A diagnostic study of the long-term budget of momentum of atmospheric large scale motion over the British Isles

By E. O. HOLOPAINE

Department of Meteorology, University of Helsinki

(Received 20 February 1979; revised 9 April 1979. Communicated by Dr D. H. McIntosh)

SUMMARY

Different terms in the equations of zonal and meridional motion for large scale flow in the atmosphere have been evaluated using British aerological data for the period 1974–76; the effect of motion on scales smaller than a few hundred kilometres on large scale flow is evaluated as the residual term in these equations.

In the long-term mean there is horizontal advection of zonal momentum into the area of the British Isles. This advection, which is characteristic of both the time-mean flow and transient disturbances and takes place mainly in the zonal plane, is to a large extent compensated for by Ferrel-type ageostrophic meridional circulation. The momentum budget in the meridional direction reflects the high degree of geostrophic balance in the time-mean zonal flow in the free atmosphere.

The mean residual forces in the atmosphere below 700 mb are used to infer the vertical sub-grid scale flux of momentum by assuming that the residual forces at these levels are due to vertical eddy stresses (i.e. stresses involving the vertical velocity component) only, and by evaluating the stress at the earth’s surface with the aid of the geostrophic drag law. The results indicate that the annual mean vertical flux of zonal momentum above about 900 mb is downwards but relatively small; the corresponding flux of meridional momentum is relatively large and upwards. Mesoscale circulations in connection with fronts are suggested as a possible explanation of the latter phenomenon which, however, may also partly arise in calculations as the result of the poor vertical resolution of the routine upper wind observations in the atmospheric boundary layer.

The residual force in the upper troposphere is very variable. However, consistent with an earlier analysis of the kinetic energy budget using the same data, it exhibits the ‘negative viscosity’ phenomenon, implying acceleration of the large scale flow by motion on scales smaller than the average distance between aerological stations.

1. INTRODUCTION

In an area containing a good network of aerological stations it is possible to investigate the balance requirements of large scale flow in more detail than in areas of more global scale. Several diagnostic studies have been made using data from North America. Some recent investigations (e.g. Blackmon et al. 1977; Holopainen 1978; Lau 1978) show, however, that there is a large longitudinal variation in the way in which different balance requirements are fulfilled, and that results for North America may not be representative of extratropical regions in general.

In one of the long-term budget studies for the European region, which has a higher density of aerological stations than North America, the author (Holopainen 1973, henceforth referred to as A) used aerological data from six British stations for the 5 years 1961–65 to study the long-term balance of momentum and kinetic energy at the 300 mb level; this study was complemented from the point of view of kinetic energy by another investigation (Holopainen and Erola 1979, henceforth referred to as B) using more recent data (3-year period 1974–76) from the same stations for the whole atmospheric column below 150 mb. Both A and B demonstrated the very different behaviour of the atmosphere over the British Isles compared with over North America. Thus, for example, while an average generation and net export of kinetic energy is characteristic of North America, the contrary is true for the British Isles. Further, while the residual term in the kinetic energy equation shows a loss of kinetic energy over North America both in the boundary layer and at the jet stream
level, corresponding results for the British Isles indicate an energy input in the upper troposphere from sub-grid scales to the synoptic scale.

This paper is a continuation of B. The same data are used to evaluate all those terms in the equations of horizontal motion which depend only upon large scale motion. The residual term necessary to make the balance is considered from the point of view of representing forcing due to processes of sub-synoptic scale.

2. Data and Method

As in B, the main data used in this study are the routine aerological observations of geopotential height, wind and temperature for the six British aerological stations shown in Fig. 1, for the 3 years 1974–76; for discussion of these data the reader is referred to B. Also,

![Figure 1](image_url)

Figure 1. The network of aerological stations used. The main discussion in the paper concerns results obtained by the 'linear method' for the polygon formed by the five triangles. The broken lines define an area for which some auxiliary calculations have been made using the 'second-degree method'.

some new results are reported from calculations for the 300 mb level using data for the period 1961–65 (for more details, see A).

There is probably some bias in certain quantities presented in this paper with respect to their long-term climatological values (see B). For example, the magnitude of the annual mean wind at 300 mb (Fig. 7) is about 2 m s$^{-1}$ smaller than that calculated in A from a more complete data sample for the period 1961–65. However, in this article we are mainly interested in the relative importance of the different terms in the momentum budget. From this point of view the deficiencies in the data are insignificant.
The equations of horizontal motion in a spherical \( p \) coordinate system can be written, with conventional notation, as

\[
\begin{align*}
R_u &= \frac{\partial u}{\partial t} + V \cdot V u + \omega \frac{\partial u}{\partial p} - (uv \tan \phi)/a + (1/a \cos \phi) (\partial \Phi/\partial \lambda) - f \nu, \\
R_v &= \frac{\partial v}{\partial t} + V \cdot V v + \omega \frac{\partial v}{\partial p} + (uv \tan \phi)/a + (1/a) (\partial \Phi/\partial \phi) + f u,
\end{align*}
\]  

(1)  

(2)

where \( R_u \) and \( R_v \) denote, respectively, the zonal and meridional components of the residual force.

In this study all the terms on the right of these equations have been evaluated from direct observations except for vertical advection terms in which \( \omega \) was obtained indirectly from the thermodynamic energy equation using the assumption of adiabatic motion:

\[
\omega_a = - (\partial T/\partial t + V \cdot V T) (\partial T/\partial p - q/c_p)^{-1}.
\]  

(3)

The sum of the six terms on the right in Eqs. (1) and (2) is then, in principle, an estimate of the zonal and meridional components of the ‘frictional’ force due to the sub-grid scale motion systems. Because \( R_u \) and \( R_v \) here contain, in addition to real sub-grid effects, any errors made in the evaluation of the right-hand side terms, it is better to refer to \( R_u \) and \( R_v \) as the zonal and meridional components of the residual force, \( R \).

The calculations were made separately for the five triangles seen in Fig. 1, which thus form our grid system. In each triangle, all variables were assumed to be linear functions of \( \lambda \) and \( \phi \). The mean value of a variable in a triangle was computed as the arithmetic mean of the variable at the three corner stations. The horizontal advection of a variable was determined as the product of the triangle-average wind and the gradient of the variable. For more details concerning this ‘linear method’ see A.

Some auxiliary calculations were made (for the area confined by the broken line in Fig. 1) using the ‘second-degree method’, in which the horizontal variation of variables was represented by second-degree polynomials. In A the reader finds a comparison between the results obtained for the momentum budget at 300 mb with the linear method and the second-degree method.

The various terms in Eqs. (1) and (2) were calculated as averages for all those 12-hour periods which had complete observations, in the same way as was done in B for kinetic energy. Making calculations on the basis of 12-hour periods, determined by aerological observations, the local time derivatives were determined from the observed change of \( u \) and \( v \) during such a period. The vertical advection terms were calculated as the product of the period-mean (adiabatic) vertical velocity and the period-mean vertical gradient of momentum. The 12-hour averages for all other quantities were calculated as the arithmetic mean of the values at the beginning and the end of the period.

In this paper, we do not discuss results for different triangles but only those obtained for the whole hexagon formed by the five triangles (Fig. 1). Results obtained by the second-degree method are averages over the region, which in Fig. 1 is confined by the broken line, and which covers approximately the same area as the hexagon.

3. Climatological upper wind conditions over the British Isles

Before discussing the momentum budget of the atmosphere over the British Isles, it may be of interest to look briefly at the general climatological upper wind conditions in this area. For this purpose, Fig. 2 shows the vertical distribution of \( \bar{u} \) and \( \bar{v} \) (as usual, a bar denotes a time average) and the corresponding standard deviations for the whole period (1974–76) considered. The British Isles is a representative mid-latitude region characterized by mean westerlies with superimposed Ferrel-type meridional circulation (northward below 500 mb,
Figure 2. Mean values and standard deviations of zonal and meridional wind components over the British Isles during 1974-76.

southward above), and by wide variability, indicated by the large values of \( \sigma_u \) and \( \sigma_v \). With regard to \( \sigma_u \) and \( \sigma_v \) it is also worth noticing that they are nearly equal at all levels. This indicates that in transient fluctuations over the British Isles the kinetic energy is almost equipartitioned between the zonal and meridional wind components.

4. MEAN ANNUAL BUDGET OF ZONAL AND MERIDIONAL MOMENTUM

The results of the analysis of the terms in Eqs. (1) and (2) are given in Tables 1 and 2 and Figs. 3(a) and (b). The mean balance of forces in the zonal direction appears to be quite different from that in the meridional direction. This is quite natural because the zonal budget roughly represents the mean forces parallel to, the meridional budget those normal to, the mean flow. One common feature in the budgets is that the values of the first term (mean local change) are practically zero.

Considering first in more detail the budget of zonal momentum, it is seen from Fig. 3(a) that the mean meridional velocity over the British Isles is to a large extent ageostrophic. (The geostrophic wind is defined in the usual way as \( V_g = (1/f)k \times \nabla \Phi \), with components \( u_g \) and \( v_g \).) As the middle part of the figure indicates, the ageostrophic meridional circulation is of Ferrel-type with northward ageostrophic flow in the friction layer and southward ageostrophic flow in the free atmosphere. (In a particular longitude sector there is not the need, as there is in zonally averaged conditions, for complete mass balance between the northward and southward branches of the ageostrophic flow.)

Blackmon et al. (1977) found in their observational study of the winter circulation in the

| TABLE 1. MEAN FORCES IN THE ZONAL DIRECTION OVER THE BRITISH ISLES DURING 1974-76. UNITS: \( 10^{-4} \text{m/s}^2 \) |
| Pressure level (mb) | \( \partial u/\partial t \) | \( + \nabla \cdot \vec{u} \) | \( + \omega \partial u/\partial p \) | \( -(\omega \tan \phi)u \) | \( +(1/\cos \phi) \partial \Phi / \partial \lambda \) | \( -f\nu \) | \( R_o \) |
|---|---|---|---|---|---|---|
| 1000 | 0.01 | -0.03 | 0.00 | 0.00 | 0.95 | -0.93 | -0.02 |
| 950 | 0.00 | -0.18 | -0.06 | -0.02 | 1.06 | -1.53 | -0.81 |
| 900 | 0.00 | -0.26 | 0.03 | -0.02 | 1.17 | -1.43 | -0.50 |
| 850 | -0.01 | -0.33 | 0.01 | -0.02 | 1.28 | -1.17 | -0.23 |
| 800 | 0.00 | -0.38 | 0.00 | -0.02 | 1.31 | -1.08 | -0.18 |
| 700 | 0.00 | -0.49 | 0.02 | -0.03 | 1.33 | -0.84 | -0.01 |
| 600 | -0.01 | -0.62 | 0.04 | -0.03 | 1.15 | -0.57 | -0.04 |
| 500 | -0.01 | -0.76 | 0.04 | -0.03 | 0.69 | 0.10 | 0.03 |
| 400 | -0.03 | -1.01 | 0.06 | -0.03 | 0.39 | 0.77 | 0.16 |
| 300 | -0.01 | -1.54 | 0.00 | -0.03 | -1.02 | 2.75 | 0.19 |
| 200 | -0.02 | -1.60 | -0.04 | 0.03 | -3.78 | 5.74 | 0.33 |
| 150 | -0.01 | -0.73 | -0.02 | 0.02 | -2.48 | 3.82 | 0.61 |
| Mean | -0.01 | -0.76 | 0.04 | -0.02 | 0.23 | 0.51 | 0.03 |
### Table 2. Mean forces in the meridional direction over the British Isles during 1974–76. Units: $10^{-4}$ m s$^{-2}$

<table>
<thead>
<tr>
<th>Pressure level (mb)</th>
<th>$\partial v / \partial t$</th>
<th>$- V \cdot \nabla v$</th>
<th>$\omega \cdot \nabla v / \partial p$</th>
<th>$-(\mu \tan \phi) a$</th>
<th>$(1/a) \partial \Phi / \partial \phi$</th>
<th>$+ f u$</th>
<th>$R_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
<td>0.05</td>
<td>-3.99</td>
<td>1.10</td>
<td>-2.84</td>
</tr>
<tr>
<td>950</td>
<td>-0.01</td>
<td>0.05</td>
<td>0.16</td>
<td>0.12</td>
<td>-4.37</td>
<td>2.83</td>
<td>-1.21</td>
</tr>
<tr>
<td>900</td>
<td>-0.01</td>
<td>0.09</td>
<td>0.07</td>
<td>0.17</td>
<td>-4.74</td>
<td>4.04</td>
<td>-0.38</td>
</tr>
<tr>
<td>850</td>
<td>-0.01</td>
<td>0.12</td>
<td>0.03</td>
<td>0.19</td>
<td>-5.12</td>
<td>4.71</td>
<td>-0.08</td>
</tr>
<tr>
<td>800</td>
<td>-0.01</td>
<td>0.10</td>
<td>0.03</td>
<td>0.21</td>
<td>-5.70</td>
<td>5.30</td>
<td>-0.08</td>
</tr>
<tr>
<td>700</td>
<td>-0.01</td>
<td>0.04</td>
<td>0.04</td>
<td>0.26</td>
<td>-6.96</td>
<td>6.47</td>
<td>-0.16</td>
</tr>
<tr>
<td>600</td>
<td>-0.01</td>
<td>-0.11</td>
<td>0.05</td>
<td>0.35</td>
<td>-8.37</td>
<td>7.84</td>
<td>-0.25</td>
</tr>
<tr>
<td>500</td>
<td>0.01</td>
<td>-0.37</td>
<td>0.06</td>
<td>0.48</td>
<td>-9.96</td>
<td>9.26</td>
<td>-0.52</td>
</tr>
<tr>
<td>400</td>
<td>0.00</td>
<td>-0.66</td>
<td>0.05</td>
<td>0.68</td>
<td>-11.82</td>
<td>10.94</td>
<td>-0.80</td>
</tr>
<tr>
<td>300</td>
<td>0.02</td>
<td>-1.11</td>
<td>0.01</td>
<td>0.97</td>
<td>-13.83</td>
<td>12.97</td>
<td>-0.98</td>
</tr>
<tr>
<td>200</td>
<td>0.00</td>
<td>-1.24</td>
<td>-0.04</td>
<td>0.83</td>
<td>-15.52</td>
<td>14.11</td>
<td>-1.86</td>
</tr>
<tr>
<td>150</td>
<td>0.00</td>
<td>-0.41</td>
<td>-0.01</td>
<td>0.48</td>
<td>-13.68</td>
<td>11.85</td>
<td>-1.77</td>
</tr>
<tr>
<td>Mean 1000–150 mb</td>
<td>0.00</td>
<td>-0.36</td>
<td>0.04</td>
<td>0.46</td>
<td>-9.23</td>
<td>8.36</td>
<td>-0.73</td>
</tr>
</tbody>
</table>

The northern hemisphere that the time-mean ageostrophic meridional circulation is of thermally direct type in the entrance region of the mean jet (eastern coasts of North America and Asia) and of Ferrel-type over the British Isles and downstream of the mean jet cores in general. Even if our results agree qualitatively with these findings, the magnitude of our direct estimate of $v - v_g$ for winter (not shown, qualitatively the same as in Fig. 3(a)) is more than twice that reported by Blackmon et al. This is natural, however, because the wind data used by Blackmon et al. are those obtained from isobaric height data via the balance equation. It is not surprising that such wind data tend to underestimate the ageostrophic wind components, a feature also pointed out by Lau (1978).

The westerlies over the British Isles are typically weaker than those over the Atlantic.
and stronger than those over continental Europe. It is therefore natural that we find \( -\nabla \cdot \nabla \mathbf{u} > 0 \). Contributions to this term from the mean flow \( -\nabla \cdot \mathbf{u} \) and transient fluctuations \( -\nabla \cdot \mathbf{u}' \) are both positive (not shown) and thus act so as to accelerate the westerlies. This accelerating advection (roughly equal to the horizontal flux convergence of zonal momentum) takes place over the British Isles mainly in the zonal, not in the meridional, plane.

It can be seen from the middle part of Fig. 3(a) that in the free atmosphere there is a quasi-balance between the advection of zonal momentum and the ageostrophic meridional circulation, a condition which has often been assumed to hold good in zonally averaged conditions, and used to estimate the mean meridional circulation from the better known horizontal advection (actually flux divergence) of zonal momentum (e.g. Lorenz 1967).

In the light of the great variability in \( u \) and \( v \), as discussed in connection with Fig. 2, it is not surprising that the variability of all terms in the momentum budget is wide. For example, when the mean value of \( -\nabla \cdot \mathbf{u} \) at 300 mb is seen, from Fig. 3(a), to be (for 1961–65) \( 1.3 \times 10^{-4} \text{m} \text{s}^{-2} \), the standard deviation of the 3319 instantaneous (i.e. 12-hour mean) values is \( 4.7 \times 10^{-4} \text{m} \text{s}^{-2} \). Even though part of this is due to noise introduced into the calculations by different kinds of data error, the larger part certainly represents real variability.

The close agreement at 300 mb between the results obtained from two independent data samples is worth emphasizing. It indicates that our calculations give a relatively accurate picture of the essential features of the annual mean momentum budget over the British Isles.

Results for the mean forces in the meridional direction (Fig. 3(b)) show, as expected, a relatively good geostrophic balance of the annual mean zonal flow \( f\tilde{u} \approx f\tilde{u}_2 \) except in the lowest kilometre, where the residual force is very important for maintaining balance. In the upper troposphere the metric acceleration term \( -(\tilde{u}u/a)\tan \phi \) is seen to counterbalance the term \( f(\tilde{u}_2 - \tilde{u}) \) to a large extent. Actually, the geostrophic balance of the zonal flow could be defined so as to include the metric term \( \{(f + (u/a)\tan \phi)\tilde{u} = -(1/a)(\partial \Phi/\partial \phi)\} \); in this sense, the mean zonal flow in the free atmosphere over the British Isles is, according to our calculations, well balanced.

In the upper troposphere, the mean advection of meridional momentum \( -\nabla \cdot \mathbf{u} \) is almost of the same magnitude as its zonal counterpart, but its relative importance in the budget of meridional momentum is much smaller than that of \( -\nabla \cdot \mathbf{u} \) in the budget of zonal momentum. Advection tends to increase the meridional velocity \( -\nabla \cdot \mathbf{u} > 0 \); this effect arises due to the frequent occurrence of upper-level highs and ridges over the British Isles.

The residual forces will be discussed in the following section.

5. **The Residual Force**

In this section we discuss in some detail the calculated values of the residual force, \( \mathbf{R} \), assuming that it consists of the real sub-grid scale forcing effect, \( \mathbf{F} \), and the noise vector, \( \mathbf{E} \):

\[
\mathbf{R} = \mathbf{F} + \mathbf{E}
\]  

(4)

A discussion of different kinds of systematic and random error, which contribute to \( \mathbf{E} \), was presented in \( \mathbf{A} \) and will not be repeated. We assume in the following that \( \mathbf{E} \) is a random vector, which implies that

\[
\mathbf{R} = \mathbf{F}
\]  

(5)

and

\[
\sigma_R^2 = \sigma_F^2 + \sigma_E^2.
\]  

(6)
We may formally consider the sub-grid scale force \( \mathbf{F} \) as consisting of two parts:

\[
\mathbf{F} = \mathbf{F}^H + \mathbf{F}^V,
\]

(7)

where \( \mathbf{F}^H \) and \( \mathbf{F}^V \) are due to sub-grid scale processes occurring in the horizontal and vertical plane, respectively. It is commonly assumed that in the lower troposphere, and particularly in the planetary boundary layer, \( \mathbf{F}^V \) is larger than \( \mathbf{F}^H \) and thus \( \mathbf{R} \approx \mathbf{F}^V \). In the upper troposphere, however, where horizontal velocities and the associated fluxes of momentum have the largest magnitude, \( \mathbf{F}^H \) may have an even larger magnitude than \( \mathbf{F}^V \). Accordingly, in the following section we discuss the results for \( \mathbf{R} \) in the lower troposphere (below 700 mb) and in the upper troposphere (300 mb) separately.

(a) Lower troposphere

Figure 4 shows area-averaged vertical profiles of the mean wind \( \mathbf{V} \), geostrophic wind \( \mathbf{V}_g \) and the calculated residual force, \( \mathbf{R} \), for the whole hexagonal area of the present study.

![Figure 4](image)

Figure 4. Hodograph of the actual wind, \( \mathbf{V} \), geostrophic wind, \( \mathbf{V}_g \), and the residual force, \( \mathbf{R} \), below 700 mb for annual mean conditions over the British Isles. \( \mathbf{V} \) is the mean surface wind (at anemometer level), \( \mathbf{V}_g \) the mean geostrophic wind at 1000 mb. All values are area averages over the polygon seen in Fig. 1. Units for \( \mathbf{V} \) and \( \mathbf{V}_g \): \( \text{ms}^{-1} \); for \( \mathbf{R} \): \( 10^{-4}\text{ms}^{-2} \).

The mean wind, \( \mathbf{V} \), below 850 mb forms a clear friction layer spiral. At the surface, the mean wind makes an angle of 25° with the geostrophic wind, with \( |\mathbf{V}|/|\mathbf{V}_g| = 0.35 \). Above 850 mb the mean wind deviates to the right from the mean geostrophic wind, in agreement with the ageostrophic wind components shown in Fig. 3.

The baroclinicity of the mean atmospheric conditions over the British Isles can be seen from the change of \( \mathbf{V}_g \) with height.

The hodograph of \( \mathbf{R} \) is such that at the surface (where, approximately, \( \mathbf{R} = f \mathbf{k} \times (\mathbf{V} - \mathbf{V}_g) \)) \( \mathbf{R} \) is 150° to the right of \( \mathbf{V} \) and is reduced to very small values already at 850 mb.

Assuming that the frictional force is solely due to stresses acting across a horizontal surface, and taking into account Eq. (5), we can write

\[
\mathbf{R}_u = \mathbf{F}_u^\gamma = -g \frac{\partial \gamma}{\partial p},
\]

(8)

\[
\mathbf{R}_v = \mathbf{F}_v^\gamma = -g \frac{\partial \tau_v}{\partial p},
\]

(9)

where \( \gamma \) and \( \tau_v \) are the zonal and meridional components of the ‘eddy stress’ (in our case, the vertical flux of momentum due to scales smaller than the average distance between the stations). Having determined \( \mathbf{R}_u \) and \( \mathbf{R}_v \), we can calculate from Eqs. (8) and (9) the vertical
distribution of $\tau_x$ and $\tau_\phi$, provided their values are known at some level. We have estimated the mean stress at the earth's surface, $\tau_y$, using the geostrophic drag law,

$$|\tau_y| = \rho C_{D_0} V^2,$$

and requiring that the direction of $\tau_y$ is the same as that of the surface wind.

$C_{D_0}$ in Eq. (10) is the geostrophic drag coefficient which depends upon the roughness of the underlying surface and the static stability of the air mass. In our calculations, however, we used a constant value, $1.5 \times 10^{-3}$. This value is obtained by taking $C_{D_0}$ to be $2 \times 10^{-3}$ over land (this is a rough area average from Smith and Carson (1977) who present, for neutral stability conditions, a map of $C_{D_0}$ over the British Isles) and $0.5 \times 10^{-3}$ over sea, and taking into account that about 1/3 of our area (Fig. 1) is sea. We then calculate from Eq. (10) (instead of $V$, we actually used the wind at 850 mb) for the annual mean surface stress components in our area: $\tau_x = 0.099 \text{ N m}^{-2}$ ($0.99 \text{ dyn cm}^{-2}$) and $\tau_\phi = 0.086 \text{ N m}^{-2}$ ($0.86 \text{ dyn cm}^{-2}$).

With these values as the lower boundary conditions and with the values of $\bar{R}_x$ and $\bar{R}_y$ given in Fig. 3, we derive from Eqs. (8) and (9) for the vertical distribution of $\tilde{\tau}_x$ and $\tilde{\tau}_\phi$ below 700 mb the curves shown in Fig. 5. The results indicate that the mean zonal stress decreases with height from its surface value and is relatively small but still positive (downward transfer of zonal momentum) above 850 mb. The meridional stress component, on the other hand, has relatively large negative values (upward transfer of meridional momentum) between 950 and 700 mb, a feature which has already been demonstrated using more restricted data for the same area (Holopainen 1964).

When looking for a possible physical explanation of the calculated stresses between 900 and 700 mb one has to remember that the horizontal grid size in our calculation (with the triangle grid of Fig. 1) is a few hundred kilometres. Therefore, the eddy stresses contain, in addition to the effect of small scale turbulence, the influence of mesoscale systems such as fronts. One of the basic statistical features of the fronts occurring over the British Isles is their quasi-meridional orientation. Qualitatively, one may then expect a positive correlation (in mesoscale) between meridional and vertical wind components, and thus a negative eddy stress, $\tau_\phi$, in the lower layers at some distance from the surface; the corresponding values of $\tilde{\tau}_x$ would be expected to be somewhat smaller. Let us take $3 \text{ m s}^{-1}$ and $0.1 \text{ m s}^{-1}$ as typical values of the frontal (sub-grid scale) fluctuations of meridional and vertical velocities, respectively. Assuming a relative frequency of 1/4 for the occurrence of fronts in the grid area we then get $0.1 \text{ N m}^{-2}$ for the order of magnitude of the frontal contribution to $\tilde{\tau}_\phi$, which agrees with the $\tilde{\tau}_\phi$ given in Fig. 5.
With meridionally oriented fronts the frontal contribution to the vertical transfer of zonal momentum would be expected to be somewhat smaller than that of meridional momentum. However, in the diffuent flow pattern which quite often occurs over the British Isles, westerlies are likely to be somewhat stronger and vertical motion smaller on the western side of the front than on the eastern side. Thus, downward transfer of zonal momentum (or positive $\tau_x$, as in Fig. 5) due to frontal circulations is not unreasonable.

Frontal contributions to the vertical transfer of momentum was hypothesized by Priestley (1967). However, to the author's knowledge, no direct or indirect estimate of their climatological effect on the momentum budget in middle latitudes has so far been made.

![Diagram](image)

**Figure 6.** Relationship between the surface wind, $\mathbf{V}_s$, surface stress, $\tau_s$, stress at 850 mb, $\tau_{850}$, and the average 'frictional' force between the surface and 850 mb (proportional to $\tau_{850} - \tau_s$) for annual mean conditions over the British Isles.

The relationship between the mean surface wind, $\mathbf{V}_s$, the mean surface stress, $\tau_s$, and the mean stress at 850 mb, $\tau_{850}$, is shown in Fig. 6. The mean surface stress is in practically the same direction as the mean wind at the surface. This is not a result of general validity but is simply a coincidence: there is no need for such a general relationship between mean stress and mean wind, though they are so related at any particular time.

Due to the non-zero value of $\tau_{850}$ the average 'frictional' force, $\mathbf{R}$, between the surface and 850 mb, proportional to $\tau_{850} - \tau_s$, has a relatively large component to the right of $\tau_s$ (and $\mathbf{V}_s$).

Too much importance should not be put on the quantitative value obtained for $\tau_{850}$. We have to remember that routine aerological data are not very suitable for studies of the atmospheric boundary layer because of rather poor vertical resolution. Furthermore the surface wind, the lowest wind in the upper air report, is measured differently from the rest of the wind profile, and is rather sensitive to details of local terrain and the height of the anemometer. For this reason, the result presented above for $\tau_{850}$ should be taken as only a rough first-order picture, to be verified by additional studies.

**(b) Upper troposphere**

In this section we consider in some detail the results for $\mathbf{R}$ at 300 mb. Attention is given not only to mean values of $\mathbf{R}$ but also to transient fluctuations. Besides the period 1974–76, some results for 1961–65 are presented, the main ones having already been discussed in A.
Figure 7. The area-averaged annual mean horizontal velocity, \( \mathbf{V} \), and the residual force, \( \mathbf{R} \), at 300 mb for 1961-65 (suffix 'A') and 1974-76 (suffix 'B'). Units: m s\(^{-1}\) for \( \mathbf{V} \), 10\(^{-5}\)m s\(^{-2}\) for \( \mathbf{R} \).

Fig. 7 shows the annual mean averaged vectors \( \mathbf{V} \) and \( \mathbf{R} \) from this study for 1974-76 and from A for 1961-65. It can be seen that results for the two periods are practically the same, except that for 1974-76 both vectors deviate to the right of the corresponding vectors for 1961-65 and have a slightly smaller magnitude (some comments concerning the differences in \( \mathbf{V} \) were made in section 2). The similarity also applies to the results (not shown) for all five individual triangles in Fig. 1.

In both periods, \( \mathbf{R} \) has a large component to the right of \( \mathbf{V} \), but also a component in the direction of \( \mathbf{V} \). Thus, \( \mathbf{V} \cdot \mathbf{R} > 0 \) and the time-mean residual force tends to increase the kinetic energy of the time-mean flow. The time-mean value of \( \mathbf{V} \cdot \mathbf{R} \) (the rate of change of kinetic energy of the large scale motion due to the residual force) can formally be written in two parts:

\[
\mathbf{V} \cdot \mathbf{R} = \mathbf{V} \cdot \mathbf{R} + \mathbf{V} \cdot \mathbf{R}', \tag{11}
\]

where the second term represents the work done by day-to-day fluctuations in \( \mathbf{R} \). The l.h.s. of Eq. (11) has been found to be positive (formally indicating an increase in the kinetic energy of the large scale motion by sub-grid processes) both for 1961-65 (see A) and for 1974-76 (see B).

By using data for 1961-65 we have calculated (using the second-degree method, see A) numerical values of each of the terms in Eq. (11) for 300 mb with the following results, in W m\(^{-2}\)(100 mb\(^{-1}\)):

\[
\mathbf{V} \cdot \mathbf{R} = 1.36; \quad \mathbf{V} \cdot \mathbf{R} = 0.95; \quad \mathbf{V} \cdot \mathbf{R}' = 0.41.
\]

Thus, both the mean residual force and its transient fluctuations increase the kinetic energy of the large scale flow (up-scale transfer of kinetic energy).

In order to study in more detail the day-to-day fluctuations in \( \mathbf{R} \) (which undoubtedly consist mainly of random noise due, for example, to random observational errors) we investigate the possibility of parametrizing it in terms of the expressions:

\[
R_u = A_u \mathbf{V}^2 u \quad \text{and} \quad R_v = A_v \mathbf{V}^2 v, \quad \tag{12}
\]
where \( A_u \) and \( A_v \) (lateral eddy viscosities) are constants. It is thus assumed that in Eq. (7) \( \mathbf{F}^H \) makes the largest contribution to \( \mathbf{F} \). The slope of the regression lines of the 3319 instantaneous (i.e. 12-hour mean) values was used to deduce for \( A_u \) and \( A_v \) the values:

\[
A_u = -2.4 \times 10^5 \text{ m}^2 \text{s}^{-1}; \quad A_v = -1.9 \times 10^5 \text{ m}^2 \text{s}^{-1}.
\]

We thus get negative viscosity coefficients, which formally implies up-scale (from unresolved to resolved) transfer of kinetic energy in transient disturbances. This is, of course, consistent with the earlier considerations of the sub-grid scale effects on the kinetic energy balance.

It should be emphasized that the (negative) correlation coefficients between 3319 individual values of the two sides of Eqs. (12) are very small (\(-0.07\) for both components). This smallness is, however, not surprising because the dominating part of the variance of \( R_u \) and \( R_v \) is random noise, i.e. \( \sigma^2_E \gg \sigma^2_f \) in Eq. (6).

Our calculations thus show that on average over the whole area in Fig. 1 there seems to be a definite tendency in the upper troposphere for the large scale flow to be accelerated by the unresolved scales. No simple explanation can be offered for this, which is contrary to what one would expect the effect of small scale turbulence, CAT and mountain waves, to be: they should, on average, cause (via \( \mathbf{F}^* \) in Eq. (7)) a deceleration of the large scale flow in the upper troposphere.

Holopainen and Nurmi (1979) have shown, by employing a horizontal filtering technique as the means of scale separation and using an explicit expression for the lateral sub-grid scale force, that in the case of a diffuent jet stream (which is quite common over the British Isles), horizontal sub-grid scale processes tend to accelerate the flow on resolved scales. They have also shown that in this particular flow pattern the classical parametrizations, like that represented by Eqs. (12), actually require negative viscosity coefficients.

Thus, horizontal sub-grid scale processes are possibly a partial explanation of the results obtained in this study for the upper troposphere. However, one of the results, for which we have not been able so far to find any plausible physical explanation, is the large southward component of \( \mathbf{R} \) in the upper troposphere. Just as in \( A \), we cannot completely exclude the possibility that (unknown) systematic errors influence our results.

6. Concluding Remarks

The most interesting result in this paper is the evidence obtained that in the lower troposphere, at a height of 1–3 km, there are over the British Isles (and probably in middle latitudes in general) important vertical sub-grid scale vertical momentum fluxes and that in the upper troposphere over this area the climatological effect of sub-grid scale processes seems to be a positive acceleration of the synoptic scale flow. These conclusions are of some significance for the numerical modelling of the large scale behaviour of the atmosphere. If frontal circulations are as important in the vertical momentum transfer as the indirect evidence presented in section 5(a) indicates, the treatment of the boundary layer in coarse resolution models may require reconsideration. Similarly, results for the upper troposphere speak for a reconsideration of internal friction in the free atmosphere.

A critical reader may wonder whether the whole discussion concerning the calculated residual forces is just putting too much meaning on numbers obtained from low quality data (routine aerological data with their many deficiencies) with a rather dubious technique (residual method). Our answer to this is that the data used are as good as the best available for long-term budget studies, and unless some source of systematic error is discovered in routine upper air data in general, or some systematic erroneous difference between British aerological stations (which all use the same type of measuring system) in particular, data
deficiencies can hardly explain the results obtained. Concerning criticism of the residual method, a longer and more general comment may be appropriate.

In meteorology we are almost always confronted with the problem that the 'resolved' scales which we are interested in are influenced by 'unresolved' (sub-grid) scales. The equations for the resolved scales are, as recently pointed out by Robinson (1978), essentially empirical due to the fact that there is no unique, deterministic way of taking into account the dynamic and thermal forcing effect of the sub-grid scale flow. This effect depends upon the grid system (in space and time) and the finite difference scheme used, and naturally also on the level of sophistication with which different physical processes are included in the model. (Thus, the dynamics of large scale flow depends, so to speak, on the glasses through which we look at the flow.) Whatever the model for the resolved scales, there will always be a difference between changes observed in the real atmosphere and those predicted by the model. This difference is the residual forcing needed in the model. Diagnostic studies of this forcing can be used to parametrize some of its statistical properties. There will, however, always remain a stochastic forcing component. The prediction of the flow on resolved scales will thus be basically nondeterministic and the efficacy of the particular model used will depend upon the size of the stochastic forcing it needs. With this in mind one may say that the residual method is very useful and has perhaps been used too little in the past.

Our approach in this study is comparable with diagnosing a kind of primitive equation model which uses a rather special finite difference technique, with data which are not initialized in any way. If we used another 'model', or data which were filtered in some way, results for the residual force would be different from those reported here. However, the similarity of the results obtained in A with the linear and second-degree methods indicates (but by no means guarantees) that the basic statistical effects of the residual force would be the same as obtained in this paper.

ACKNOWLEDGMENTS

A large part of the programming for this study was done by Mr Kalle Eerola, drafting by Mr Pertti Nurmi and typing by Mrs Pirkko Pertula.

REFERENCES


A 1973 An attempt to determine the effects of turbulent friction in the upper troposphere from the balance requirements of the large-scale flow: a frustrating experiment, Geophysica, 12, 151–176.


