \( S \) using Eulerian wind direction data. Two forms for \( S \), Eq. (7), and that corresponding to an exponential correlogram,

\[
S^2 = (2t^2/T^2)\{\exp(-T/t_c) + T/t_L - 1\},
\]

(14)

have been tested against observation. These forms were chosen because they roughly encompass the range previously considered (see, for example, Pasquill 1975a). Despite the fact that the present method relates only indirectly to the true Lagrangian \( S \), it has the advantage that the measurements required are relatively much simpler and more reliable than diffusion measurements and thus should provide a clearer test of the functional form of \( S \).

2. Experiments

Eight wind direction time series of nominally one hour sampling duration have been measured in near-neutral conditions. Runs 1–6 were measured at a height of 2.3 m over flat grassland (roughness length \( \approx 1 \) cm) whilst runs 7–8 were measured at a height of 1.5 m at a similar site. For runs 1–6, net radiation at 1 m and wind speed at 2.3 m were also recorded. For all runs wind speed was near 7 m s\(^{-1}\), while net radiation varied from 80 to 500 W m\(^{-2}\).

A sensitive vane of time constant about 1 s was used and its output recorded on a chart recorder and on a portable cassette analogue recording system (Jordan 1974). Tests indicate error contributions by the recording system of less than 1 \( \% \) to \( \sigma_\theta \).

Further error is likely because of high frequency cut-off due to the limited response of the vane. Measurements by Jones (1963) suggest that this is not more than 10 \( \% \) of the measured variance (or 5 \( \% \) of \( \sigma_\theta \)).

3. Data treatment

Data were extracted from the cassette recording system at 1 s intervals. Most runs show trends despite attempts to choose stationary conditions. Linear trends were removed before analysis in order to improve stationarity but no attempt was made to identify and remove low frequency sinusoids.

The data were then treated as prescribed by Smith (1962). First a new time series was constructed by taking running means of the original series over length \( t \). Then the variance over a period \( \tau \) was calculated from the derived series. The original series were nominally one hour long and the maximum \( t \) considered was 450 s so that \( \tau \) was typically about 50
minutes, which is adequate to sample most of the microscale variance (Pasquill 1975b). For each of the eight runs $\sigma_\theta$ was evaluated for averaging intervals $t = 0, 1, 3, 7, 14, 29$ s and then at steps of 30 s up to 449 s.

4. FUNCTIONAL FORM OF $S$

Both Draxler’s (Eq. (7)) and the exponential (Eq. (14)) forms of $S$ were fitted by the method of least squares to the data to determine $\sigma_{\theta,0}$, and the timescale $t_1$, the time for which $S$ equals one half. The least-squares parameters, $\sigma_{\theta,0}$ and $t_1$, are shown in Table 1 for all runs. Note that the relation between $t_1$ and the Eulerian integral timescale, $t_{Ei}$, depends on the form of $S$. For Eq. (7), $t_1 = 2t_{Ei}$, while for Eq. (14), $t_1 = 6.83t_{Ei}$.

Data from the eight runs, normalized with respect to $\sigma_{\theta,0}$ and $t_1$, are plotted with the appropriate form in Figs. 1(a) and (b). In both cases, deviations from the fitted curve for individual runs tend to be systematic rather than random. However, deviations of the ensemble of runs tend to be fairly random about Draxler’s form in contrast to the exponential fit where even the ensemble of runs shows systematic deviations. The observed deviations from Draxler’s form for individual runs may result from residual low frequency sinusoids in the original time series. Such low frequency elements, being uncorrelated from run to run, would result in a random scatter about the true stationary form of $S$. In the present case Eq. (7) seems to be a better representation of that form than Eq. (14).

Of particular interest is the relative performance of the two forms in estimating $\sigma_\theta$. Figures 2(a) and (b) show the mean relative deviation between measured and fitted values of $\sigma_\theta$, as a function of averaging time, $t = T/\beta$. In effect, this represents the relative error in $\sigma_\theta$ as a function of distance downwind, $x$, since the relative errors for $\sigma_x$ and $\sigma_\theta$ are identical and $x = \beta \bar{u} t$, where $\bar{u}$ is the mean wind speed. Equations (7) and (14) represent the wind direction observations on average to within 3% and 10%, respectively, and may thus be expected to represent crosswind dispersion, on average, to similar accuracy. There will be scatter between measured and predicted dispersion in individual cases due to errors in diffusion measurements and due to variations in $\beta$ arising from statistical variation in the spectra from run to run. The results of Islitzer and Dumbauald (1963) suggest a scatter of up to 50%.

If used with the same (independently derived, say) value of $t_1$, Eqs. (7) and (14) yield even more disparate dispersion estimates (see, for example, Fig. 4 of Pasquill (1975a)). In particular, Eq. (14) approaches the large-time limit, (5), more rapidly. Thus at $T = 10t_1$, the $S$ calculated from the limit (5) is only 5% larger than that from Eq. (14), but nearly 45% larger than that from Eq. (7).

<table>
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<tr>
<th>Run</th>
<th>Measured $\sigma_{\theta,0}$</th>
<th>$t_1$</th>
<th>Least-squares fit $\sigma_{\theta,0}$</th>
<th>$t_1$</th>
<th>$\sigma_{\theta,0}$</th>
<th>$t_1$</th>
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<tr>
<td>1</td>
<td>0.1834</td>
<td>80</td>
<td>0.1799</td>
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Figure 1. Nondimensional plot of dispersion $S = \sigma_{\theta_e} / \sigma_{\theta_o}$ as a function of averaging time for (a), Eq. (7) and (b), Eq. (14). $\sigma_{\theta_e}$ is the measured wind direction standard deviation for averaging time $t$, $\sigma_{\theta_o}$ and $t_i$ being the appropriate least-squares parameters from Table I. The points represent data for all runs; the lines represent (a), Eq. (7) and (b), Eq. (14).
Figure 2. Mean relative deviation of $\sigma_{\theta}$ measurements from fitted values as a function of averaging time $t$, for (a), Eq. (7) and (b), Eq. (14). Error bars represent one standard error.

In practice, however, integral timescales are not estimated independently, but require the assumption of a particular spectral form. In representing the present observations, Eq. (7) implies a timescale about four times larger than does Eq. (14), which thus approaches the large-time limit relatively even more rapidly. Consequently, although the exponential form has been forced by the fitting procedure to represent the observations reasonably well, it cannot be expected to extrapolate very well.

Also shown in Table 1 are the 'actual' values of $\sigma_{\theta,0}$ and $t_1$ – the measured $\sigma_{\theta,0}$, and $t_1$ estimated directly from the data as the averaging time $t$ for which $\sigma_{\theta,t} = \sigma_{\theta,0}/2$. These values are close to the least-squares values corresponding to Eq. (7). If the actual values of $\sigma_{\theta,0}$ and $t_1$ are used to normalize the data, figures similar to Fig. (a) and (b) result, but with Eq. (7) slightly more obviously a better representation of the data.

The present data can be used to infer a Lagrangian timescale from each run, from the Eulerian $t_1$ together with a value of $\beta$ determined from Eq. (10) using the measured value of $\sigma_{\theta,0}$. The values of $\beta t_1$ found range from 87 to 440 s and are thus consistent with the Lagrangian values determined by Draxler (1975) (see his Figure 3) from diffusion data.

5. DEPENDENCE OF $\sigma_{\theta,0}$ AND $t_1$ ON STABILITY

Unfortunately, stability was not measured directly. However, runs 1–6 were over the same surface with very similar mean wind velocities, so net radiation is taken as a measure of stability. Because of the relatively high wind speeds (~7 m s$^{-1}$), all runs fall within Pasquill's D stability class and thus cover only a small range of stabilities near neutral (slightly unstable). Figure 3(a) suggests a reasonable correlation of both $\sigma_{\theta,0}$ and $t_1$ with stability over this range. The dependence of $\sigma_{\theta,0}$ on stability is well known (Smith and Abbott 1961) but there is little support in the literature for a relationship between the scale of turbulence and stability, for lateral velocity spectra in unstable conditions. Eulerian spectra analysed by Busch et al. (1968) show an increase in scale with increasing instability but Kaimal et al. (1972) report a lack of systematic dependence on stability in unstable conditions. The estimates of the Lagrangian $t_1$ by Draxler also show only a weak tendency to increase with increasing instability, but the scatter is large.
For the present data the correlation of both $t_1$ and $\sigma_{g,0}$ with stability means that these parameters correlate with each other (Fig. 3(b)). If such a relationship indeed exists (over a larger range of stability), then for many purposes measurement of the total intensity of turbulence, $\sigma_{g,0}$, may be adequate to specify the distance dependence of $\sigma_\varphi$. Such a relation would also depend on the properties of the surface.

6. Conclusions

This limited study of fixed point wind direction fluctuations suggests the following conclusions.

1. The empirical form (7), proposed by Draxler for the effect of travel time on dispersion, is also a good representation of the effect of averaging time on the variance of fixed-point wind direction fluctuations. In contrast, Eq. (14), which corresponds to an exponential correlogram, not unexpectedly is a poorer representation of these data.

2. According to the Hay–Pasquill hypothesis, Draxler’s form should also be a good representation of the effect of averaging time on the variance of Lagrangian fluctuations, and hence of the effect of travel time on dispersion. In so far as the analysis of diffusion data supports this conclusion, the Hay–Pasquill hypothesis, in the weaker sense discussed in section 1, also receives some justification. In other words, the consistency (including the range of timescales observed) between Eulerian data discussed here and the Lagrangian diffusion data analysed in the literature, supports both the Hay–Pasquill hypothesis and the Draxler dispersion relation.

3. The Draxler relation may thus be recommended for practical estimates of dispersion when values for $\sigma_{g,0}$ and $t_1$ are known. Alternatively, it may be used to interpolate and extrapolate limited wind direction statistics such as those available from the band-pass filter technique developed by Jones and Pasquill (1959) and Jones (1963), and used by Moore (1967).

4. Over the limited stability range considered, both $\sigma_{g,0}$ and $t_1$ correlate with stability, thus suggesting the possibility of a useful working relationship between them. Further observations over a wider range of stabilities are desirable in order to test this relationship.
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